Introduction to Quantum Computing

Through the lenses of Physicists & Computer Scientist

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Introduction to Quantum Computing

Through the lenses of Physicists & Computer Scientist

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○ Quantum states & gates
○ Quantum algorithms
○ Quantum communication concepts</td></tr>
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○ Simulations & Programming
○ Information theory</td><td>○ Bell inequalities, EPR paradox
○ No-cloning, teleportation
○ Cryptography</td></tr>
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Introduction to Quantum Computing

Through the lenses of Physicists & Computer Scientist

1. Quantum Computing Background
2. Complex Numbers

Quantum Computing Background

Quantum Computing

Classical Computing

Deterministic

"OR"

Binary Language

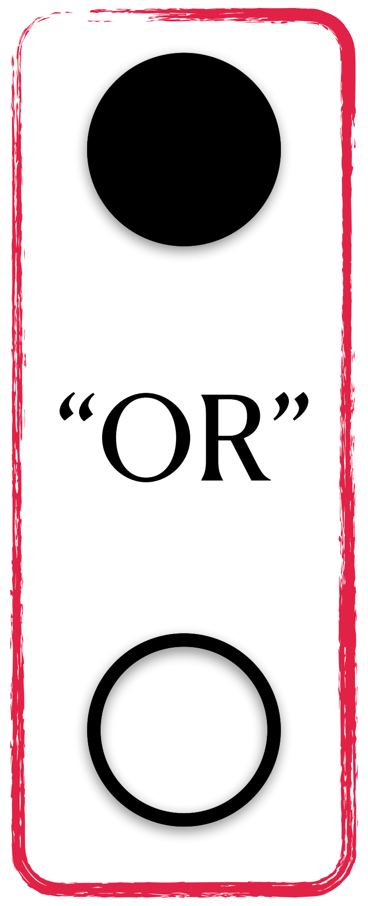
10001 + 110100 = 1000101

You measure what you have

Quantum Computing

Classical Computing

Deterministic



Binary Language

10001 + 110100 = 1000101

The system exist in *one* state

Quantum Computing

Classical Computing

Deterministic

“OR”

Binary Language

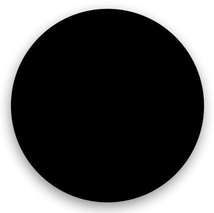
10001 + 110100 = 1000101

Boolean logic

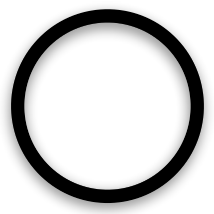
Quantum Computing

Classical Computing

Deterministic



"OR"

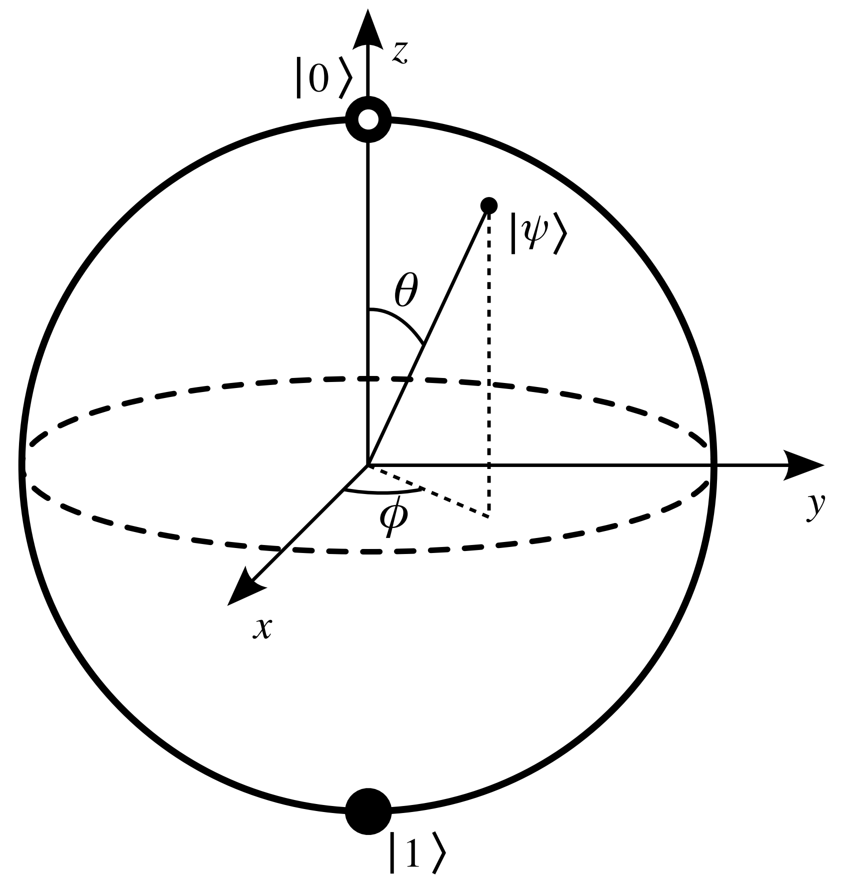


Binary Language

10001 + 110100 = 1000101

Quantum Computing

Probabilistic



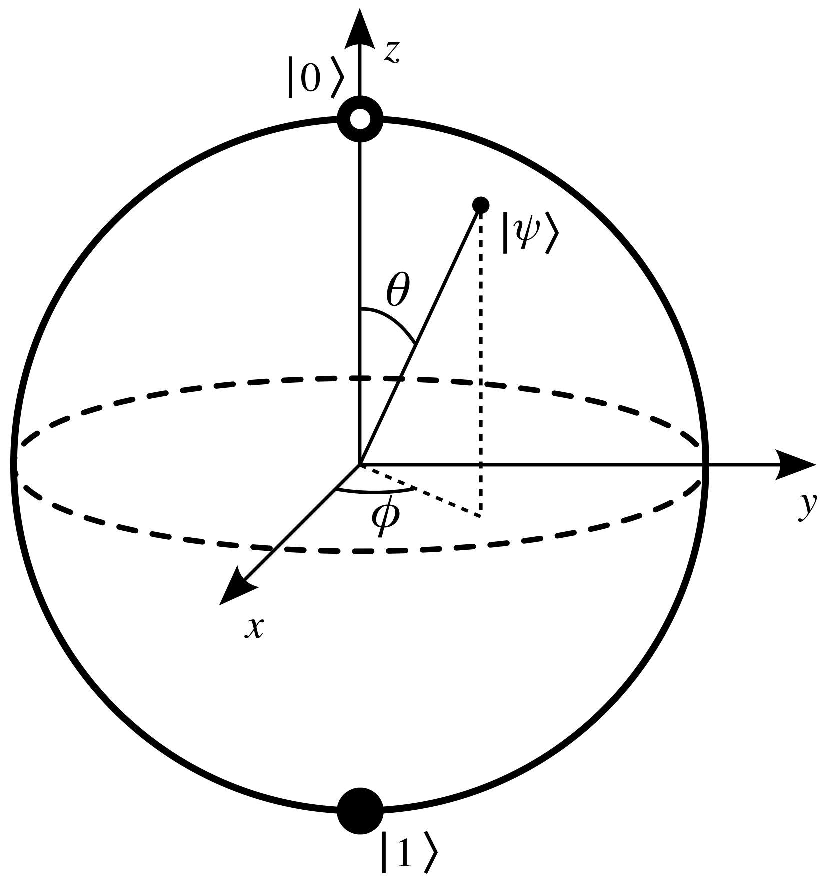
Linear Algebra

$$\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

Quantum Computing

Quantum Computing

Probabilistic

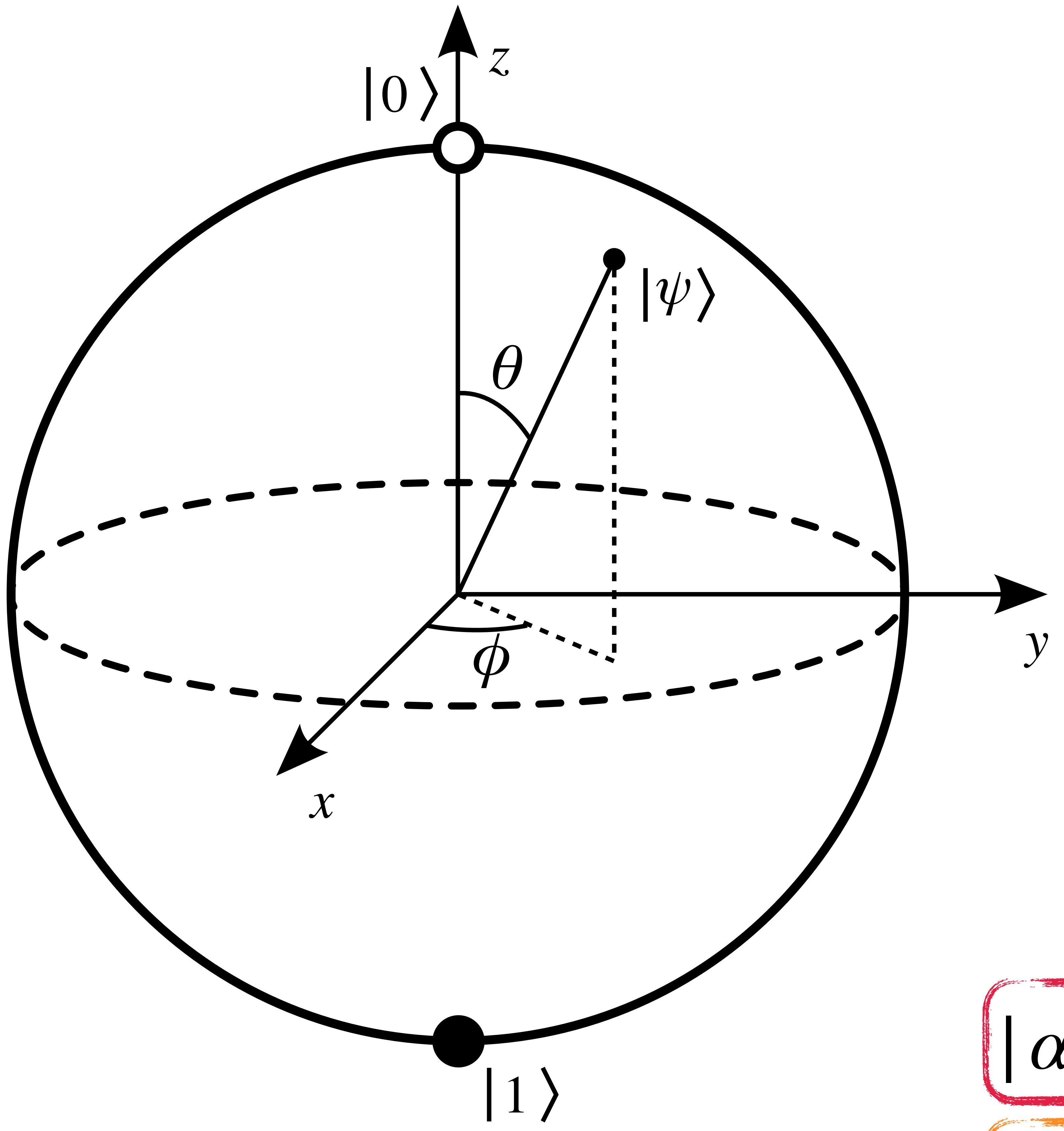


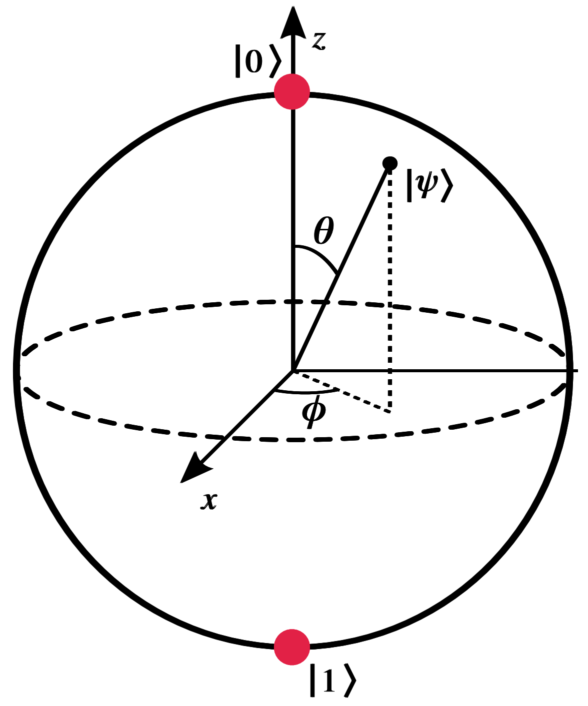
Linear Algebra

$$\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

Measurement has an effect

Quantum Computing





- Superposition

$$|\Psi\rangle = \boxed{\alpha|0\rangle} + \boxed{\beta|1\rangle}$$

α, β are complex probability *amplitudes*

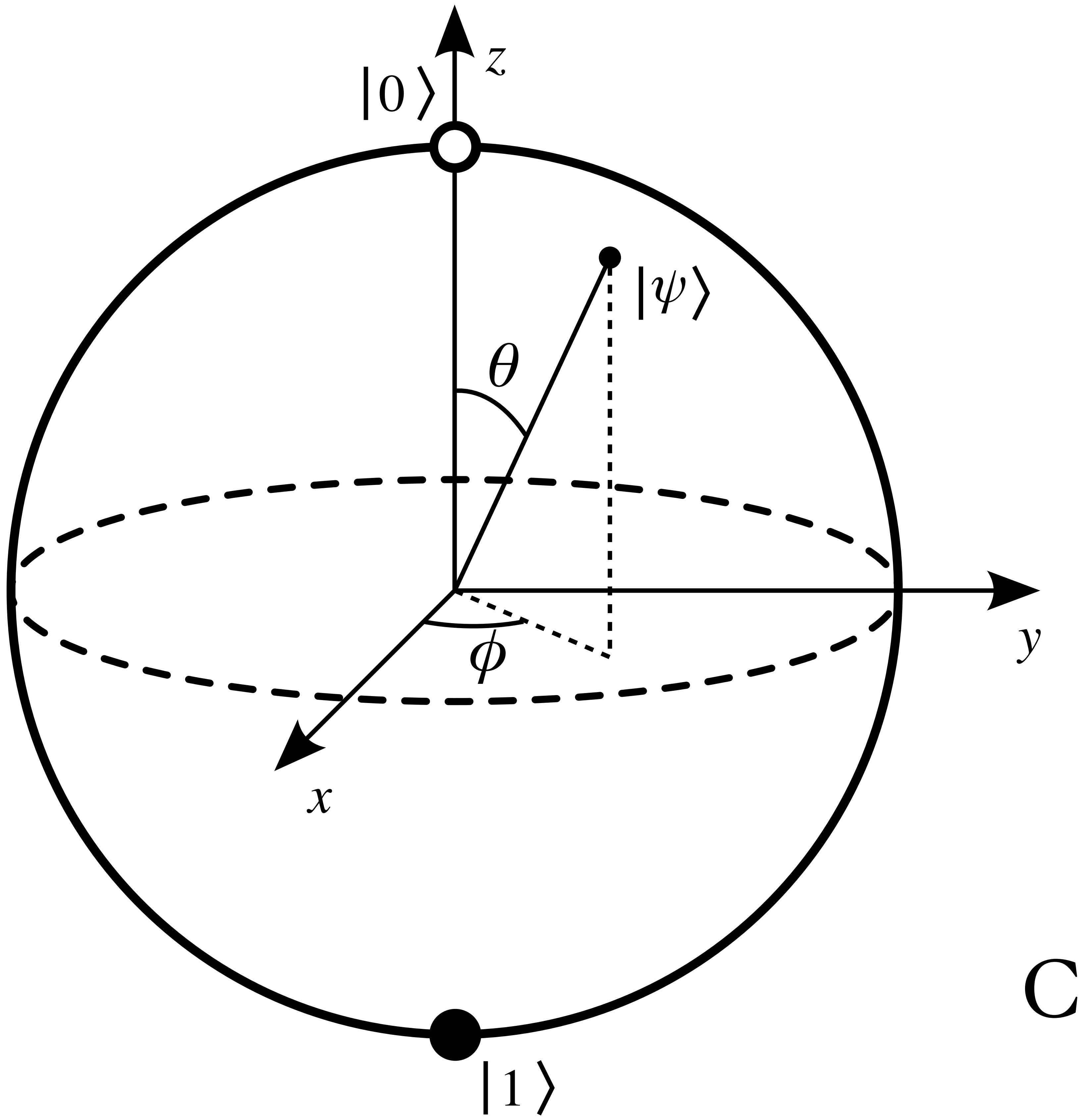
$\boxed{|\alpha|^2}$ Chance to measure $|0\rangle$

$\boxed{|\beta|^2}$ Chance to measure $|1\rangle$

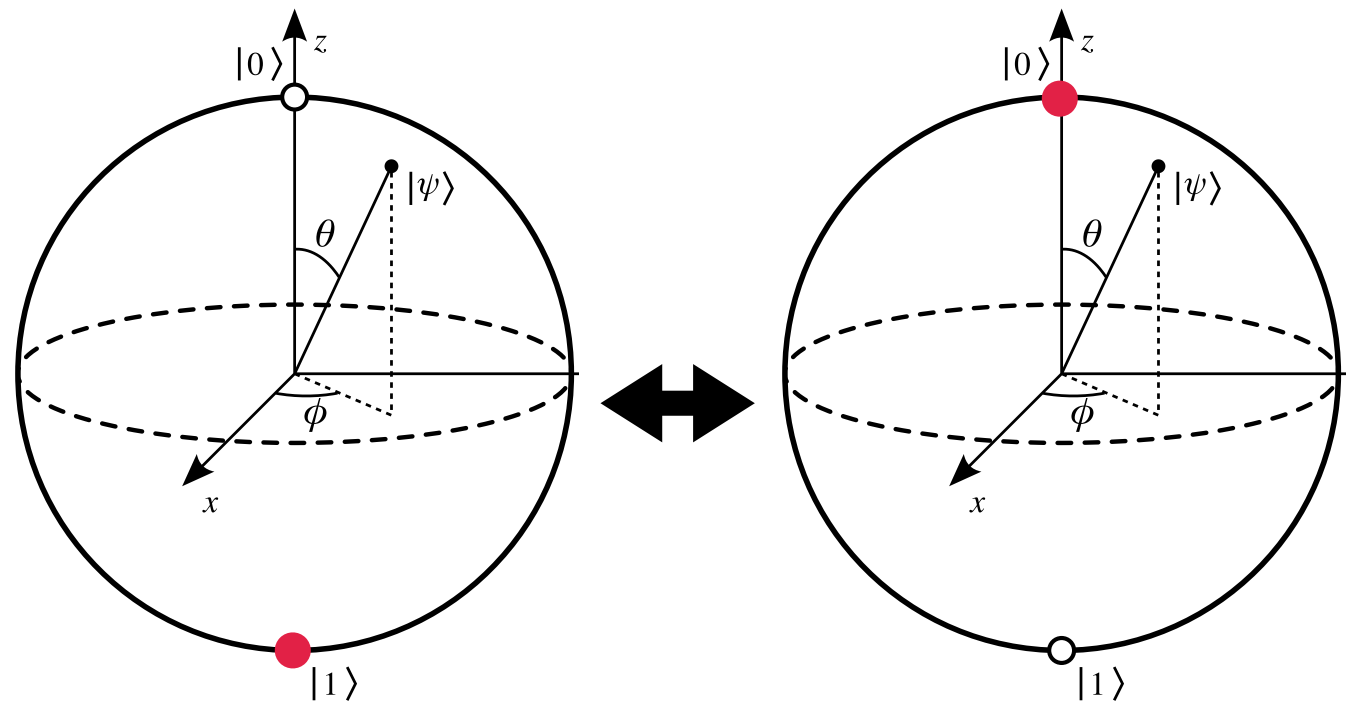
$$\langle\Psi|\Psi\rangle = \alpha\alpha^*\langle 0|0\rangle + \beta\beta^*\langle 1|1\rangle$$

$$1 = |\alpha|^2 + |\beta|^2$$

Quantum Computing

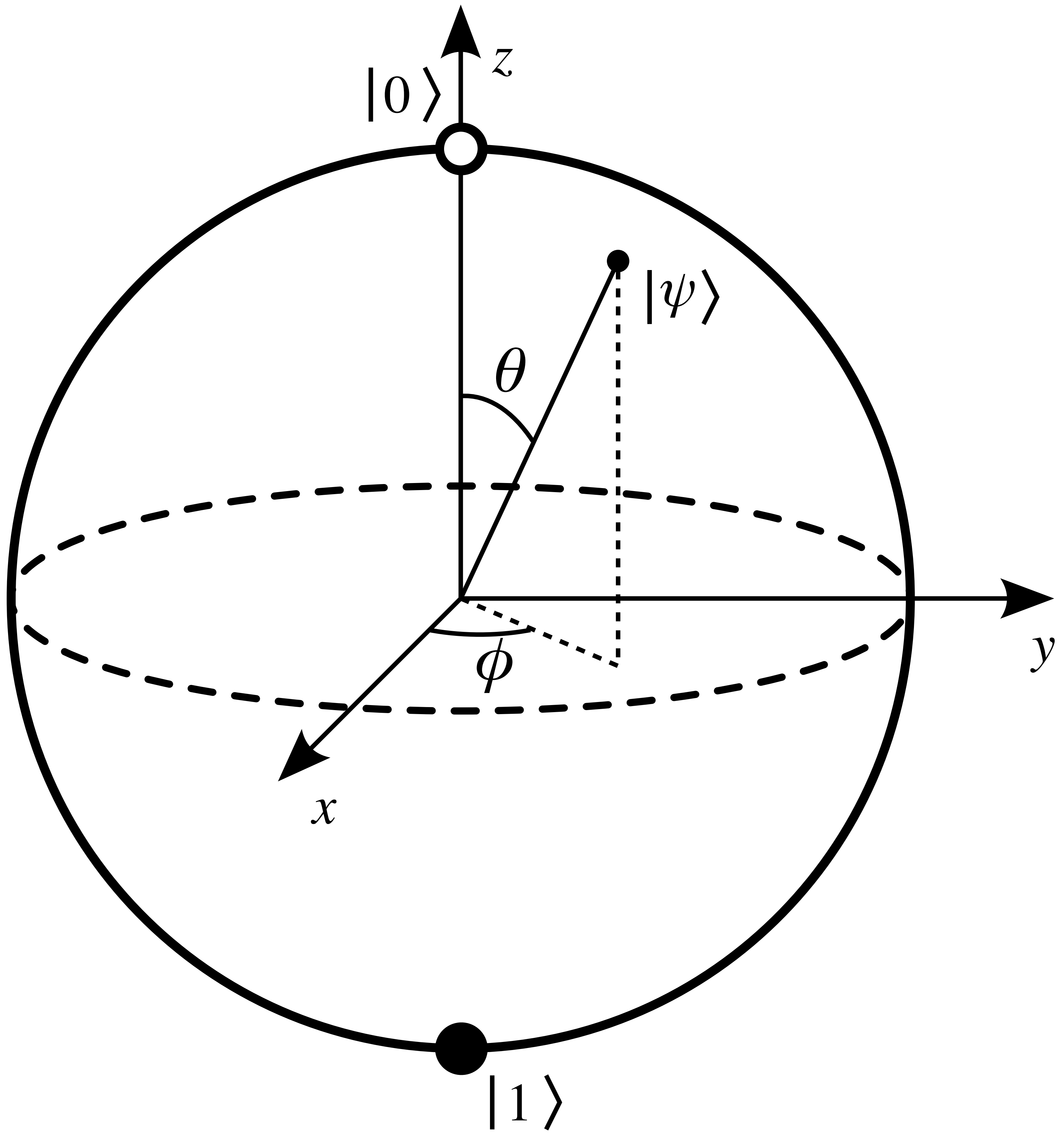


- Interference



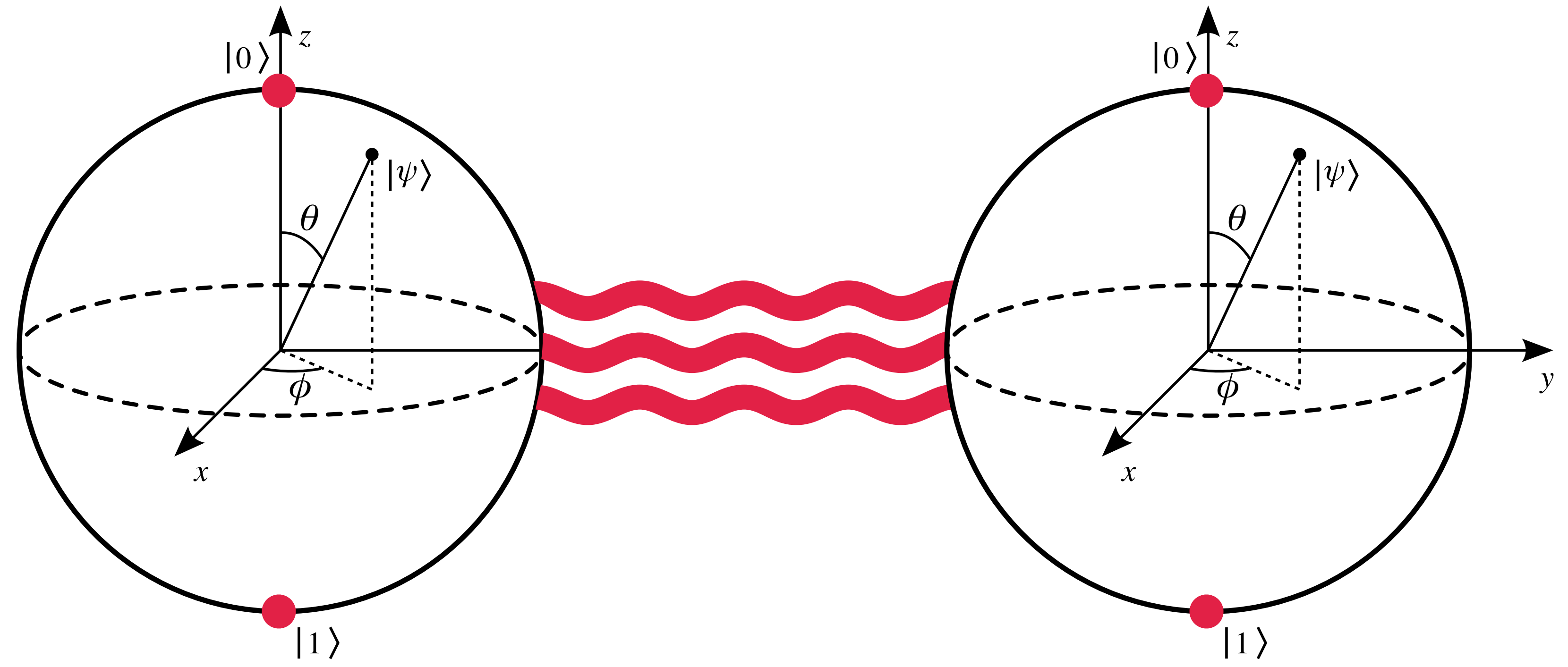
Constructive/Destructive interference of states

Quantum Computing



- Entanglement

Inability to describe the parts independently

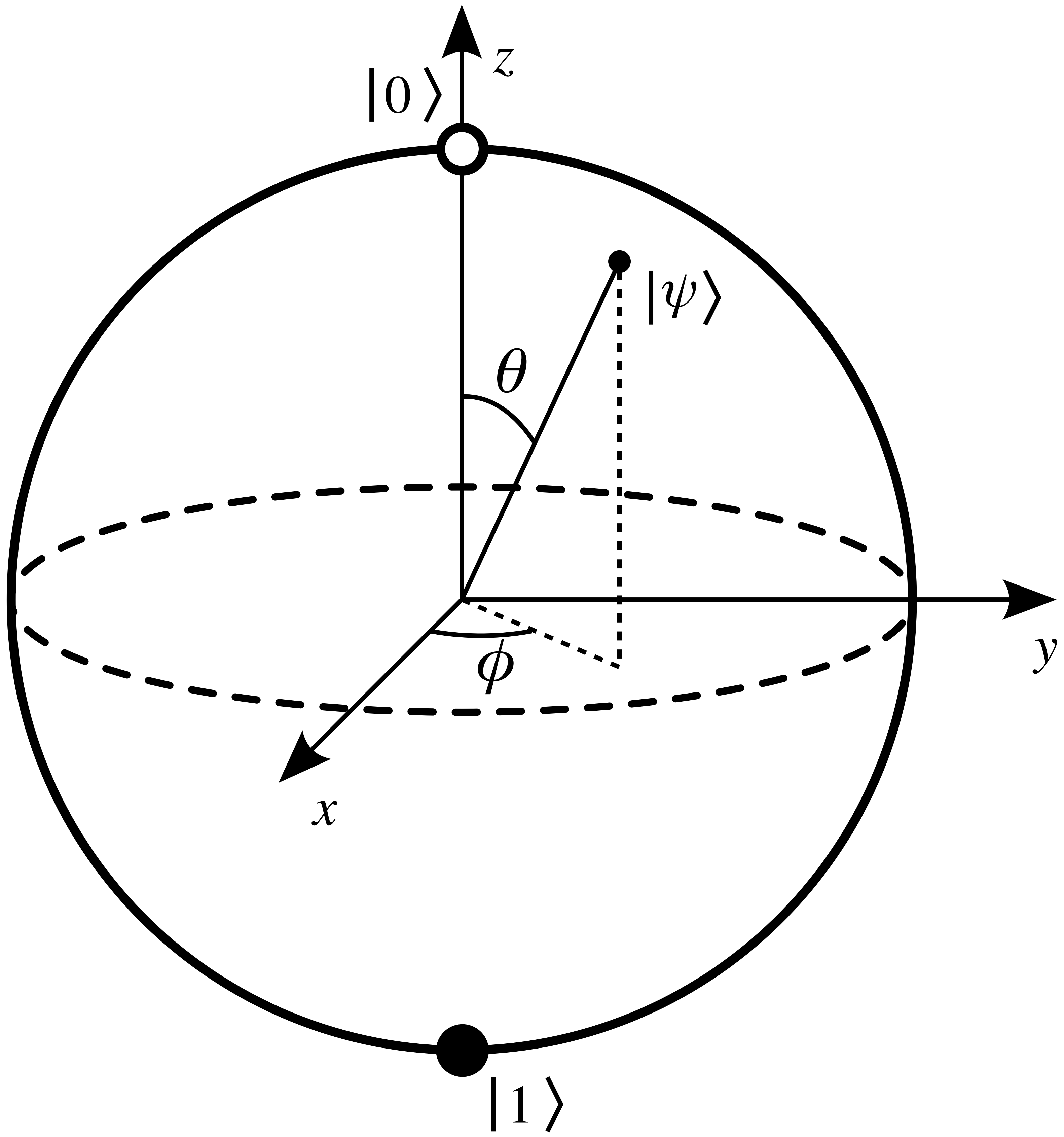


$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

You can't write this as:

$$|\Phi^+\rangle \neq |\Psi_0\rangle \otimes |\Psi_1\rangle$$

Quantum Computing



○ Measurement

Quantum Computing

The reason you need clever tricks
for exponential speedups

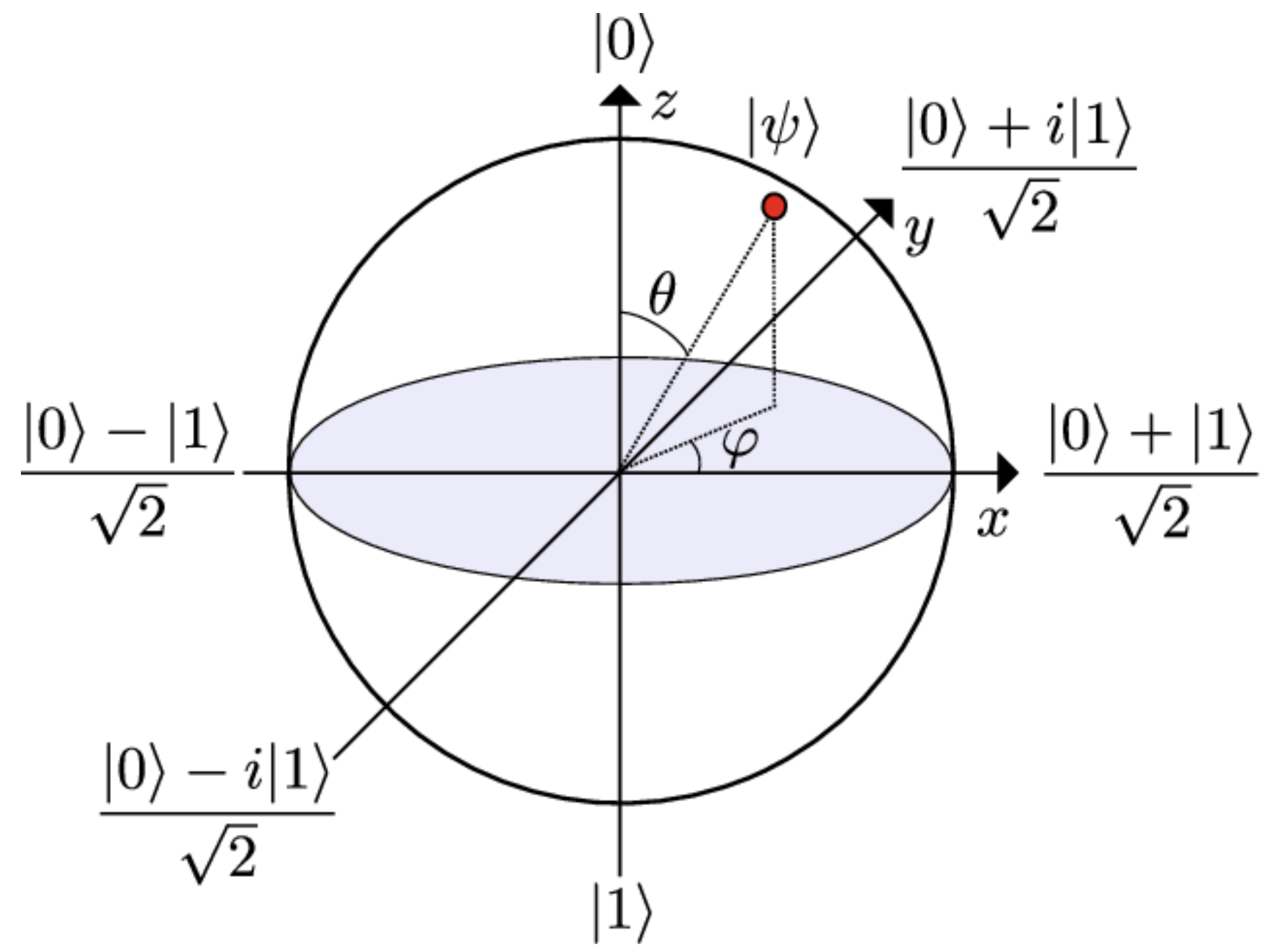
- Measurement

The collapse into classical bits

Quantum Computing

Quantum Computing

Probabilistic



Linear Algebra

$$\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

States live in complex Hilbert space

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix}; \quad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix} \qquad X = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \qquad Z = \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$$

$$X|0\rangle = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix} \times \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix} = |1\rangle$$

Complex Numbers

Complex Numbers

Why Complex Numbers in Quantum Computing?

The Foundation of Quantum Mechanics

- **Quantum amplitudes:** State $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where $\alpha, \beta \in \mathbb{C}$
- **Probability:** $P(0) = |\alpha|^2$, $P(1) = |\beta|^2$ with $|\alpha|^2 + |\beta|^2 = 1$
- **Phase matters:** $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ vs $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$
- **Unitary operations:** Quantum gates preserve normalization via complex rotations

Basic Operations

Core Complex Arithmetic

Let $z = a + bi$ where $i^2 = -1$

Operation	**Formula**
Addition	$(a+bi) + (c+di) = (a+c) + (b+d)i$
Multiplication	$(a+bi)(c+di) = (ac-bd) + (ad+bc)i$
Conjugate	$\overline{a+bi} = a - bi$
Modulus	$\lvert a+bi\rvert = \sqrt{a^2+b^2}$
Division	$\frac{z_1}{z_2} = \frac{z_1 \cdot \overline{z_2}}{\lvert z_2\rvert^2}$

QC Example: For $\alpha = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$, we have $|\alpha|^2 = \frac{1}{2} + \frac{1}{2} = 1$ (normalized)

Complex Numbers

Euler's Formula and Polar Form

Euler's Formula

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Key Values:

$$\begin{array}{l|l} e^{i0} = 1 & e^{i\pi/2} = i \\ e^{i\pi} = -1 & e^{i3\pi/2} = -i \\ e^{i2\pi} = 1 & e^{i\pi/4} = \frac{1+i}{\sqrt{2}} \end{array}$$

Polar Form

$$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

Conversions:

- $r = |z| = \sqrt{a^2 + b^2}$
- $\theta = \arg(z) = \operatorname{atan2}(b, a)$
- $a = r\cos\theta,\ b = r\sin\theta$

Multiplication in polar form: $z_1 \cdot z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$ (multiply magnitudes, add phases)

Quantum Gates as Complex Operations

Common Gate Phases

Gate	Action	Phase
X	Bit flip	Real (± 1)
Y	Bit + phase flip	$\pm i$
Z	Phase flip	± 1
S	$\sqrt{Z}$	$e^{i\pi/2} = i$
T	$\sqrt{S}$	$e^{i\pi/4}$
H	Superposition	$\frac{1}{\sqrt{2}}$

Bloch Sphere

General qubit state:

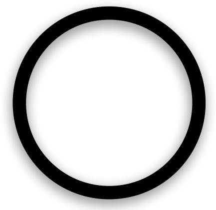
$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

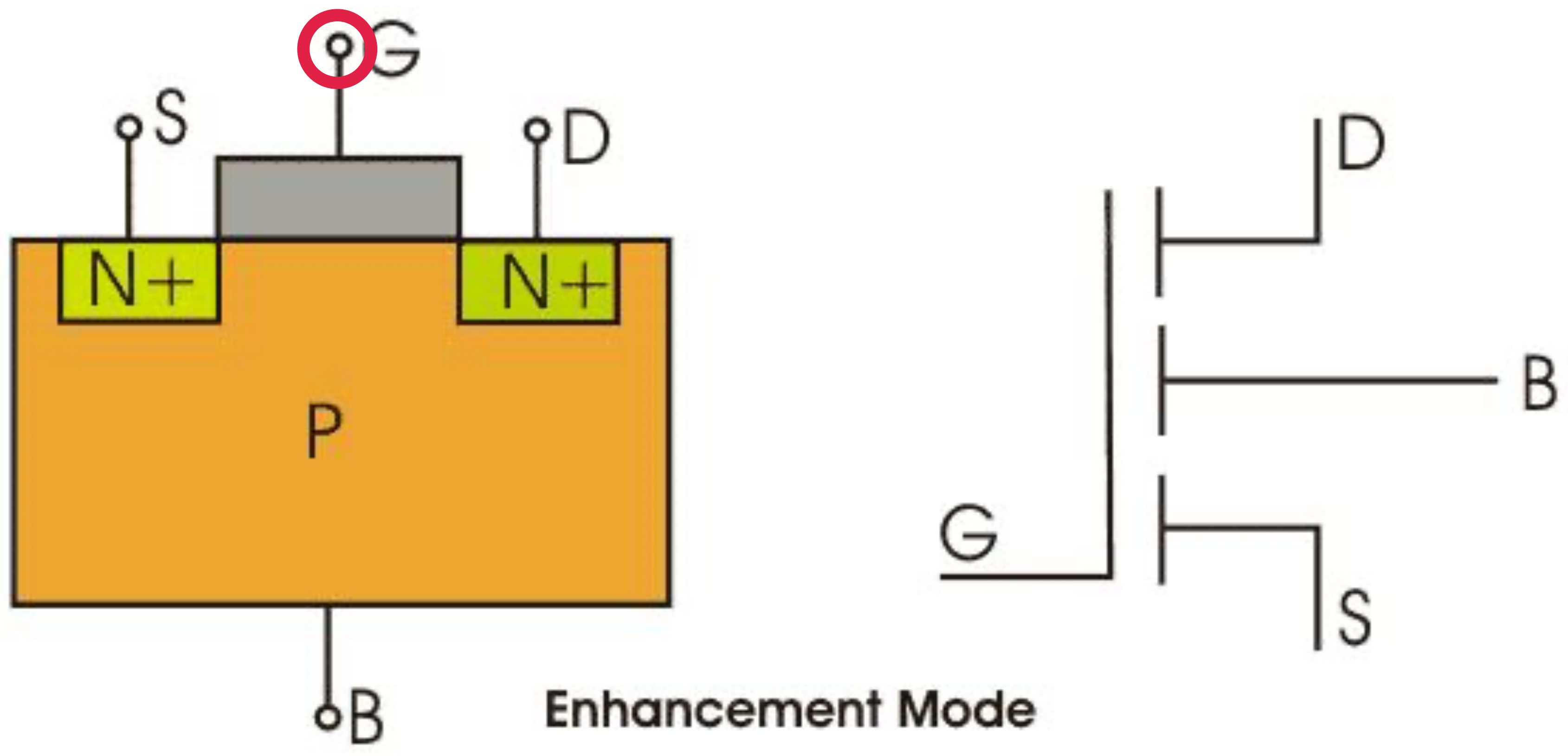
- θ: polar angle (0 to π)
- ϕ: azimuthal angle (0 to 2π)
- Poles: $|0\rangle$ (north), $|1\rangle$ (south)
- Equator: equal superpositions

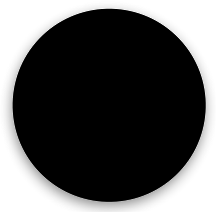
Physical Realization

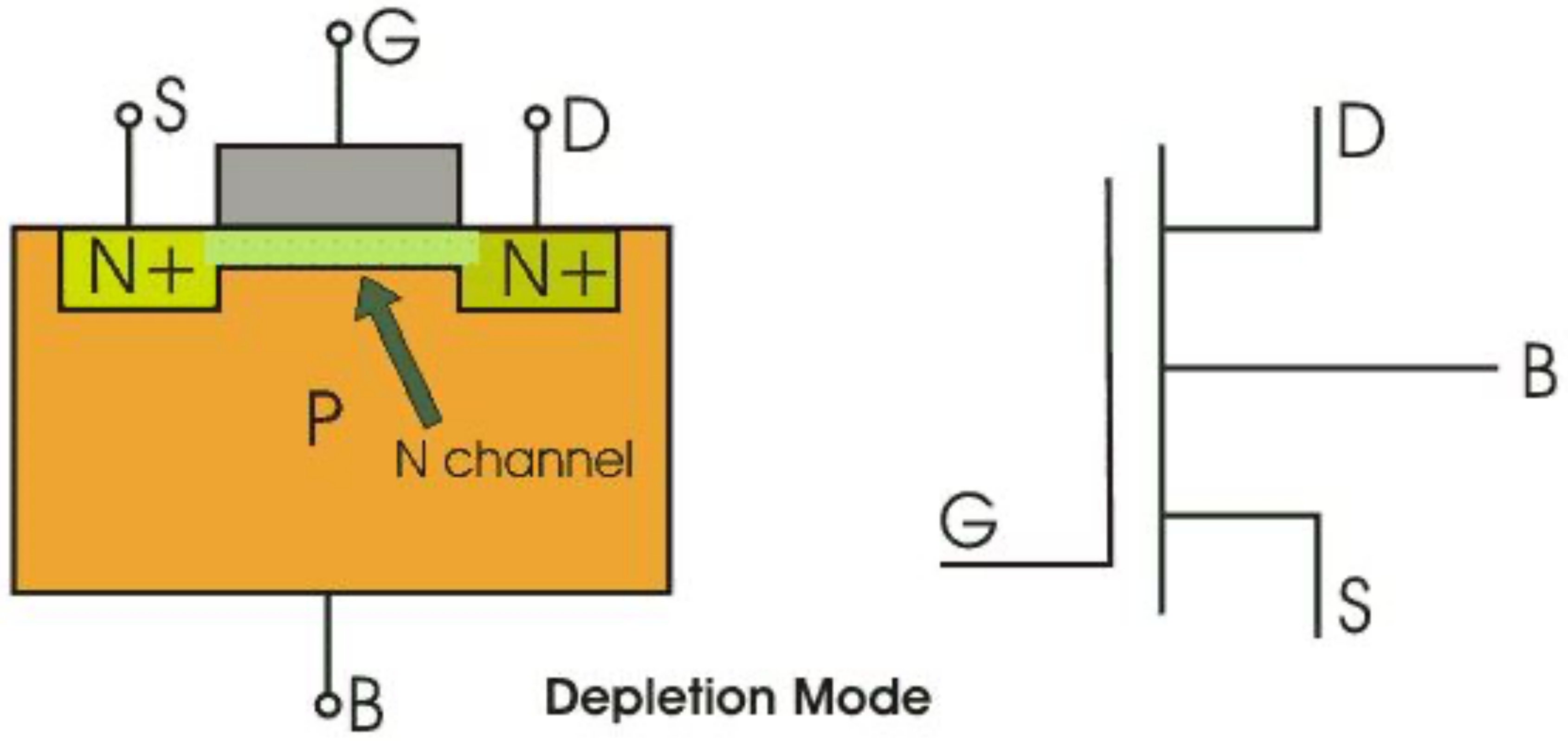
Physical Realization

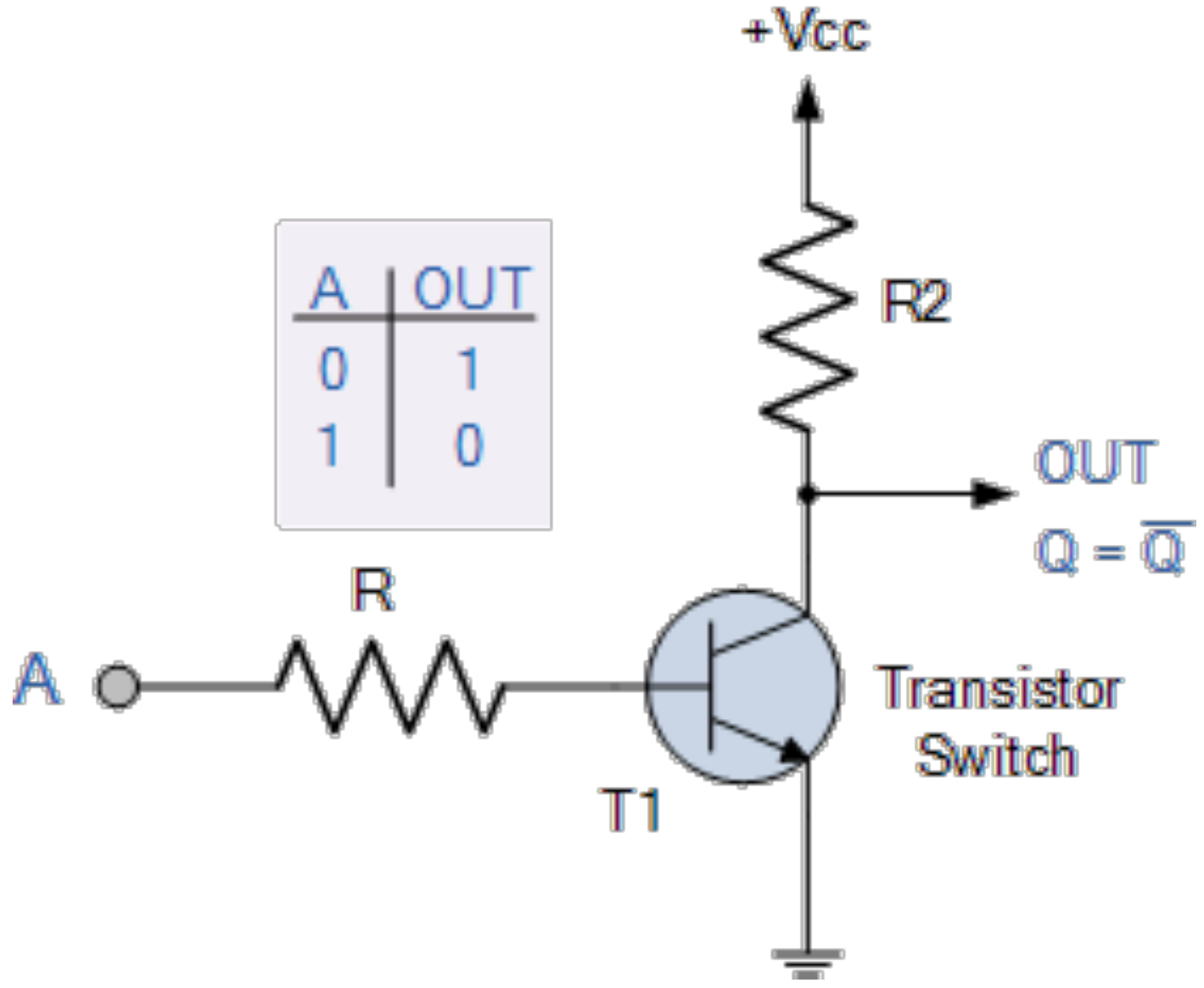
Classical Bit (MOSFET)

“OFF” 



“ON” 



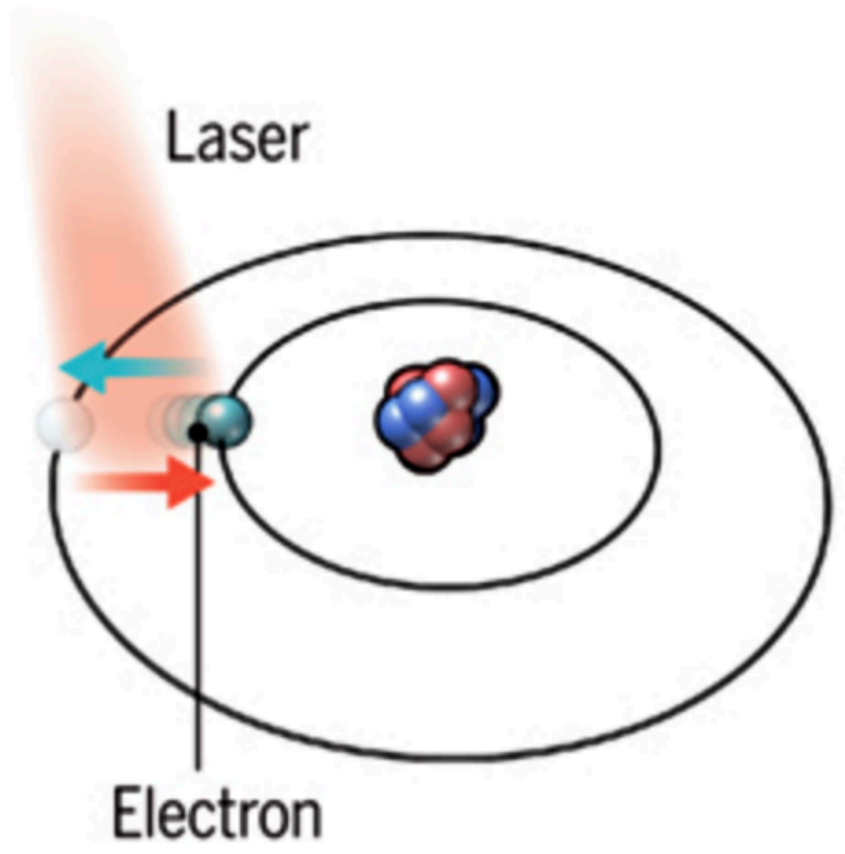


Classical NOT

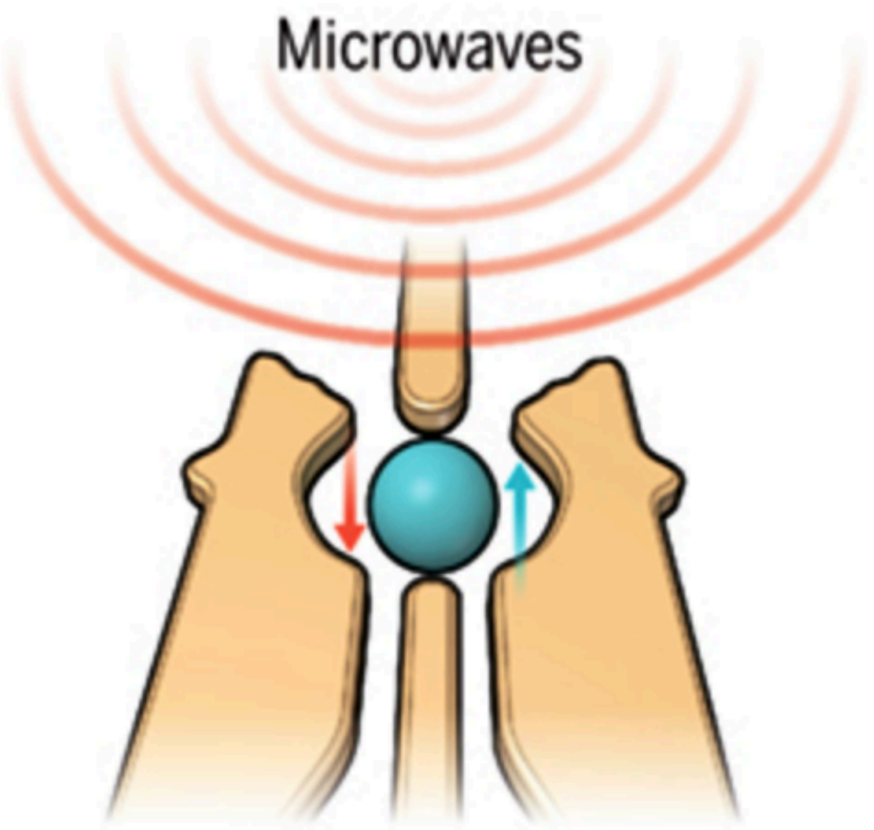
Physical Realization

What about the Qubit?

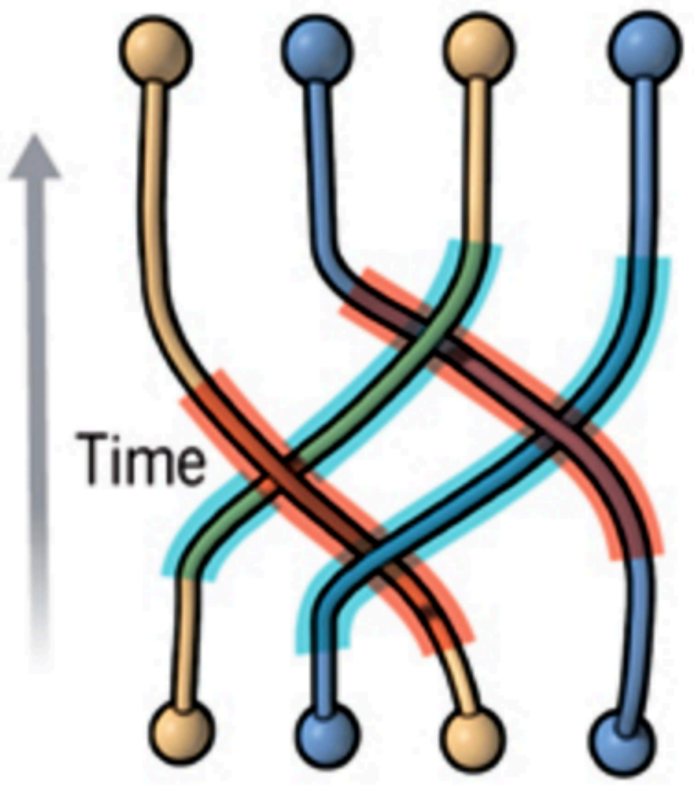
Qubit Platforms



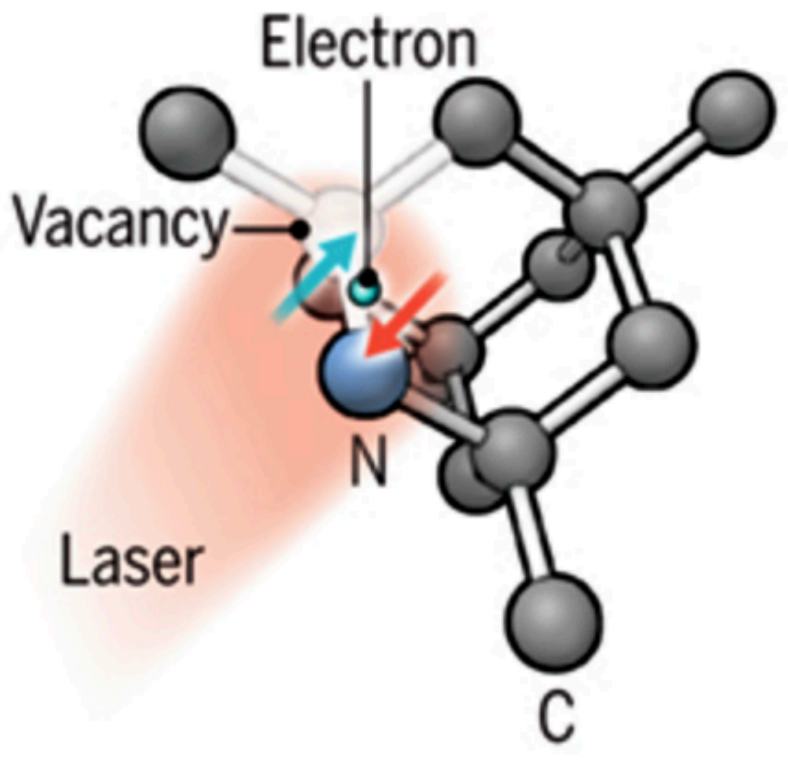
Trapped ions



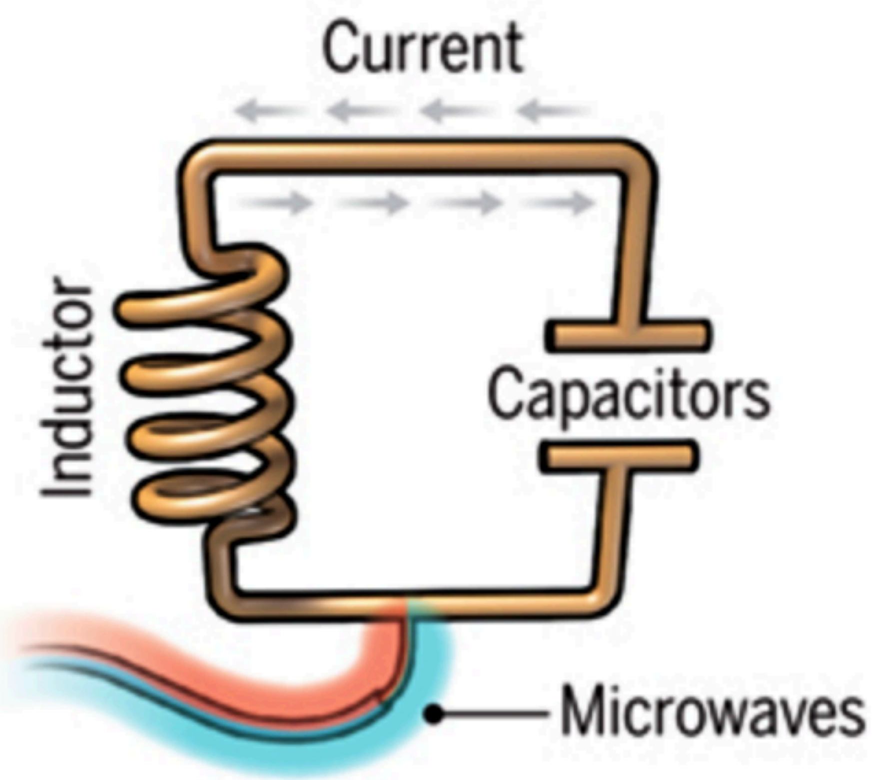
Silicon quantum dots



Topological qubits



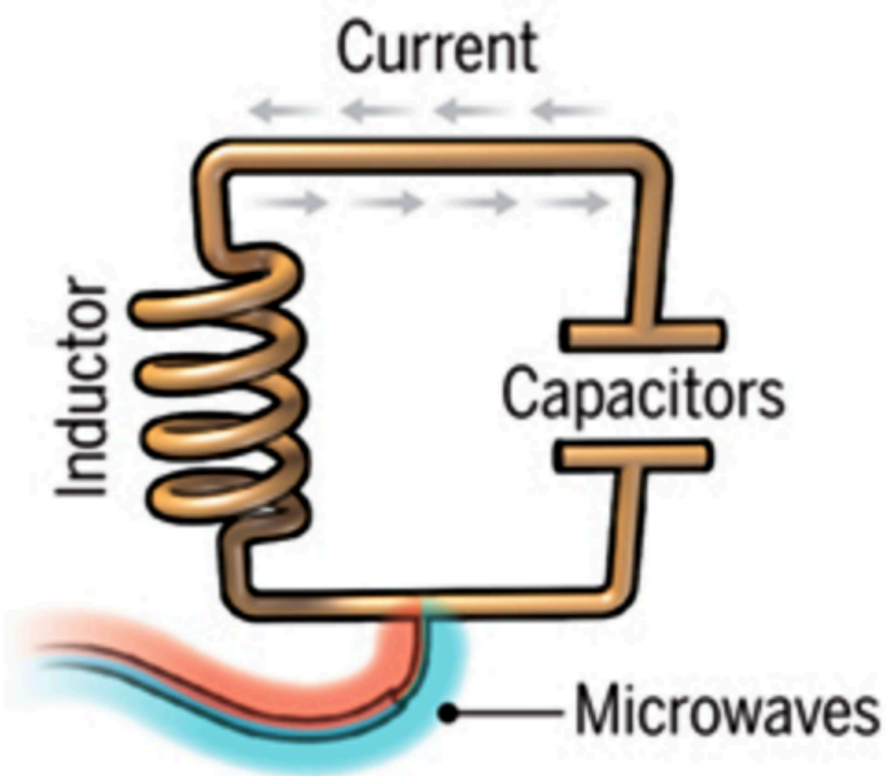
Diamond vacancies



Superconducting loops

No Consensus Yet!

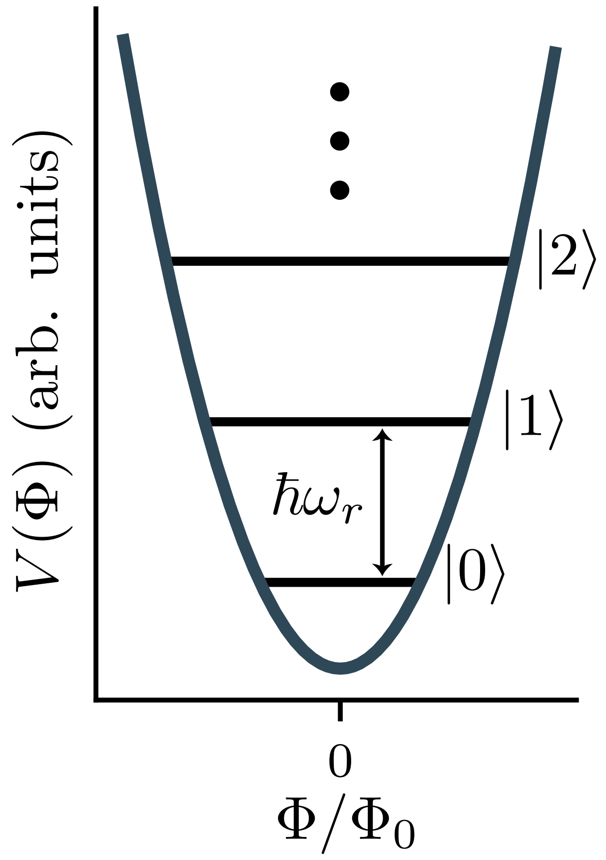
Qubit Platforms

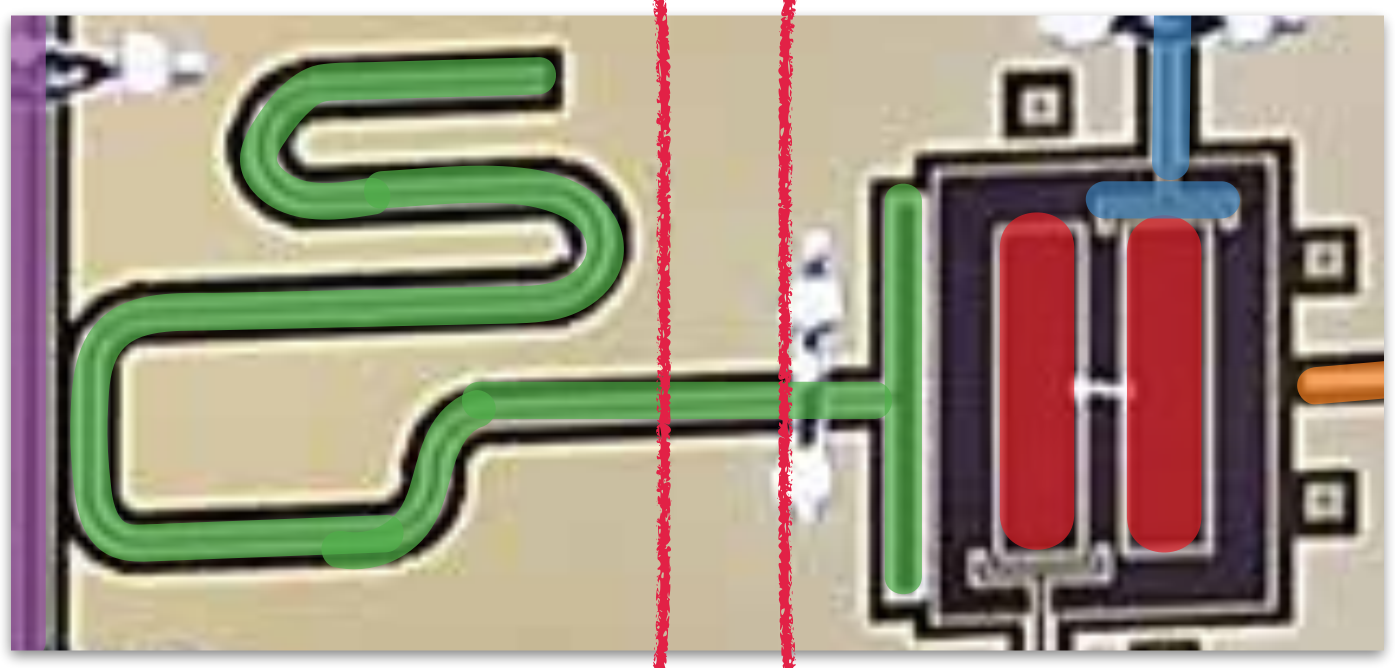


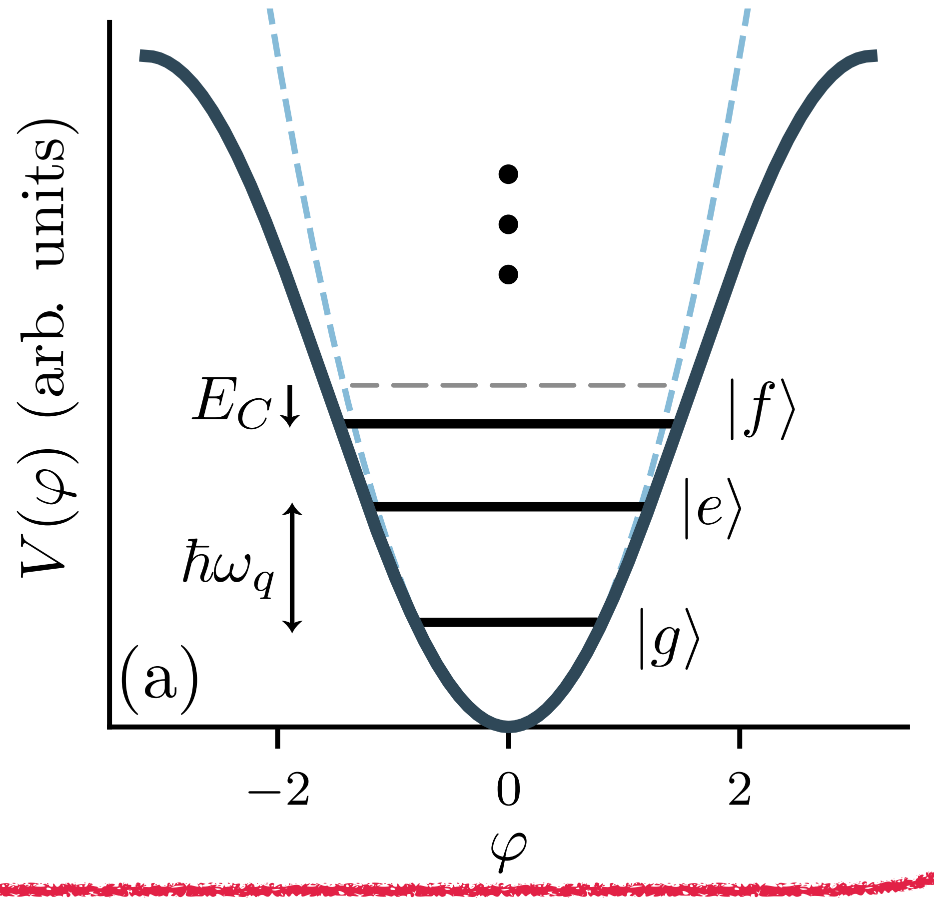
Superconducting loops

Resonator

Transmon



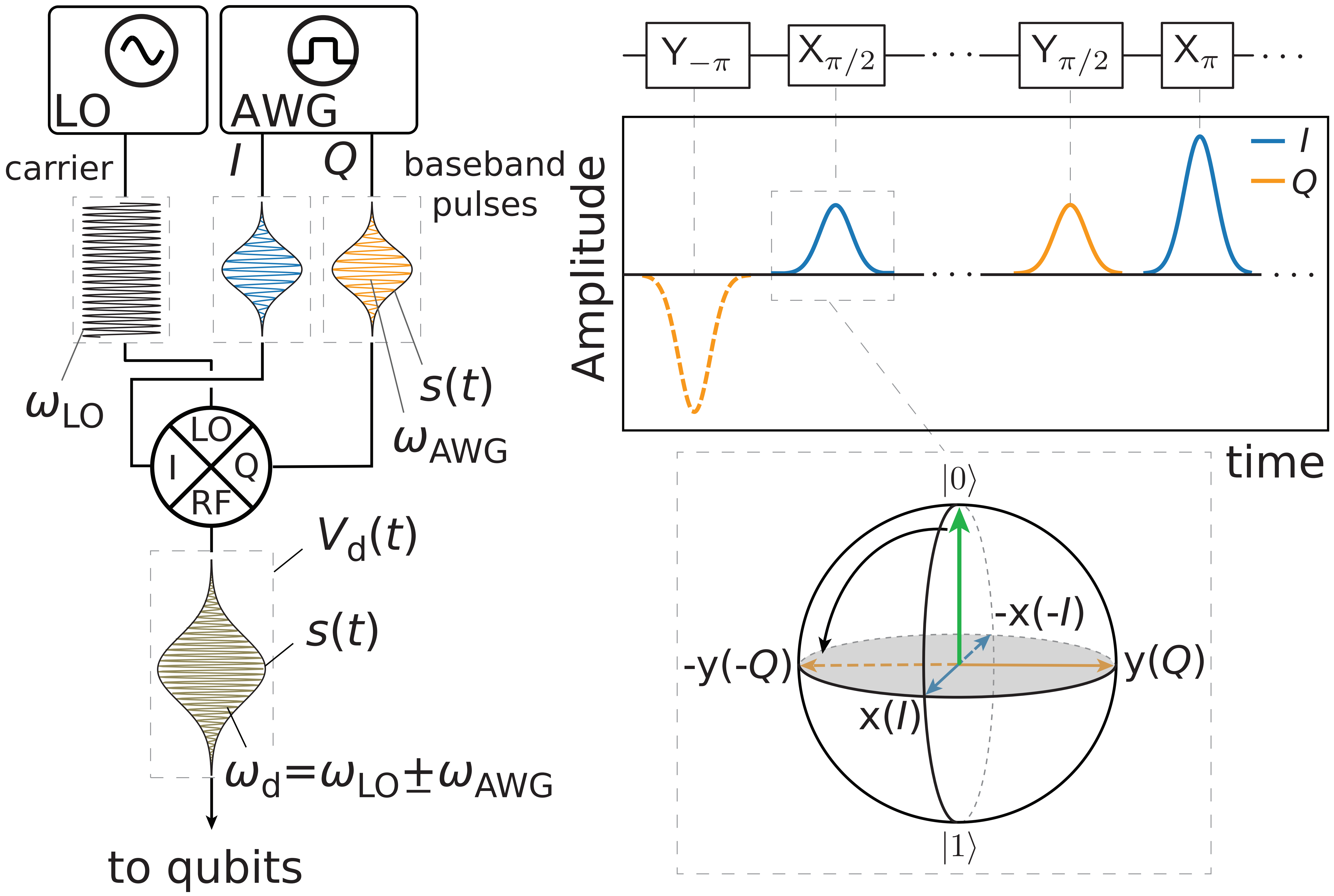




Measure / Read

Manipulate

Qubit Platforms



Using Microwave Signals!