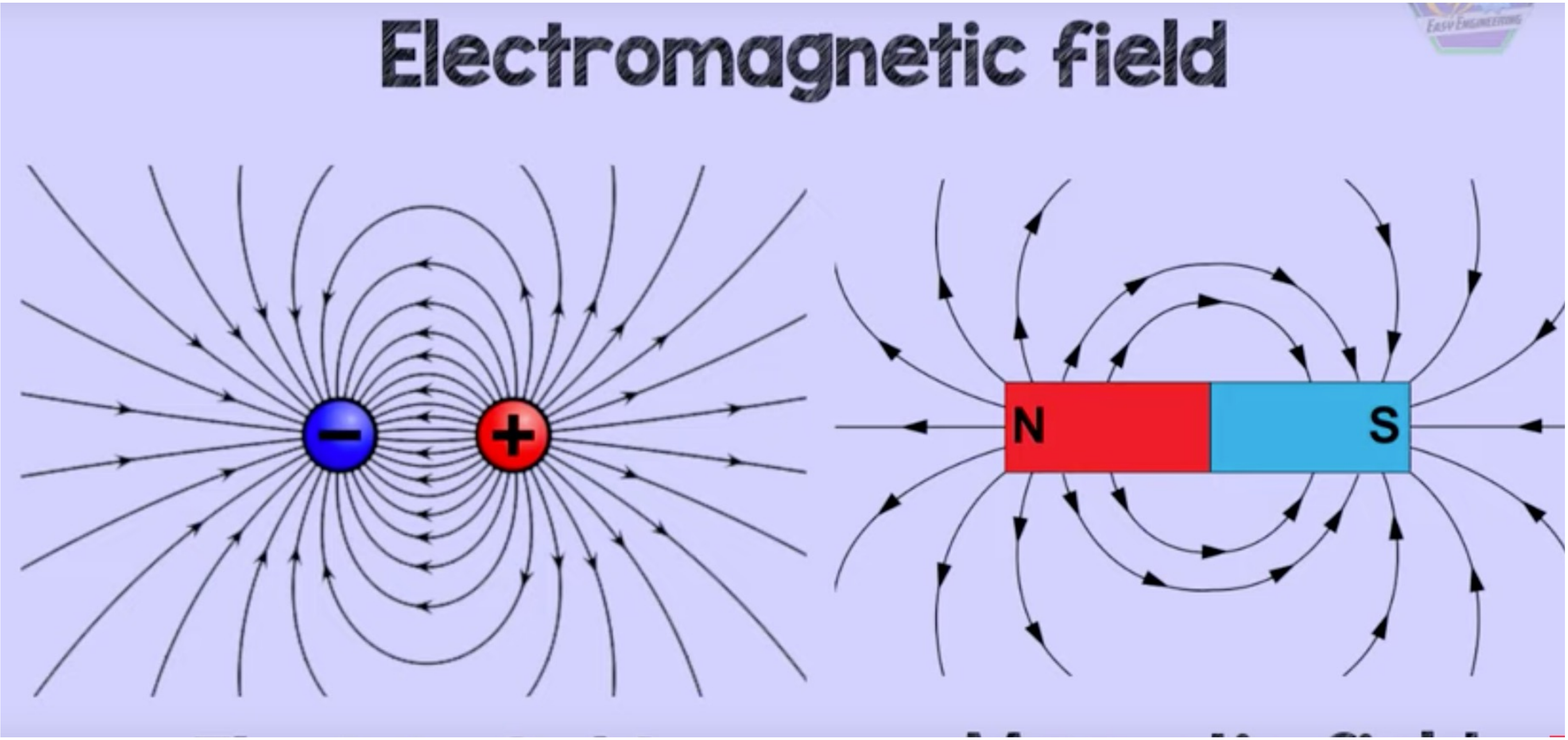
Chapter 32

32-1 GAUSS' LAW FOR MAGNETIC FIELDS

Maxwell's Equations

Magnetic Materials

Maxwell Equations; Magnetism of Matter

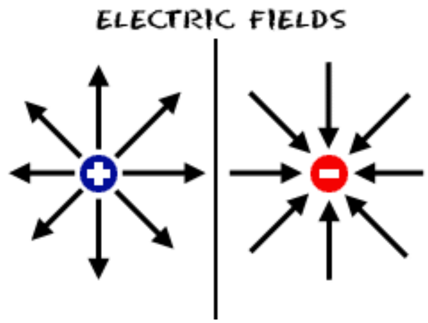


Electric field **Magnetic field**

Review

Gauss's Law.

Uses to find $\vec{E}$



For symmetric situations

$$\oint_S \vec{E}.\overrightarrow{dA} = \frac{q_{en}}{\varepsilon_\circ}$$

The integral over closed surface

Electric field force ($\vec{F}_E$):

$$\vec{F}_E = q\vec{E} \quad \Longrightarrow \quad \vec{F}_E \ // \ \vec{E}$$

Work:

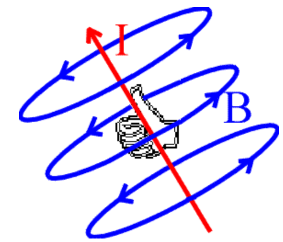
$$\boxed{\begin{array}{l} dW_E = \vec{F}_E.\overrightarrow{dx} \\ \Longrightarrow W_E \neq 0 \end{array}}$$

The work done by the electric field is <u>not</u> zero

The electric field can change the direction of a particle's motion and change its speed.

Ampère's Law

Uses to find $\vec{B}$



For symmetric situations

$$\oint_C \vec{B}.\overrightarrow{dl} = \mu_\circ I$$

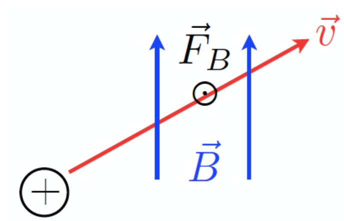
The integral over closed path

Magnetic field force ($\vec{F}_B$):

$$\vec{F}_B = q\left(\vec{V} \times \vec{B}\right) \quad \Longrightarrow \quad \vec{F}_B \perp \vec{B} \,\&\, \vec{V}$$

Work:

$$\boxed{\begin{array}{l} dW_B = \vec{F}_B.\overrightarrow{dx} \\ \Longrightarrow W_B = 0 \quad (\vec{F}_B \perp \vec{V}) \end{array}}$$

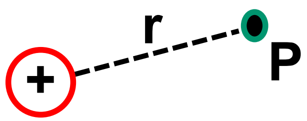


The work done by a magnetic field is always zero

A magnetic field can change the direction of a particle's motion, but cannot change its speed.

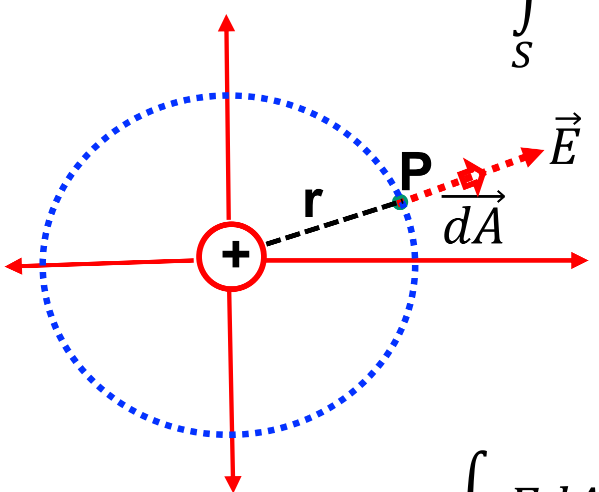
Gauss's Law.

Ex: Find the $\vec{E}$ at point P away from positive charge Q



Solution:

$$\oint_S \vec{E}.\overrightarrow{dA} = \frac{q_{en}}{\varepsilon_\circ}$$



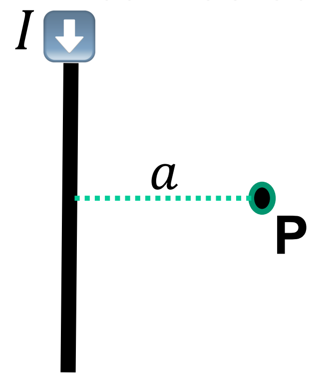
$$\oint_S EdA = \frac{Q}{\varepsilon_\circ}$$

$$\mathrm{E} \oint_S dA = \frac{Q}{\varepsilon_\circ} \implies E\,(4\pi\, r^2) = \frac{Q}{\varepsilon_\circ}$$

$$E = \frac{Q}{4\pi\varepsilon_\circ r^2}$$

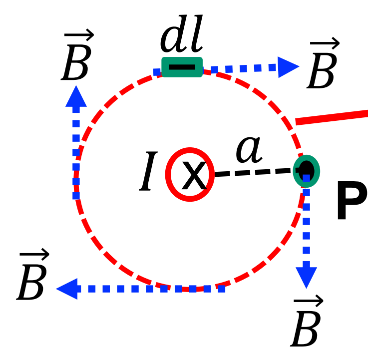
Ampère's Law

Ex: Find the $\vec{B}$ at point P away from a wire carries current I



Solution:

$$\oint_C \vec{B}.\overrightarrow{dl} = \mu_\circ I$$



Closed loop

$dl\ //\ B$

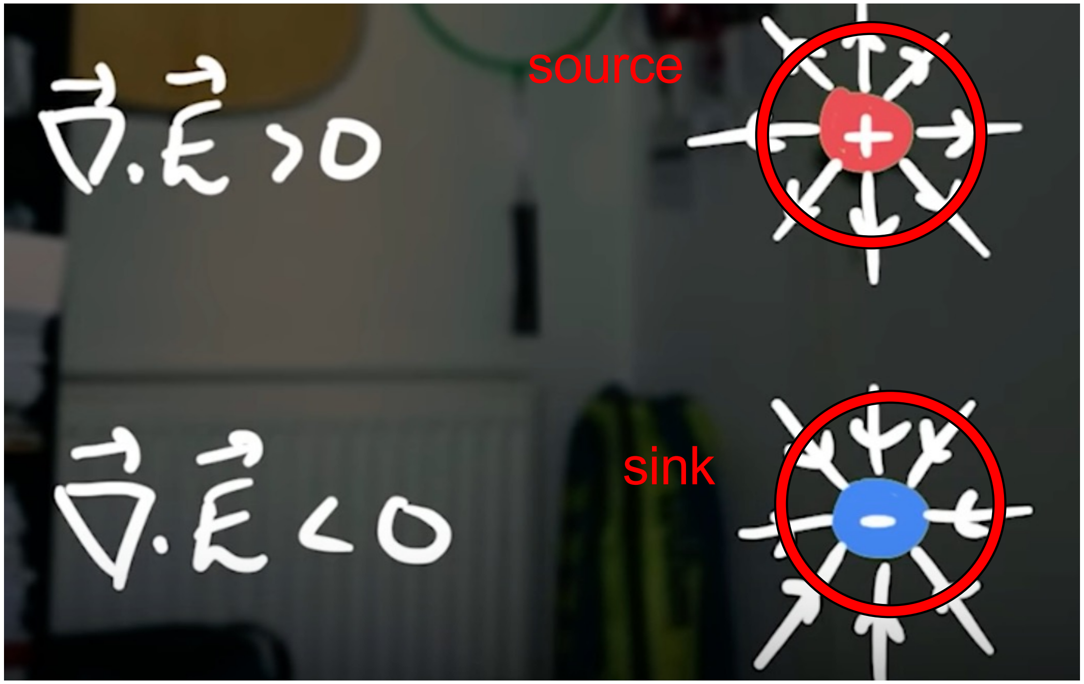
$$\oint_C Bdl = \mu_\circ I$$

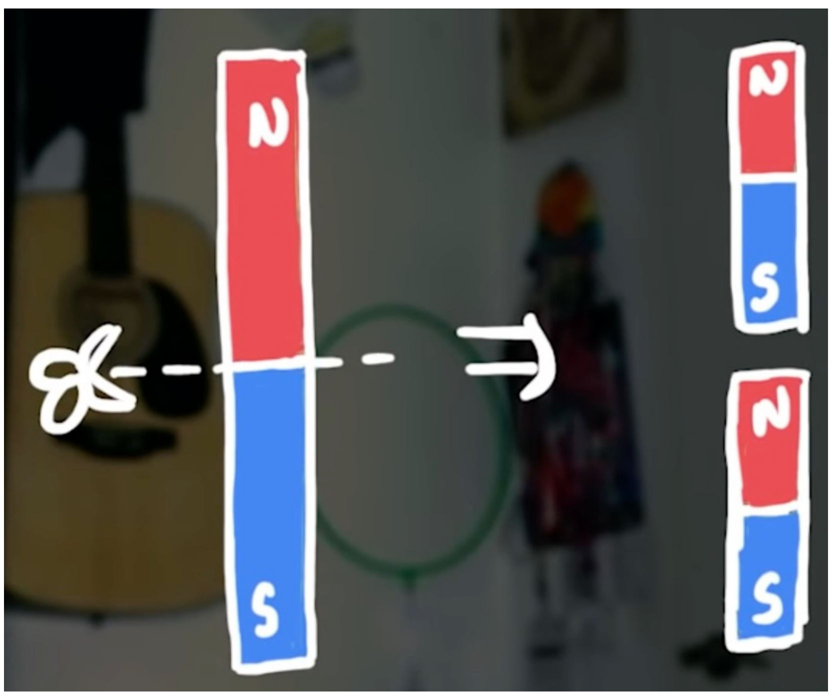
B

$$\oint_C dl = \mu_\circ I$$

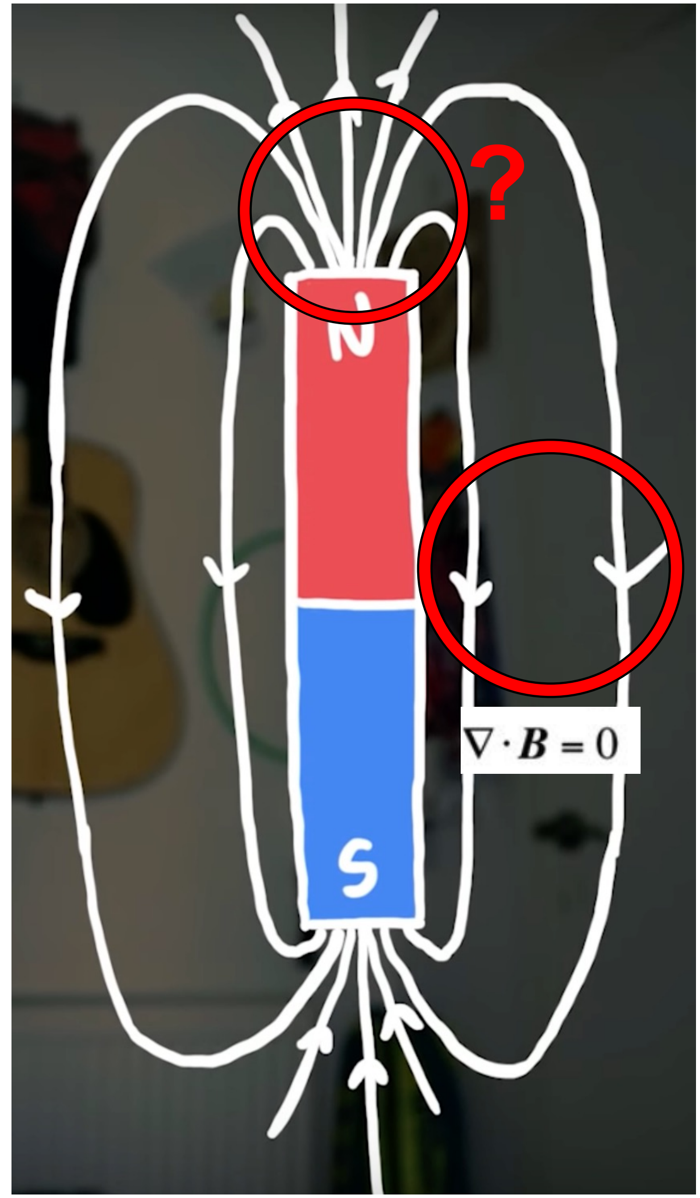
$$\mathrm{B}\,(2\pi a) = \mu_\circ I$$

$$\boxed{\mathrm{B} = \frac{\mu_\circ I}{2\pi a}}$$

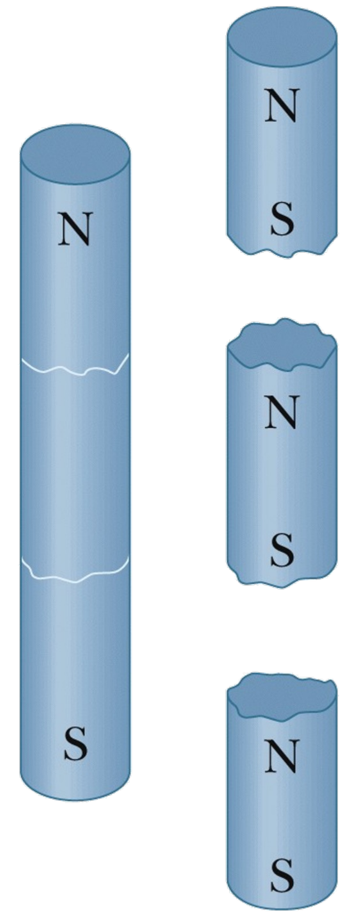




Magnetic monopoles do not exist.



If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.



The simplest magnetic structure that can exist is a magnetic dipole. Magnetic **monopoles do not exist** (as far as we know).

Maxwell Equations:

1) GAUSS' LAW FOR MAGNETIC FIELDS

[2] $$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0,$$

The magnetic field lines is always closed field. The net magnetic flux through any (closed) Gaussian surface is zero.

It implies that magnetic monopoles do not exist.

Contrast this with Gauss' law for electric fields, $$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$ [1]

Whenever there is a charge, there is electric field

2) INDUCED MAGNETIC FIELDS

In Chapter 30 you saw that a changing magnetic flux induces an electric field, and we ended up with Faraday's law of induction in the form

[3] $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(Faraday's law of induction).}$$

Now

A changing electric flux induces a magnetic field $\vec{B}$ Maxwell's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} \quad \text{(Maxwell's law of induction)},$$

Ampere–Maxwell Law

- Ampere's law, $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$, gives the magnetic field generated by a current i_{enc} encircled by a closed loop. Maxwell's law and Ampere's law can be written as the single equation

[4]

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \quad (\text{Ampere} - \text{Maxwell law})\,.$$

Maxwell Equations:

* Written on the assumption that no dielectric or magnetic materials are present.

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

Maxwell's equations summary:

Maxwell's equations are a set of fundamental equations in electromagnetism that describe how electric and magnetic fields interact and propagate through space. They were formulated by the Scottish physicist James Clerk Maxwell in the 19th century and represent a cornerstone of classical electromagnetism. Maxwell's equations provide a comprehensive framework for understanding the behavior of electric and magnetic fields and have profound implications for our understanding of light, electricity, and magnetism. Here's a summary of the meaning and significance of Maxwell's equations:

1) Gauss's Law for Electricity (Gauss's Law for Electric Fields):
   - Equation: $\nabla \cdot E = \rho / \varepsilon_0$
   - This equation relates the electric field (E) to the electric charge density (ρ) within a closed surface.
   - It states that the electric field lines emanate from positive charges and terminate on negative charges, obeying the inverse square law.

2) Gauss's Law for Magnetism (Gauss's Law for Magnetic Fields):
   - Equation: $\nabla \cdot B = 0$
   - This equation states that there are no magnetic monopoles; magnetic field lines always form closed loops and do not start or end.

Maxwell's equations summary:

3) Faraday's Law of Electromagnetic Induction:

- Equation: $\nabla \times E = -\partial B/\partial t$
- This equation describes how a changing magnetic field induces an electric field. It is the basis for understanding the generation of electrical currents in coils and transformers.

4) Ampère's Circuital Law (with Maxwell's addition):

- Equation: $\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \partial E/\partial t$
- Ampère's Circuital Law, when coupled with Maxwell's addition (the last term), relates the circulation of the magnetic field (B) to the current density (J) and the rate of change of the electric field (E).
- This equation explains how electric currents create magnetic fields and how changing electric fields generate magnetic fields.

In summary, Maxwell's equations are a set of four fundamental equations that describe the behavior of electric and magnetic fields and their relationship to electric charges and currents. They have played a pivotal role in shaping our understanding of electromagnetism and have had profound implications for physics and technology.

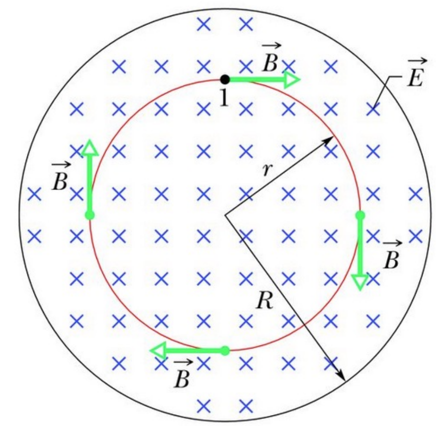
Maxwell Equations:

Ex-01:

(a) Derive an expression for the magnetic field at radius r for the case $r \le R$ and $r \ge R$.

(b) Evaluate the field magnitude B for r = R/5 = 11.0 mm and dE/dt = 1.50 × 1012 V/m· s.

Solution:



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0^\circ = \oint B\, ds.$$

$$(B)(2\pi r) = \mu_0 \varepsilon_0 \frac{d(EA)}{dt}.$$

$$B = \frac{\mu_0 \varepsilon_0 r}{2} \frac{dE}{dt}. \quad \text{(Answer)} \quad r \le R.$$

$$B = \frac{\mu_0 \varepsilon_0 R^2}{2r} \frac{dE}{dt}. \quad \text{(Answer)} \quad r \ge R$$

(b)

$$\begin{aligned} B &= \tfrac{1}{2}\mu_0 \varepsilon_0 r \tfrac{dE}{dt} \\ &= \tfrac{1}{2}(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}) \\ &\quad \times (11.0 \times 10^{-3}\ \mathrm{m})(1.50 \times 10^{12}\ \mathrm{V/m \cdot s}) \\ &= 9.18 \times 10^{-8}\ \mathrm{T}. \qquad \text{(Answer)} \end{aligned}$$

Ex-02: Gauss' law for magnetism:

A) can be used to find $\vec{B}$ due to given currents provided there is enough symmetry

B) is false because there are no magnetic poles

C) can be used with open surfaces because there are no magnetic poles

D) contradicts Faraday's law because one says $\Phi_B = 0$ and the other says $\mathcal{E} = -d\Phi_B/dt$

E) none of the above

Ans: E

Maxwell Equations:

Ex-03: The statement that magnetic field lines form closed loops is a direct consequence of:

A) Faraday's law

B) Ampere's law

C) Gauss' law for electricity

D) Gauss' law for magnetism

Ans: D

E) the Lorentz force

Ex-04: Gauss' law for magnetism, $\oint \vec{B} \cdot d\vec{A} = 0$, tells us:

A) the net charge in any given volume

B) that the line integral of a magnetic field around any closed loop must vanish

C) the magnetic field of a current element

D) that magnetic monopoles do not exist

Ans: D

E) charges must be moving to produce magnetic fields

Ex-05: One of the Maxwell equations begins with $\oint \vec{B} \cdot d\vec{s} =$ .... The symbol $d\vec{s}$ means:

A) an infinitesimal displacement of a charge

B) an infinitesimal displacement of a magnetic pole

C) an infinitesimal inductance

D) an infinitesimal surface area

Ans: E

E) none of the above

Maxwell Equations:

Ex-06: One of the Maxwell equations begins with $\oint \vec{E} \cdot d\vec{s} =$ .... The º symbol in the integral sign means:

A) the same as the subscript in μ_0

B) integrate clockwise around the path

C) integrate counterclockwise around the path

D) integrate around a closed path

E) integrate over a closed surface

Ans: D

Ex-07: Which of the following equations can be used, along with a symmetry argument, to calculate the electric field of a point charge?

A) $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$

B) $\oint \vec{B} \cdot d\vec{A} = 0$

C) $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

D) $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

E) none of these

Ans: A

Q1 (*4pts*): Consider the four Maxwell equations:

I. $\oint \vec{E} \cdot d\vec{A} = q/\varepsilon_0$

II. $\oint \vec{B} \cdot d\vec{A} = 0$

III. $\oint \vec{E} \cdot d\vec{s} = -\dfrac{d\Phi_B}{dt}$

IV. $\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$

Which of these must be modified if magnetic poles are discovered?

A) only I
B) only II
C) only II and III
D) only III and IV
E) only II, III, IV

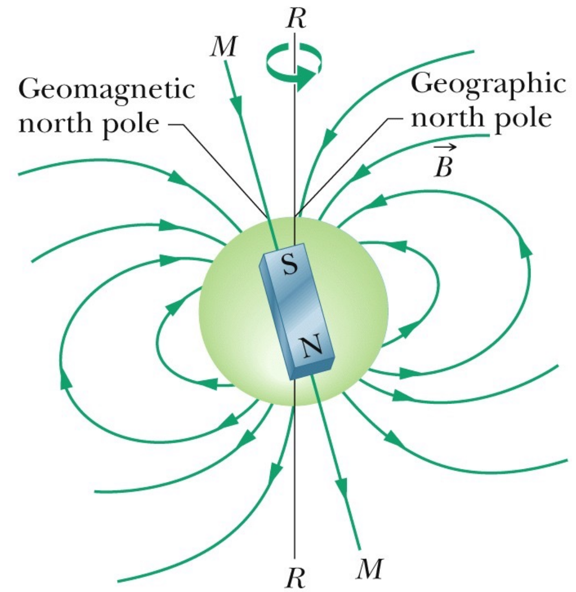
Q1): Answer: C

Law of Physics	Standard Form w/o Magnetic Monopoles	With Magnetic Monopoles (simple symmetrization)
Gauss' Law	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho_e}{\epsilon_0}$
Gauss' Law for Magnetism	$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{\nabla} \cdot \vec{B} = \mu_0 \rho_m$
Faraday's Law of Induction	$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$	$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\mu_0 \vec{J}_m$
Ampere's Circuital Law	$\vec{\nabla} \times \vec{B} - \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}$	$\vec{\nabla} \times \vec{B} - \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}_e$
Lorentz Force Equation	$\vec{F} = q_e(\vec{E} + \vec{v} \times \vec{B})$	$\vec{F} = q_e(\vec{E} + \vec{v} \times \vec{B}) + q_m\left(\vec{B} - \vec{v} \times \frac{\vec{E}}{c^2}\right)$

Magnets:

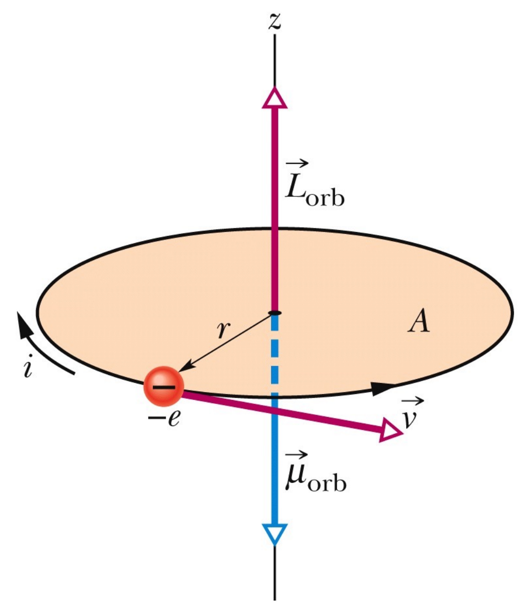
Earth's magnetic field represented as a dipole field. The dipole axis *MM* makes an angle of 11.5˚ with Earth's rotational axis *RR*. The south pole of the dipole is in Earth's Northern Hemisphere.

For Earth, the south pole of the dipole is actually in the north.



Magnetism and Electrons :

An electron moving at constant speed *v* in a circular path of radius *r* that encloses an area *A*.



Magnetic materials

Each electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment that combine vectorially. The resultant of these two vector quantities combines vectorially with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a sample of a material. If the combination of all these magnetic dipole moments produces a magnetic field, then the material is magnetic. There are three general types of magnetism:

- Diamagnetism
- Paramagnetism
- Ferromagnetism

Diamagnetism

A diamagnetic material placed in an external magnetic field develops a magnetic $\vec{B}_{\text{ext}}$ dipole moment directed opposite $\vec{B}_{\text{ext}}$ If the field is nonuniform, the diamagnetic material is repelled from a region of greater magnetic field toward a region of lesser field.

Paramagnetism:

A paramagnetic material placed in an external magnetic field $\vec{B}_{\text{ext}}$ develops a magnetic dipole moment in the direction of $\vec{B}_{\text{ext}}$

Paramagnetic materials have atoms with a permanent magnetic dipole moment but the moments are randomly oriented, with no net moment, unless the material is in an external magnetic field, where the dipoles tend to align with that field. The extent of alignment within a volume *V* is measured as the magnetization *M*, given by

$$M = \frac{\text{measured magnetic moment}}{V}.$$

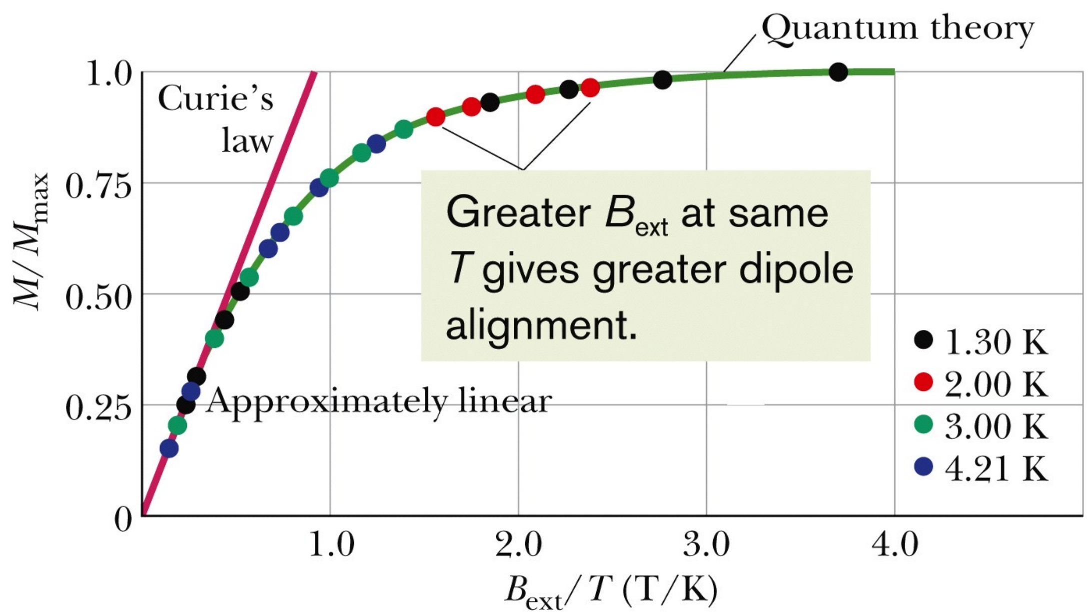
Complete alignment (saturation) of all *N* dipoles in the volume

gives a maximum value $$M_{\max} = \frac{N\mu}{V}.$$

At low values of the ratio $\frac{B_{ext}}{T}$,

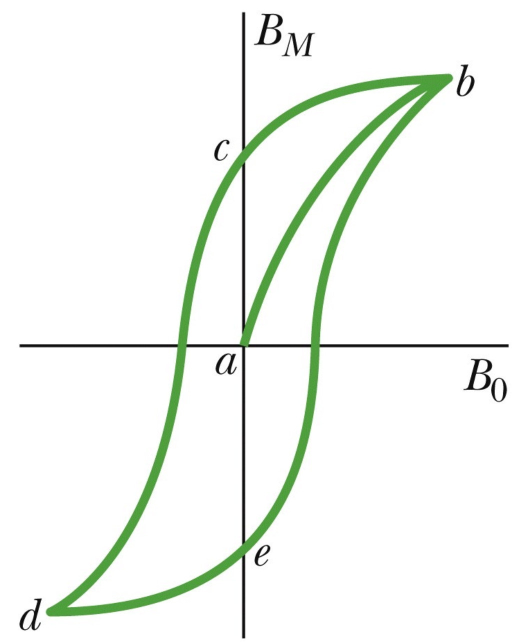
$$M = C\frac{B_{ext}}{T}$$

In a nonuniform external field, a paramagnetic material is attracted to the region of greater magnetic field.



Ferromagnetism:

The magnetic dipole moments in a ferromagnetic material can be aligned by an external magnetic field and then, after the external field is removed, remain partially aligned in regions (domains). Alignment is eliminated at temperatures above a material's Curie temperature. In a nonuniform external field, a ferromagnetic material is attracted to the region of greater magnetic field.



Typical Hysteresis

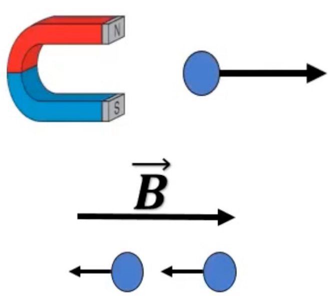
Summary:

Diamagnetic materials

↑↓ ↑↓ ↑↓

No unpaired Electrons.

Weakly repelled by a magnet



Weakly magnetized
In opposite direction
Of magnetic field.

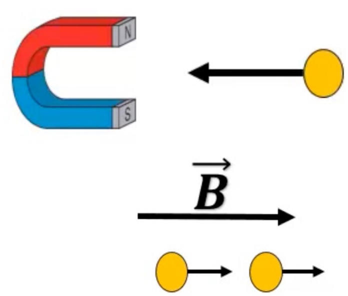
Value = −ve, small

Paramagnetic materials

↑↓ ↑↓ ↑

unpaired Electrons.

Weakly attracted by a magnet



Weakly magnetized
In the same direction
Of magnetic field.

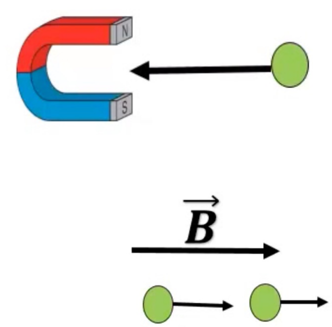
Value = +ve, small

Ferromagnetic materials

↑↓ ↑ ↑

unpaired Electrons.

Strongly attracted by a magnet

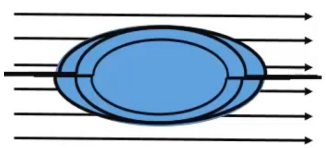


strongly magnetized
In the same direction
Of magnetic field.

Value = +ve, large

Summary:

Diamagnetic materials



Repulsion of magnetic lines of force from the center of the material.

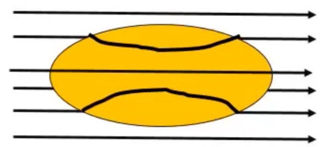
They lose their magnetism, on removal of external magnetic field.

Solid, Liquid, Gases

Examples

Water, copper
Mercury
Gold, hydrogen

Paramagnetic materials



Attraction of magnetic lines of force towards center of the material.

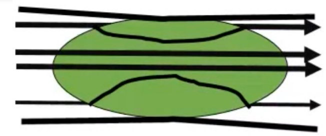
They lose their magnetism, on removal of external magnetic field.

Solid, Liquid, Gases

Examples

Aluminum, platinum, Sodium, crown glass.

Ferromagnetic materials



Heavy Attraction of magnetic lines of force toward center of the material.

They do not lose their magnetism, on removal of external magnetic field. i. e. they are permanet Magnets.

Only Solid

Examples

Iron, nickel, and cobalt