Chapter 33

Chapter 33

Electromagnetic Waves

Electromagnetic Waves

Energy Transport

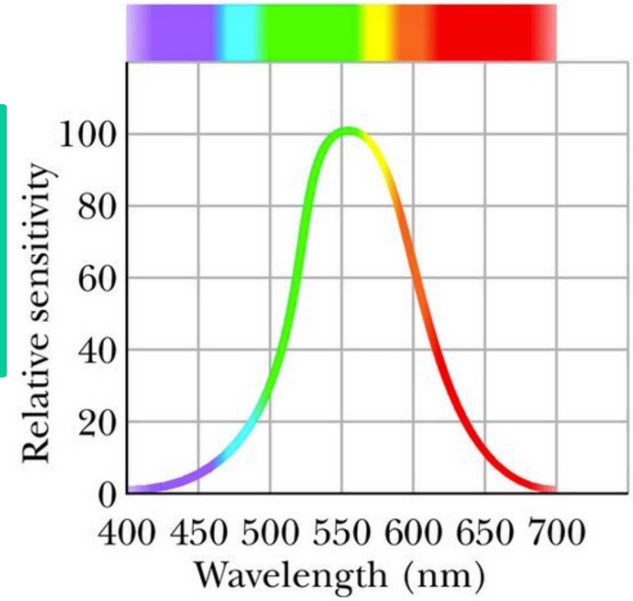
Total Internal Reflection

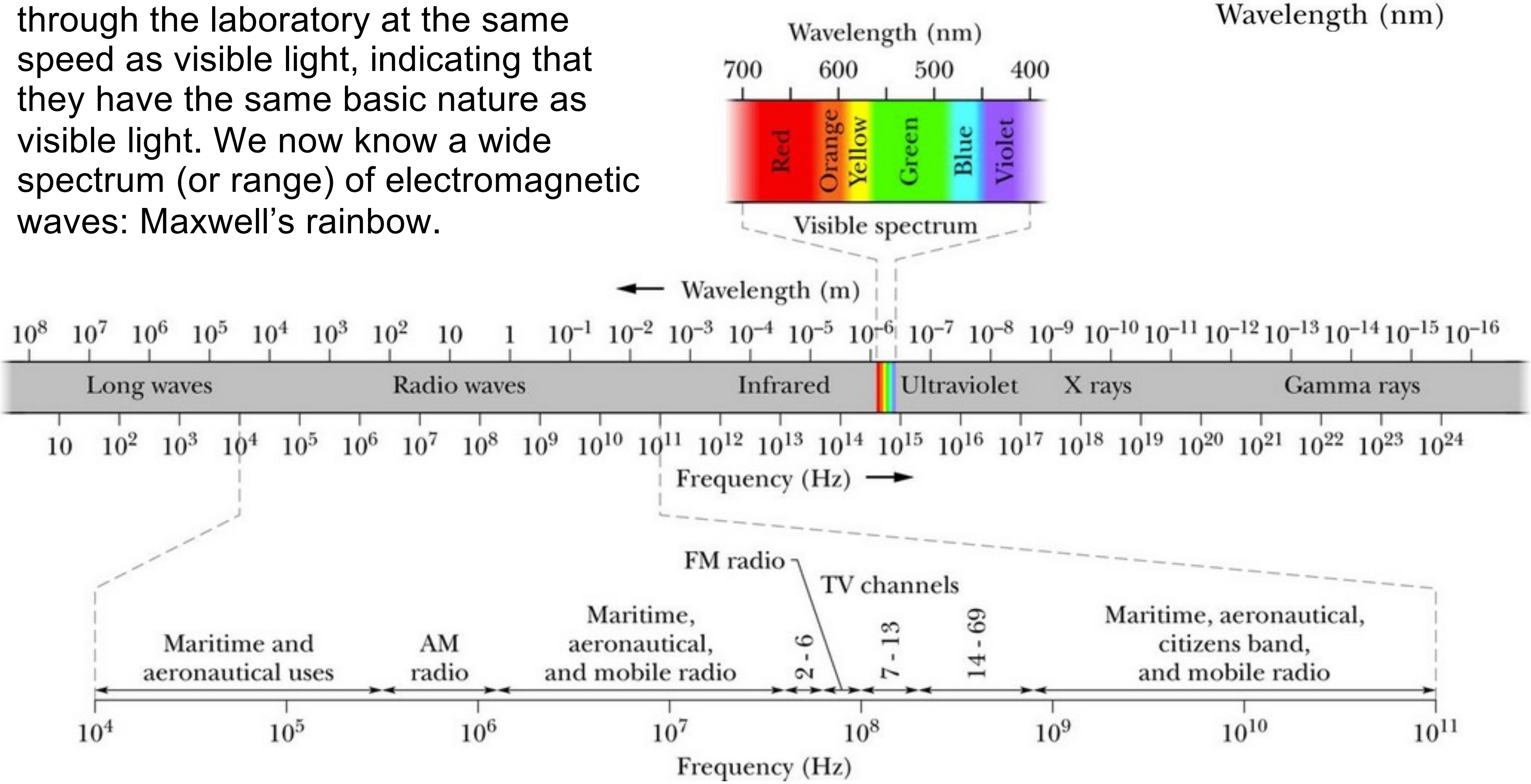
Electromagnetic Waves

Maxwell's Rainbow

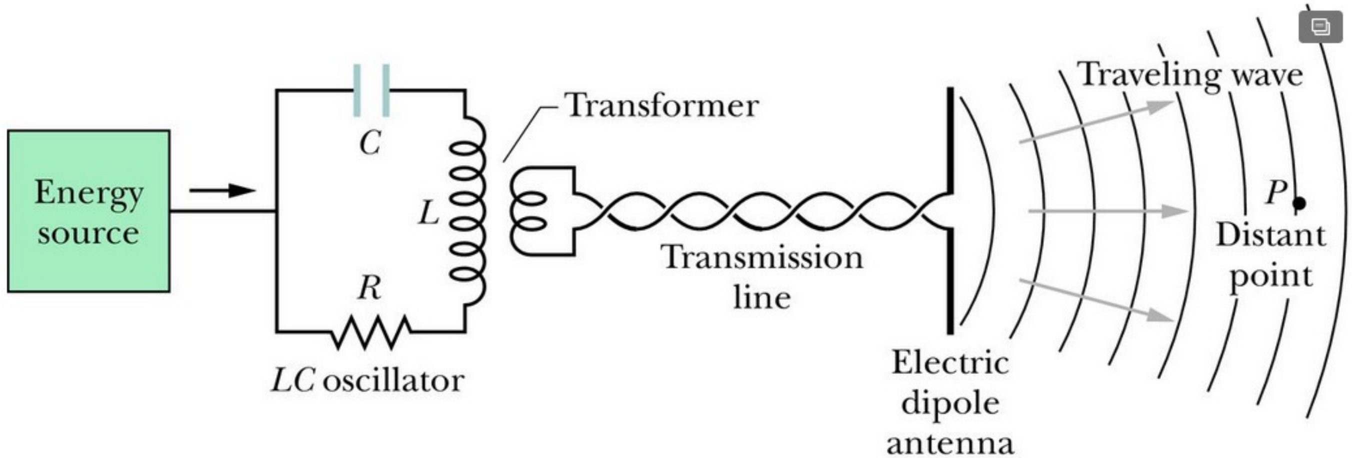
In Maxwell's time (the mid 1800s), the visible, infrared, and ultraviolet forms of light were the only electromagnetic waves known. Spurred on by Maxwell's work, however, Heinrich Hertz discovered what we now call radio waves and verified that they move through the laboratory at the same speed as visible light, indicating that they have the same basic nature as visible light. We now know a wide spectrum (or range) of electromagnetic waves: Maxwell's rainbow.

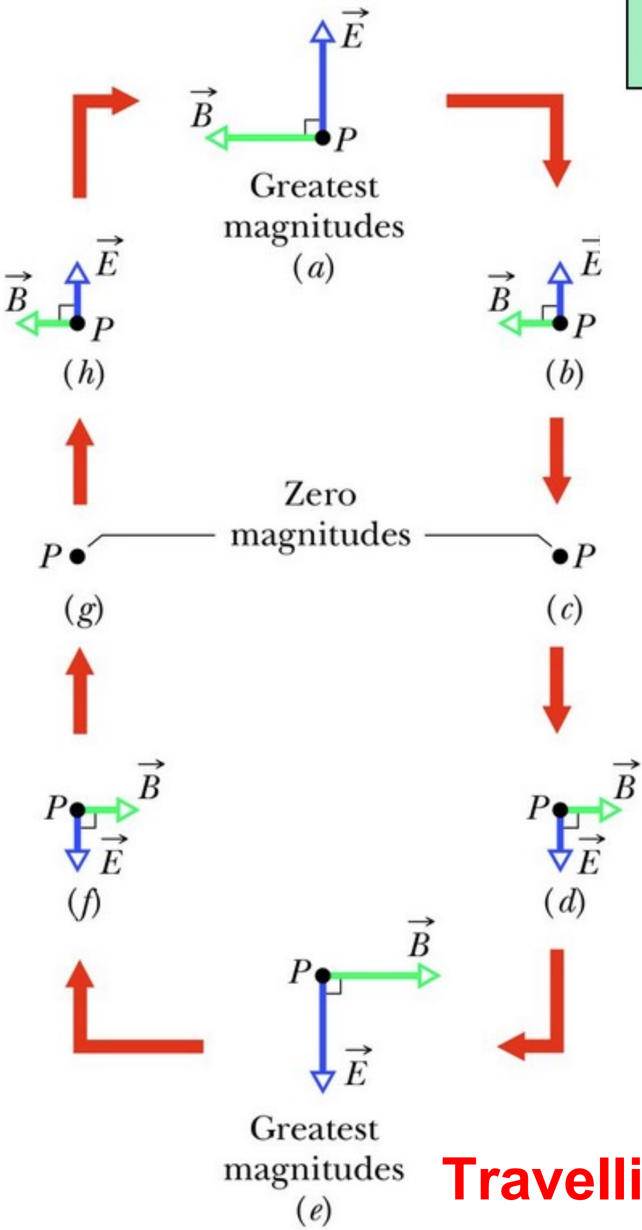
4G uses frequencies below 6 GHz, while some 5G networks use higher frequencies, like around 30 GHz or more.

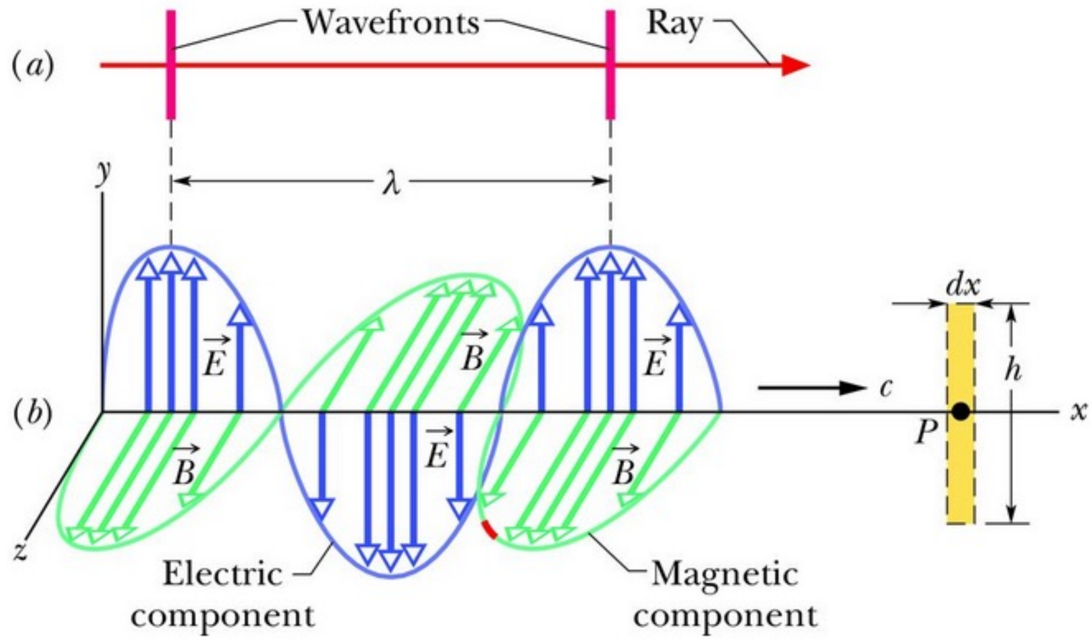




The Traveling Electromagnetic Wave, Qualitatively

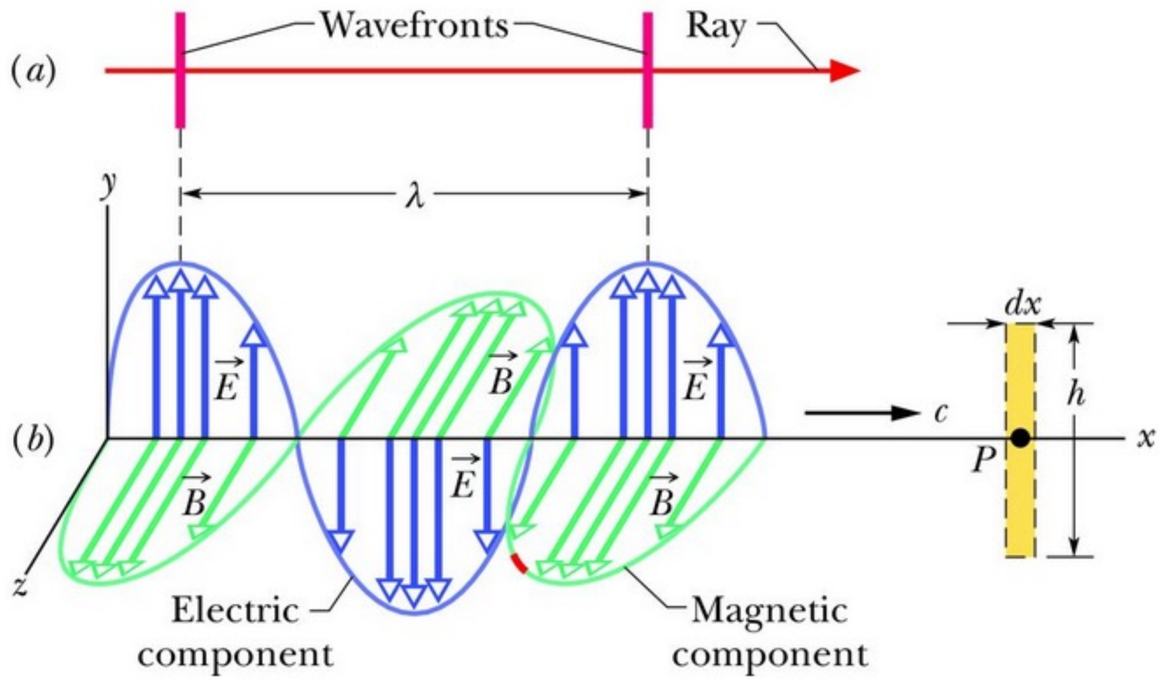






Travelling Electromagnetic Wave

The Traveling Electromagnetic Wave, Qualitatively



1. The electric and magnetic fields $\vec{E}$ and $\vec{B}$ are always perpendicular the direction in which the wave is traveling.

2. The electric field is always perpendicular to the magnetic field.

3. The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels

33-1 Electromagnetic Waves

Travelling Electromagnetic Wave

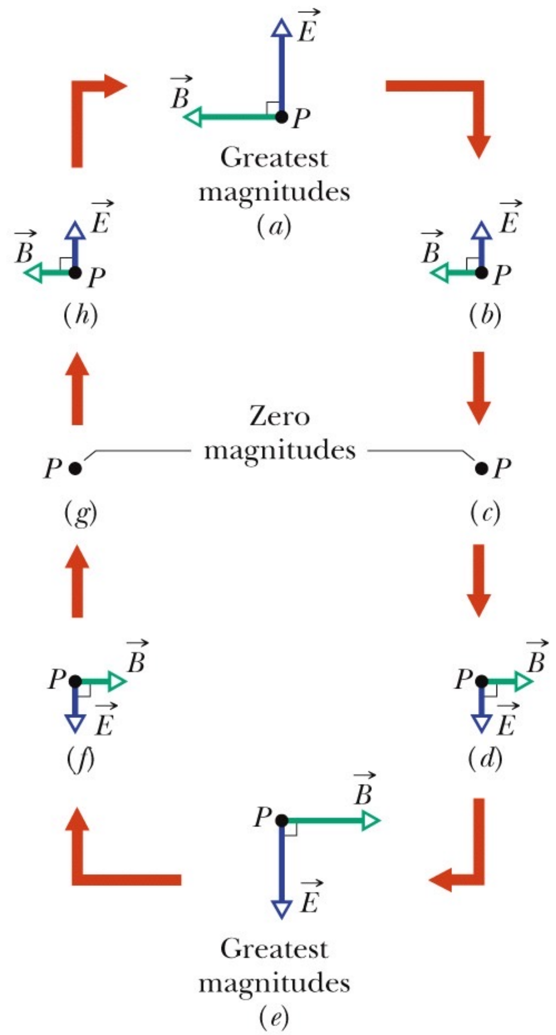
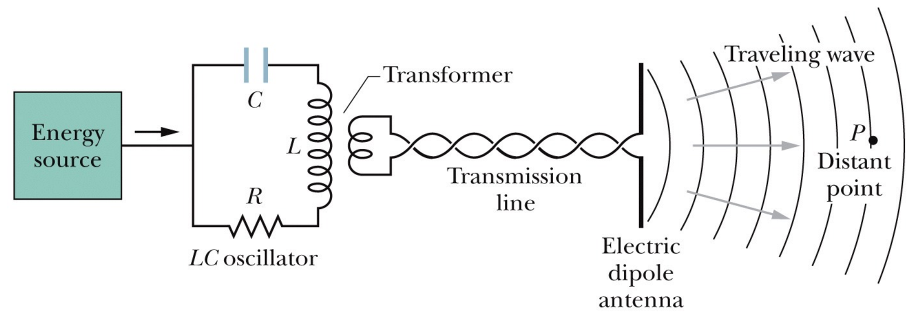


Figure 1

Figure 2



In keeping with these features, we can deduce that an electromagnetic wave traveling along an x axis has an electric field ***E*** and a magnetic field ***B*** with magnitudes that depend on x and t:

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

where E_m and B_m are the amplitudes of ***E*** and ***B***. The electric field induces the magnetic field and vice versa.

33-1 Electromagnetic Waves

Travelling Electromagnetic Wave

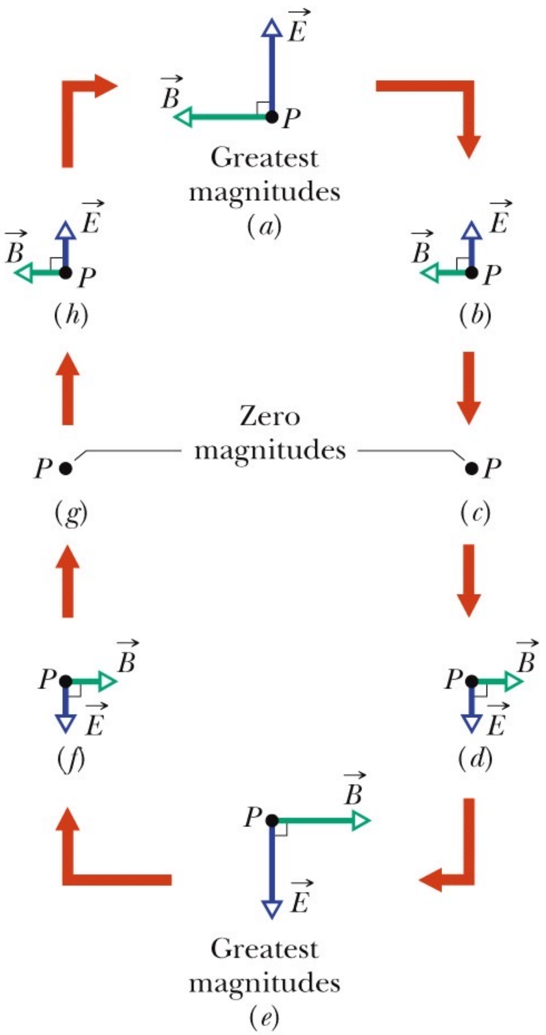
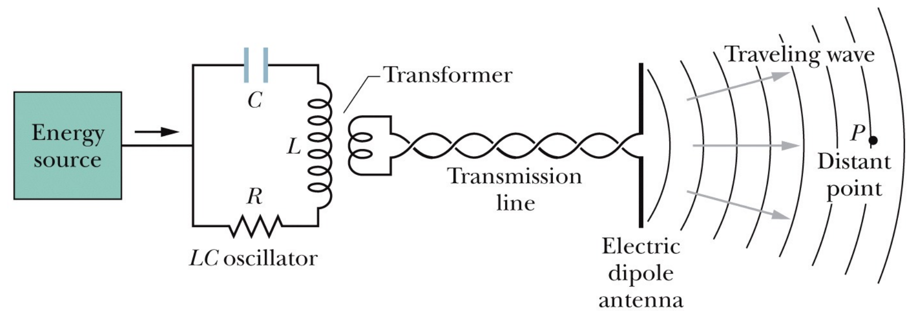


Figure 1

Figure 2



Wave Speed. From chapter 16 (Eq. 16-13), we know that the speed of the wave is ω/k. However, because this is an electromagnetic wave, its speed (in vacuum) is given the symbol c rather than v and that c has the value given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \text{(wave speed)},$$

which is about 3.0×10^8 m/s. In other words,

All electromagnetic waves, including visible light, have the same speed c in vacuum.

33.3: The Traveling Wave, Qualitatively:

We can write the electric and magnetic fields as sinusoidal functions of position x (along the path of the wave) and time t :

$$E = E_m \sin(kx - \omega t),$$

$$B = B_m \sin(kx - \omega t),$$

Here E_m and B_m are the amplitudes of the fields and, ω and k are the angular frequency and angular wave number of the wave, respectively.

All electromagnetic waves, including visible light, have the same speed c in vacuum.

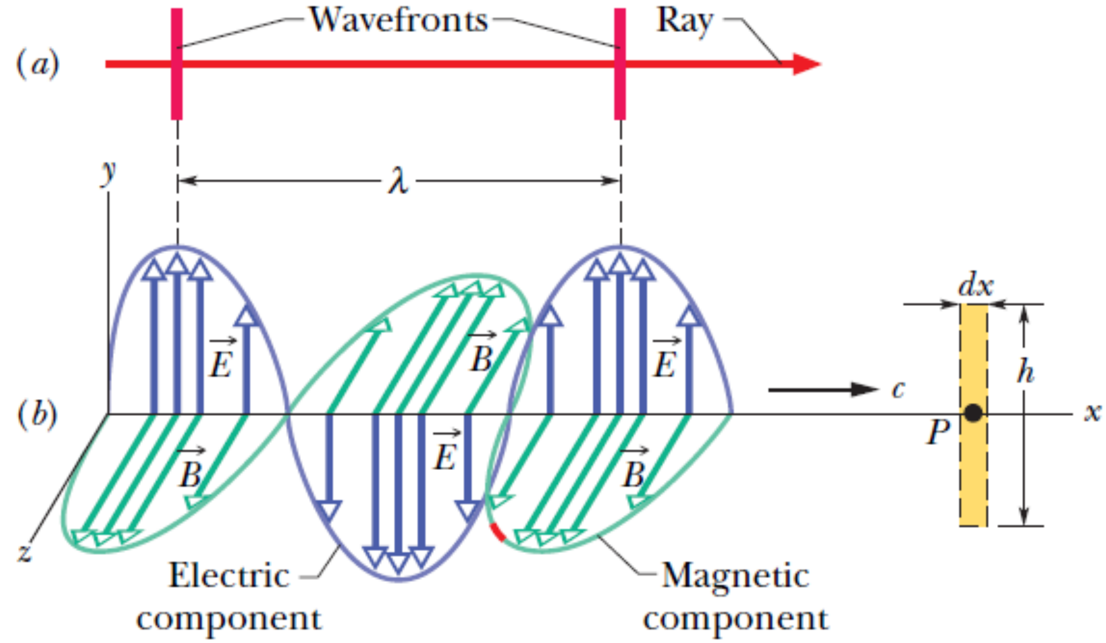
The speed of the wave (in vacuum) is given by c.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad \text{(wave speed)},$$

Its value is about 3.0 x10^8 m/s.

33.3: The Traveling Wave, Quantitativelv:

The dashed rectangle of dimensions dx and h in Fig. 33-6 is fixed at point P on the x axis and in the xy plane.



As the electromagnetic wave moves rightward past the rectangle, the magnetic flux B through the rectangle changes and—according to Faraday's law of induction—induced electric fields appear throughout the region of the rectangle. We take **E** and **E + dE** to be the induced fields along the two long sides of the rectangle. These induced electric fields are, in fact, the electrical component of the electromagnetic wave.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = (E + dE)h - Eh = h\,dE.$$

$$\frac{d\Phi_B}{dt} = h\,dx\,\frac{dB}{dt} \;\Rightarrow\; h\,dE = -h\,dx\,\frac{dB}{dt} \;\Rightarrow\; \frac{dE}{dx} = -\frac{dB}{dt}.$$

$$\frac{\partial E}{\partial x} = kE_m \cos(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = -\omega B_m \cos(kx - \omega t).$$

$$kE_m \cos(kx - \omega t) = \omega B_m \cos(kx - \omega t).$$

$$\boxed{\frac{E_m}{B_m} = c \qquad \text{(amplitude ratio)},}$$

33.4: The Traveling Wave, Quantitatively:

The oscillating electric field induces an oscillating and perpendicular magnetic field.

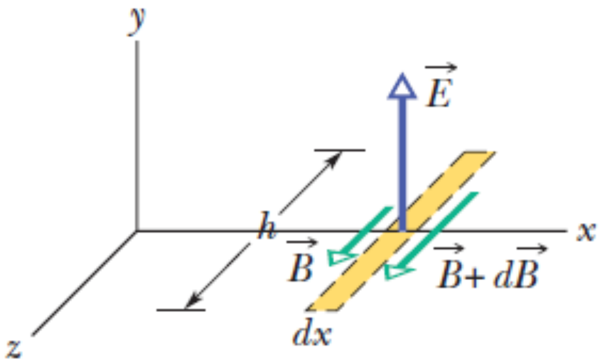


Fig. 33-7 The sinusoidal variation of the electric field through this rectangle, located (but not shown) at point *P* in *Fig. 33-5b*, ***E*** induces magnetic fields along the rectangle.The instant shown is that of Fig. 33-6: is decreasing in magnitude, and the magnitude of the induced magnetic field is greater on the right side of the rectangle than on the left.

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt},$$

$$\oint \vec{B}\cdot d\vec{s} = -(B + dB)h + Bh = -h\,dB.$$

$$\Phi_E = (E)(h\,dx), \qquad \frac{d\Phi_E}{dt} = h\,dx\frac{dE}{dt}$$

$$-h\,dB = \mu_0\varepsilon_0\left(h\,dx\frac{dE}{dt}\right)$$

$$-\frac{\partial B}{\partial x} = \mu_0\varepsilon_0\frac{\partial E}{\partial t}.$$

$$-kB_m\cos(kx - \omega t) = -\mu_0\varepsilon_0\omega E_m\cos(kx - \omega t),$$

$$\frac{E_m}{B_m} = \frac{1}{\mu_0\varepsilon_0(\omega/k)} = \frac{1}{\mu_0\varepsilon_0 c}.$$

$$\boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \qquad \text{(wave speed)},}$$

33.5: Energy Transport and the Poynting Vector:

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \text{(Poynting vector).}$$

$$S = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{inst}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{inst}}.$$

$$S = \frac{1}{\mu_0}EB, \quad \Rightarrow \quad \boxed{S = \frac{1}{c\mu_0}E^2}$$

$$I = S_{\text{avg}} = \left(\frac{\text{energy/time}}{\text{area}}\right)_{\text{avg}} = \left(\frac{\text{power}}{\text{area}}\right)_{\text{avg}} = \frac{1}{c\mu_0}[E^2]_{\text{avg}} = \frac{1}{c\mu_0}[E_m^2 \sin^2(kx - \omega t)]_{\text{avg}}.$$

$$E_{\text{rms}} = \frac{E_m}{\sqrt{2}}. \quad \Rightarrow \quad \boxed{I = \frac{1}{c\mu_0}E_{\text{rms}}^2.}$$

The energy density u (= ½ $\varepsilon_o E^2$) within an electric field, can be written as:

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2 = \tfrac{1}{2}\varepsilon_0 (cB)^2. = \tfrac{1}{2}\varepsilon_0 \frac{1}{\mu_0 \varepsilon_0}B^2 = \frac{B^2}{2\mu_0}$$

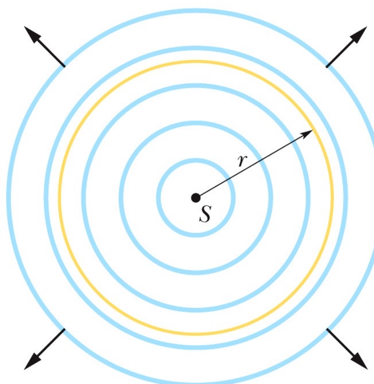
33-2 Energy Transport and The Poynting Vector

The Poynting Vector: The rate per unit area at which energy is transported via an electromagnetic wave is given by the Poynting vector

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The direction of the Poynting vector $\vec{S}$ of an electromagnetic wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

The energy emitted by light source S must pass through the sphere of radius r.



The time-averaged rate per unit area at which energy is transported is S_{avg}, which is called the intensity I of the wave:

$$S = \frac{1}{c\mu_0} E^2 \quad \text{(instantaneous energy flow rate).} \qquad (33\text{-}22)$$

A point source of electromagnetic waves emits the waves isotropically—that is, with equal intensity in all directions. The intensity of the waves at distance r from a point source of power P_s is

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2},$$

$$I = S_{avg} = \left(\frac{\text{energy/time}}{\text{area}}\right)_{avg} = \left(\frac{\text{power}}{\text{area}}\right)_{avg}.$$

From Eq. 33-22, we find

$$S = \frac{1}{c\mu_0}E^2 \quad \text{(instantaneous energy flow rate).} \tag{33-22}$$

$$I = S_{avg} = \frac{1}{c\mu_0}[E^2]_{avg} = \frac{1}{c\mu_0}[E_m^2 \sin^2(kx - \omega t)]_{avg}.$$

Over a full cycle, the average value of $\sin^2 \theta$, for any angular variable θ, is $\frac{1}{2}$ (see Fig. 31-17). In addition, we define a new quantity E_{rms}, the *root-mean-square* value of the electric field, as

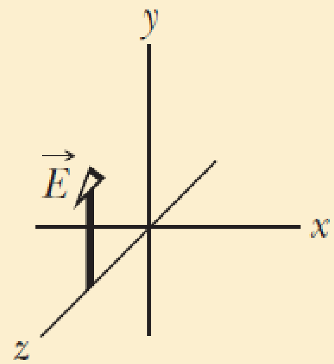
$$E_{rms} = \frac{E_m}{\sqrt{2}}. \tag{33-25}$$

$$I = \frac{1}{c\mu_0}E_{rms}^2.$$

$$I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\pi r^2},$$

Checkpoint 2

The figure here gives the electric field of an electromagnetic wave at a certain point and a certain instant. The wave is transporting energy in the negative z direction. What is the direction of the magnetic field of the wave at that point and instant?



positive direction of x

Ex: The oscillating magnetic field in a plane electromagnetic wave is given as $B_Y = 8\times10^{-6} \sin(5000\pi x - 3\times10^{11}\pi t)$ T.

(a) Frequency

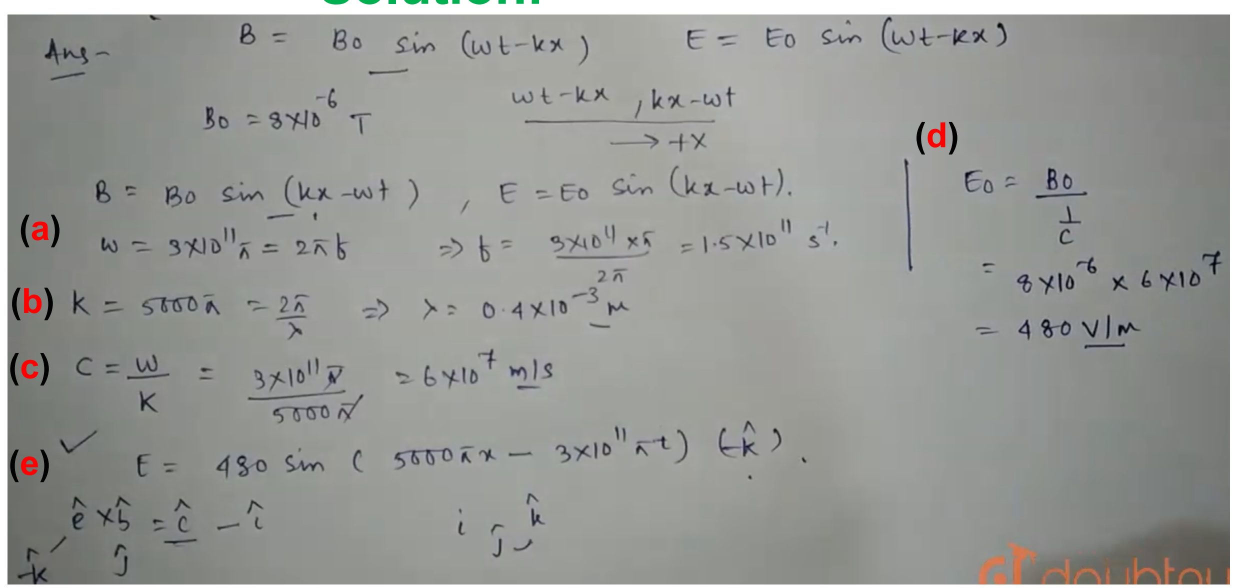
(b) Wavelength

(c) Speed of the wave

(d) Electric field

(e) Write down expression for oscillating electric field.

Solution:



Sample Problem 33.01 Light wave: rms values of the electric and magnetic fields

When you look at the North Star (Polaris), you intercept light from a star at a distance of 431 ly and emitting energy at a rate of 2.2×10^3 times that of our Sun ($P_{sun} = 3.90 \times 10^{26}$ W). Neglecting any atmospheric absorption, find the rms values of the electric and magnetic fields when the starlight reaches you.

KEY IDEAS

1. The rms value E_{rms} of the electric field in light is related to the intensity I of the light via Eq. 33-26 ($I = E_{rms}^2/c\mu_0$).
2. Because the source is so far away and emits light with equal intensity in all directions, the intensity I at any distance r from the source is related to the source's power P_s via Eq. 33-27 ($I = P_s/4\pi r^2$).
3. The magnitudes of the electric field and magnetic field of an electromagnetic wave at any instant and at any point in the wave are related by the speed of light c according to Eq. 33-5 ($E/B = c$). Thus, the rms values of those fields are also related by Eq. 33-5.

Electric field: Putting the first two ideas together gives us

$$I = \frac{P_s}{4\pi r^2} = \frac{E_{rms}^2}{c\mu_0}$$

and

$$E_{rms} = \sqrt{\frac{P_s c\mu_0}{4\pi r^2}}.$$

By substituting $P_s = (2.2 \times 10^3)(3.90 \times 10^{26}\ \text{W})$, $r = 431\ \text{ly} = 4.08 \times 10^{18}$ m, and values for the constants, we find

$$E_{rms} = 1.24 \times 10^{-3}\ \text{V/m} \approx 1.2\ \text{mV/m}. \qquad \text{(Answer)}$$

Magnetic field: From Eq. 33-5, we write

$$B_{rms} = \frac{E_{rms}}{c} = \frac{1.24 \times 10^{-3}\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}}$$
$$= 4.1 \times 10^{-12}\ \text{T} = 4.1\ \text{pT}.$$

Cannot compare the fields: Note that E_{rms} (= 1.2 mV/m) is small as judged by ordinary laboratory standards, but B_{rms} (= 4.1 pT) is quite small. This difference helps to explain why most instruments used for the detection and measurement of electromagnetic waves are designed to respond to the electric component. It is wrong, however, to say that the electric component of an electromagnetic wave is "stronger" than the magnetic component. You cannot compare quantities that are measured in different units. However, these electric and magnetic components are on an equal basis because their average energies, which *can* be compared, are equal.

Checkpoint 3

Light of uniform intensity shines perpendicularly on a totally absorbing surface, fully illuminating the surface. If the area of the surface is decreased, do (a) the radiation pressure and (b) the radiation force on the surface increase, decrease, or stay the same?

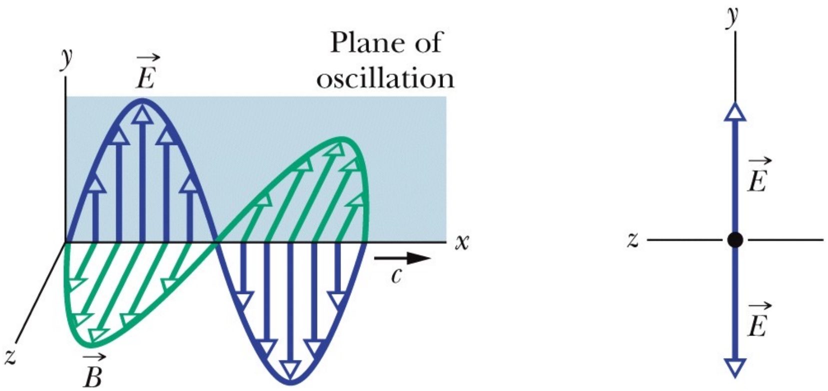
(a) same; (b) decrease

$$F = \frac{IA}{c}$$

$$p_r = \frac{I}{c}$$

33-4 Polarization

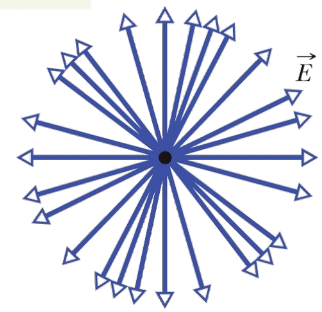
Electromagnetic waves are polarized if their electric field vectors are all in a single plane, called the plane of oscillation. Light waves from common sources are not polarized; that is, they are unpolarized, or polarized randomly.



(a) (b)

Vertically polarized light headed toward you—the electric fields are all vertical.

Unpolarized light headed toward you—the electric fields are in all directions in the plane.



(a)

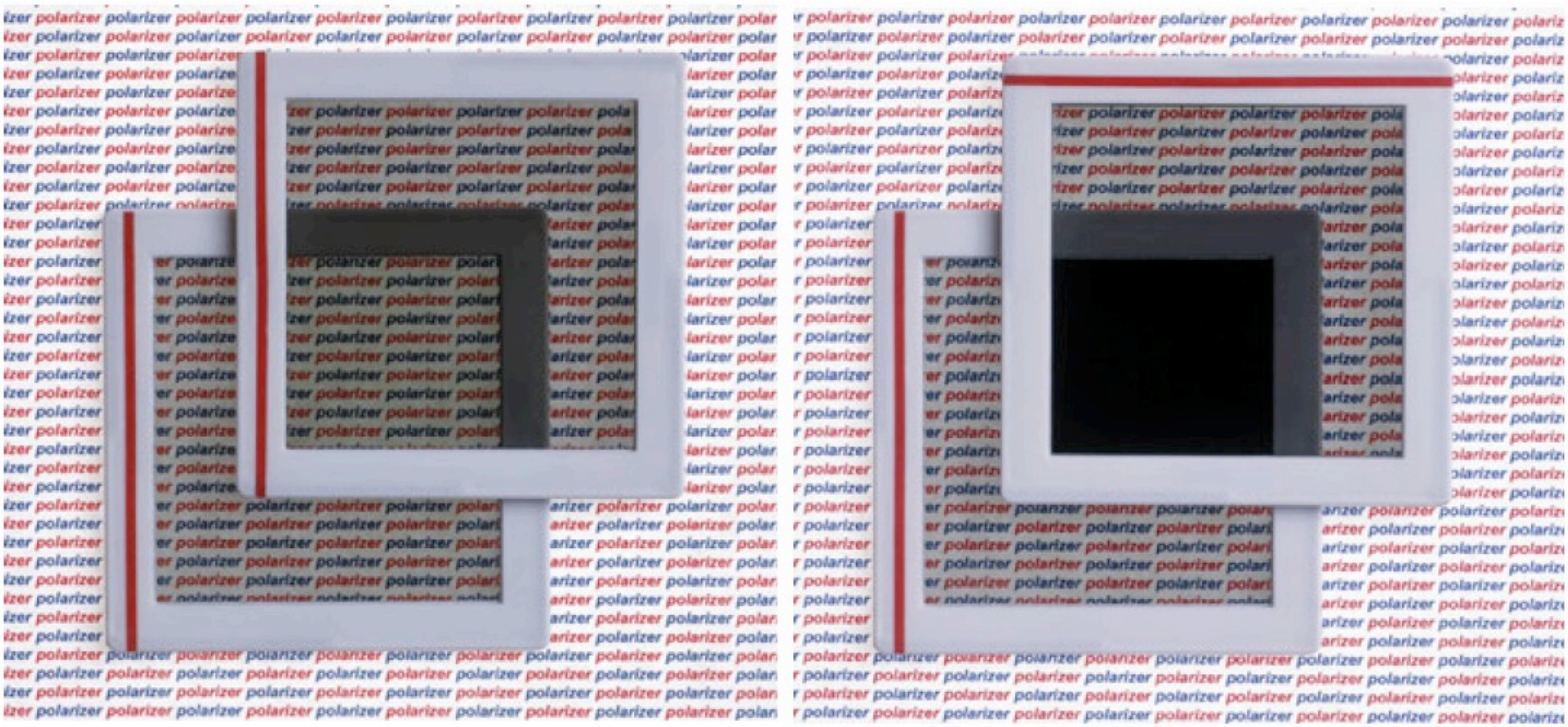
An electric field component parallel to the polarizing direction is passed (*transmitted*) by a polarizing sheet; a component perpendicular to it is absorbed.

If the original light is initially unpolarized, the transmitted intensity I is half the original intensity I_0:

$$I = \tfrac{1}{2} I_0 \quad \text{(one-half rule).}$$

If the original light is initially polarized, the transmitted intensity depends on the angle θ between the polarization direction of the original light and the polarizing direction of the sheet:

$$I = I_0 \cos^2 \theta \quad \text{(cosine-squared rule).}$$



Richard Megna/Fundamental Photographs

(*a*) (*b*)

Figure 33-14 (*a*) Overlapping polarizing sheets transmit light fairly well when their polarizing directions have the same orientation, but (*b*) they block most of the light when they are crossed.

The sheet's polarizing axis is vertical, so only vertically polarized light emerges.

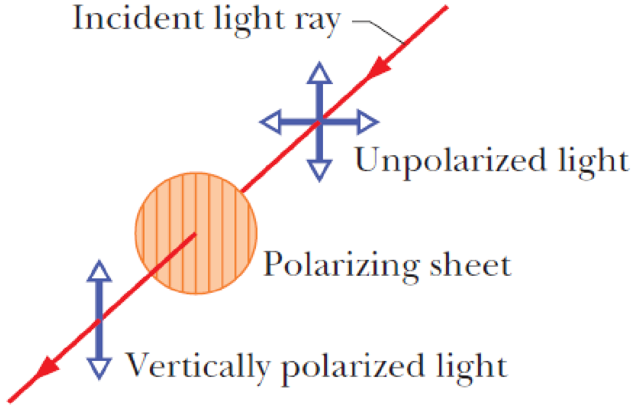
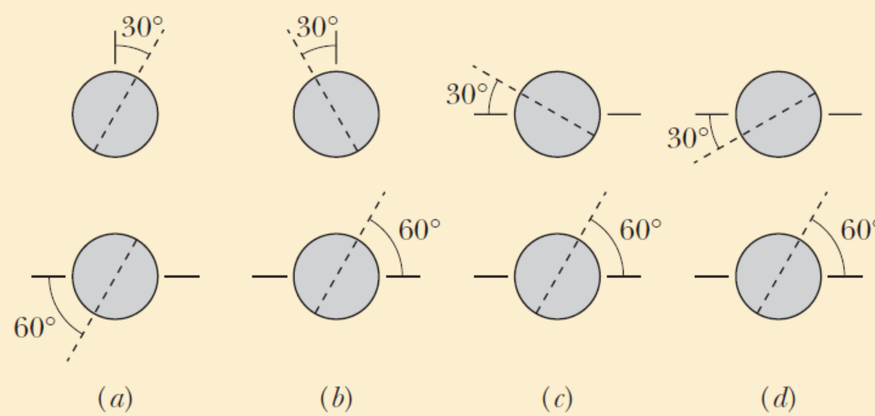


Figure 33-11 Unpolarized light becomes polarized when it is sent through a polarizing sheet. Its direction of polarization is then parallel to the polarizing direction of the sheet, which is represented here by the vertical lines drawn in the sheet.

Checkpoint 4

The figure shows four pairs of polarizing sheets, seen face-on. Each pair is mounted in the path of initially unpolarized light. The polarizing direction of each sheet (indicated by the dashed line) is referenced to either a horizontal x axis or a vertical y axis. Rank the pairs according to the fraction of the initial intensity that they pass, greatest first.



a, d, b, c (zero)

$$I = I_0 \cos^2 \theta \quad \text{(cosine-squared rule).}$$

Sample Problem 33.02 Polarization and intensity with three polarizing sheets

Figure 33-15*a*, drawn in perspective, shows a system of three polarizing sheets in the path of initially unpolarized light. The polarizing direction of the first sheet is parallel to the y axis, that of the second sheet is at an angle of 60° counter-clockwise from the y axis, and that of the third sheet is parallel to the x axis. What fraction of the initial intensity I_0 of the light emerges from the three-sheet system, and in which direction is that emerging light polarized?

KEY IDEAS

1. We work through the system sheet by sheet, from the first one encountered by the light to the last one.
2. To find the intensity transmitted by any sheet, we appl either the one-half rule or the cosine-squared rul depending on whether the light reaching the sheet i unpolarized or already polarized.
3. The light that is transmitted by a polarizing sheet is alway polarized parallel to the polarizing direction of the sheet.

First sheet: The original light wave is represented i Fig. 33-15*b*, using the head-on, double-arrow representatio of Fig. 33-10*b*. Because the light is initially unpolarized, th intensity I_1 of the light transmitted by the first sheet is give by the one-half rule (Eq. 33-36):

$$I_1 = \tfrac{1}{2} I_0.$$

Ex: A unpolarised light is passed through 3 polarises. If the second polariser is at an angle 30° with the first and the third polariser is at an angle 60° with the second. Find the final intensity of the light passed through this combination if initial intensity was I.

Solution

After passing through first polarizer $I = \dfrac{I_0}{2}$

After second polarizer $I = \dfrac{I_0}{2}\cos^2(30^\circ) = \dfrac{3}{8} I_0$

After third polarizer $I = \dfrac{3I_0}{8}\cos^2(60^\circ) = \dfrac{3I_0}{32}$

So final intensity $= \dfrac{3I_0}{32}$.

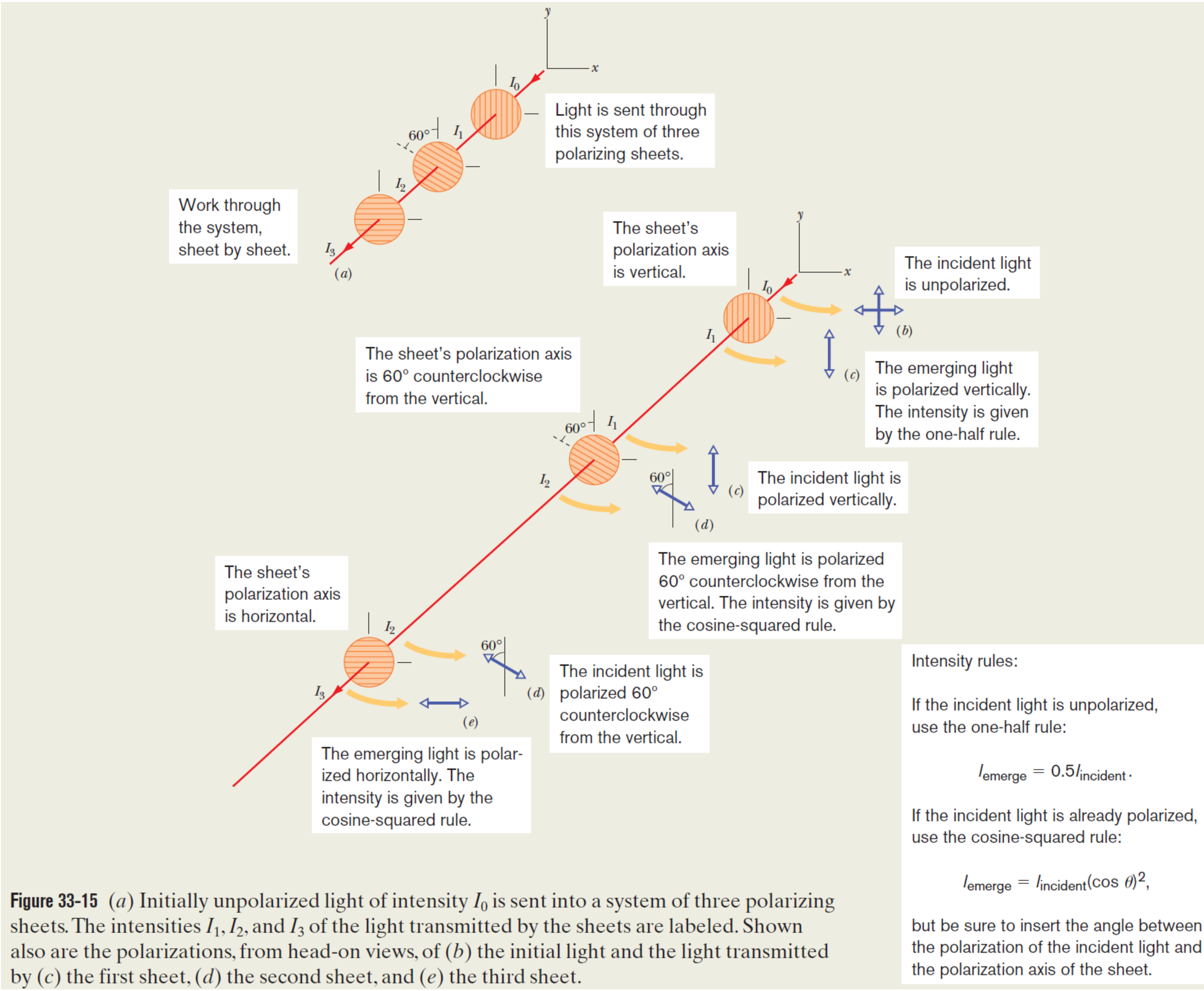


Figure 33-15 (*a*) Initially unpolarized light of intensity I_0 is sent into a system of three polarizing sheets. The intensities I_1, I_2, and I_3 of the light transmitted by the sheets are labeled. Shown also are the polarizations, from head-on views, of (*b*) the initial light and the light transmitted by (*c*) the first sheet, (*d*) the second sheet, and (*e*) the third sheet.

Because the polarizing direction of the first sheet is parallel to the y axis, the polarization of the light transmitted by it is also, as shown in the head-on view of Fig. 33-15*c*.

Second sheet: Because the light reaching the second sheet is polarized, the intensity I_2 of the light transmitted by that sheet is given by the cosine-squared rule (Eq. 33-38). The angle θ in the rule is the angle between the polarization direction of the entering light (parallel to the y axis) and the polarizing direction of the second sheet (60° counterclockwise from the y axis), and so θ is 60°. (The larger angle between the two directions, namely 120°, can also be used.) We have

$$I_2 = I_1 \cos^2 60°.$$

The polarization of this transmitted light is parallel to the polarizing direction of the sheet transmitting it—that is, 60° counterclockwise from the y axis, as shown in the head-on view of Fig. 33-15*d*.

Third sheet: Because the light reaching the third sheet is polarized, the intensity I_3 of the light transmitted by that sheet is given by the cosine-squared rule. The angle θ is now the angle between the polarization direction of the entering light (Fig. 33-15*d*) and the polarizing direction of the third sheet (parallel to the x axis), and so $\theta = 30°$. Thus,

$$I_3 = I_2 \cos^2 30°.$$

This final transmitted light is polarized parallel to the x axis (Fig. 33-15*e*). We find its intensity by substituting first for I_2 and then for I_1 in the equation above:

$$I_3 = I_2 \cos^2 30° = (I_1 \cos^2 60°) \cos^2 30°$$
$$= (\tfrac{1}{2} I_0) \cos^2 60° \cos^2 30° = 0.094 I_0.$$

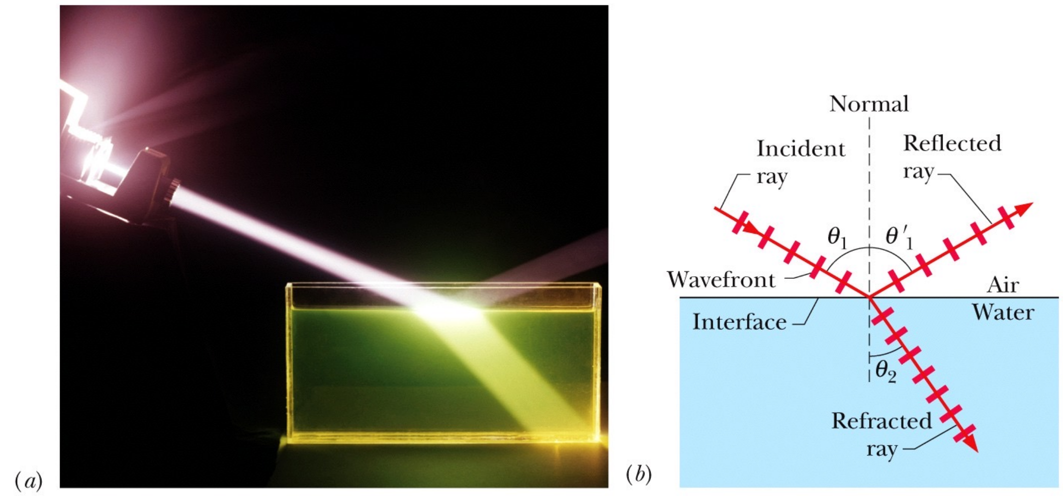
Thus, $$\frac{I_3}{I_0} = 0.094. \qquad \text{(Answer)}$$

That is to say, 9.4% of the initial intensity emerges from the three-sheet system. (If we now remove the second sheet, what fraction of the initial intensity emerges from the system?)

33-5 Reflection and Refraction

(a) A photograph showing an incident beam of light reflected and refracted by a horizontal water surface.

(b) A ray representation of (a). The angles of incidence (θ_1), reflection (θ'_1), and refraction (θ_2) are marked.



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When a light ray encounters a boundary between two transparent media, a reflected ray and a refracted ray generally appear as shown in figure above.

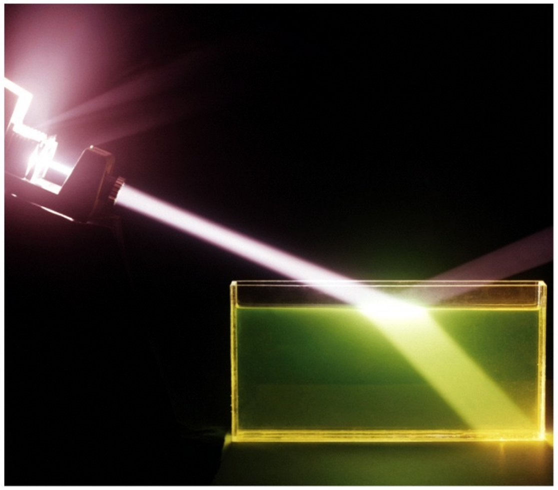
Law of reflection: A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence (both relative to the normal). In Fig. (b), this means that

$$\theta'_1 = \theta_1 \quad \text{(reflection).}$$

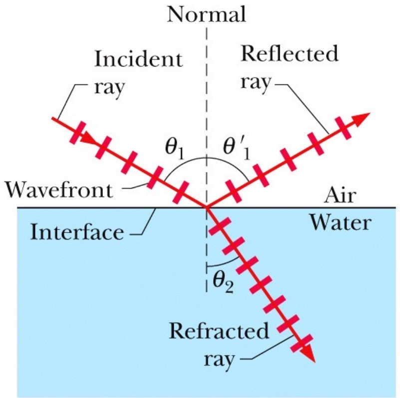
33-5 Reflection and Refraction

(a) A photograph showing an incident beam of light reflected and refracted by a horizontal water surface.
(b) A ray representation of (a). The angles of incidence (θ_1), reflection (θ'_1), and refraction (θ_2) are marked.

(a)



(b)



Law of refraction: A refracted ray lies in the plane of incidence and has an angle of refraction θ_2 that is related to the angle of incidence θ_1 by

$$n_2 \sin \theta_2 = n_1 \sin \theta_1$$

Here each of the symbols n_1 and n_2 is a dimensionless constant, called the **index of refraction**, that is associated with a medium involved in the refraction.

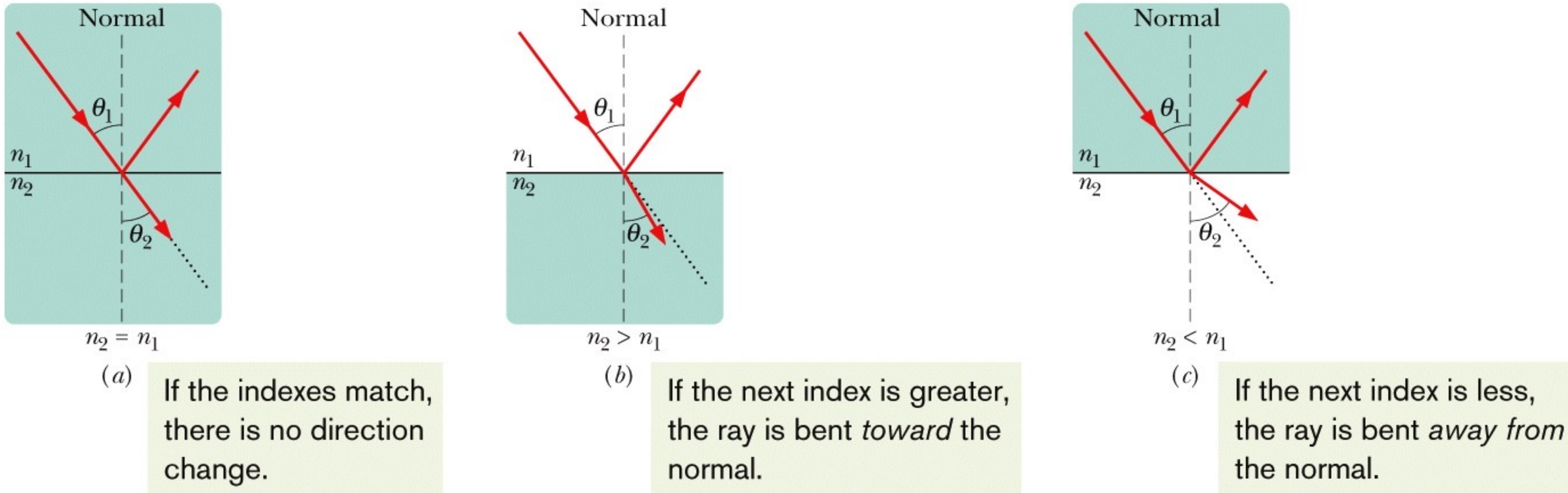
Table 33-1 Some Indexes of Refraction[a]

Medium	Index	Medium	Index
Vacuum	Exactly 1	Typical crown glass	1.52
Air (STP)[b]	1.00029	Sodium chloride	1.54
Water (20°C)	1.33	Polystyrene	1.55
Acetone	1.36	Carbon disulfide	1.63
Ethyl alcohol	1.36	Heavy flint glass	1.65
Sugar solution (30%)	1.38	Sapphire	1.77
Fused quartz	1.46	Heaviest flint glass	1.89
Sugar solution (80%)	1.49	Diamond	2.42

[a]For a wavelength of 589 nm (yellow sodium light).

[b]STP means "standard temperature (0°C) and pressure (1 atm)."

33-5 Reflection and Refraction

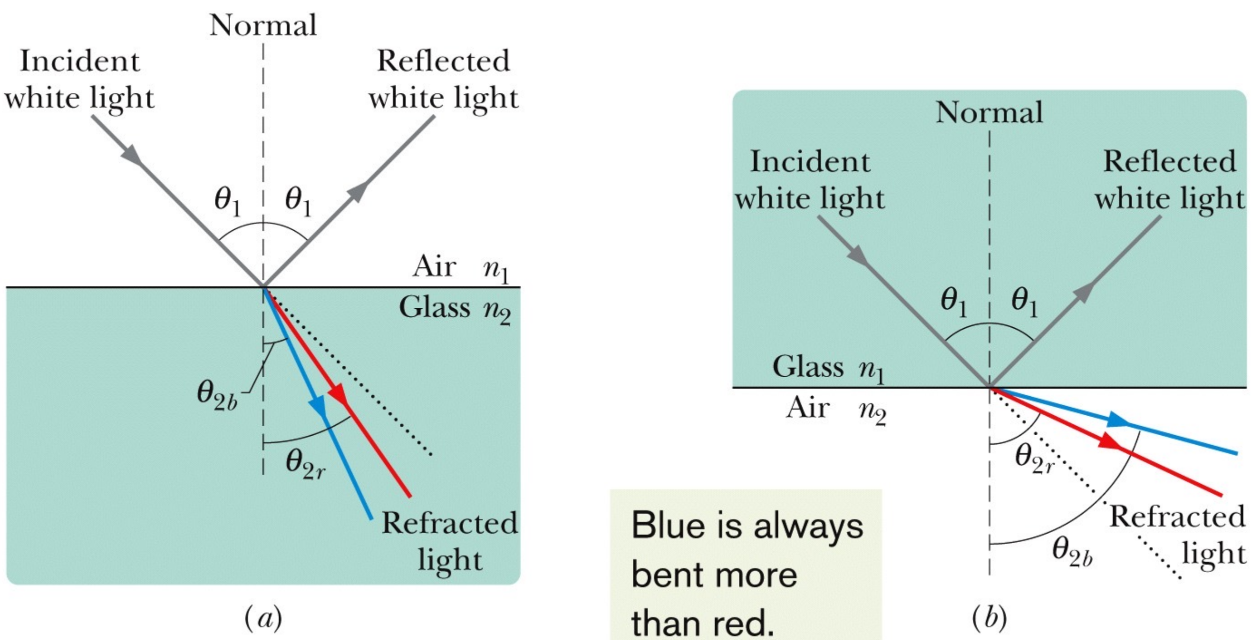


$$n_2 \sin \theta_2 = n_1 \sin \theta_1$$

1. If n_2 is equal to n_1, then θ_2 is equal to θ_1 and refraction does not bend the light beam, which continues in the undeflected direction, as in Fig. (a).
2. If n_2 is greater than n_1, then θ_2 is less than θ_1. In this case, refraction bends the light beam away from the undeflected direction and toward the normal, as in Fig. (b).
3. If n_2 is less than n_1, then θ_2 is greater than θ_1. In this case, refraction bends the light beam away from the undeflected direction and away from the normal, as in Fig. (c).

33-5 Reflection and Refraction

Chromatic dispersion of white light. The blue component is bent more than the red component. (a) Passing from air to glass, the blue component ends up with the smaller angle of refraction. (b) Passing from glass to air, the blue component ends up with the greater angle of refraction. Each dotted line represents the direction in which the light would continue to travel if it were not bent by the refraction.



Blue is always bent more than red.

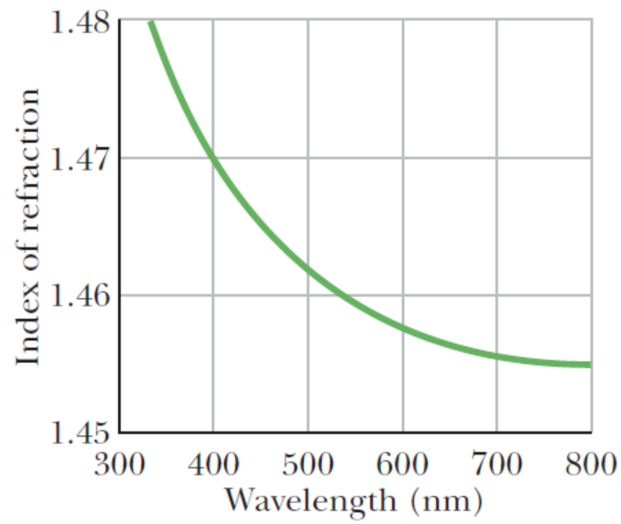
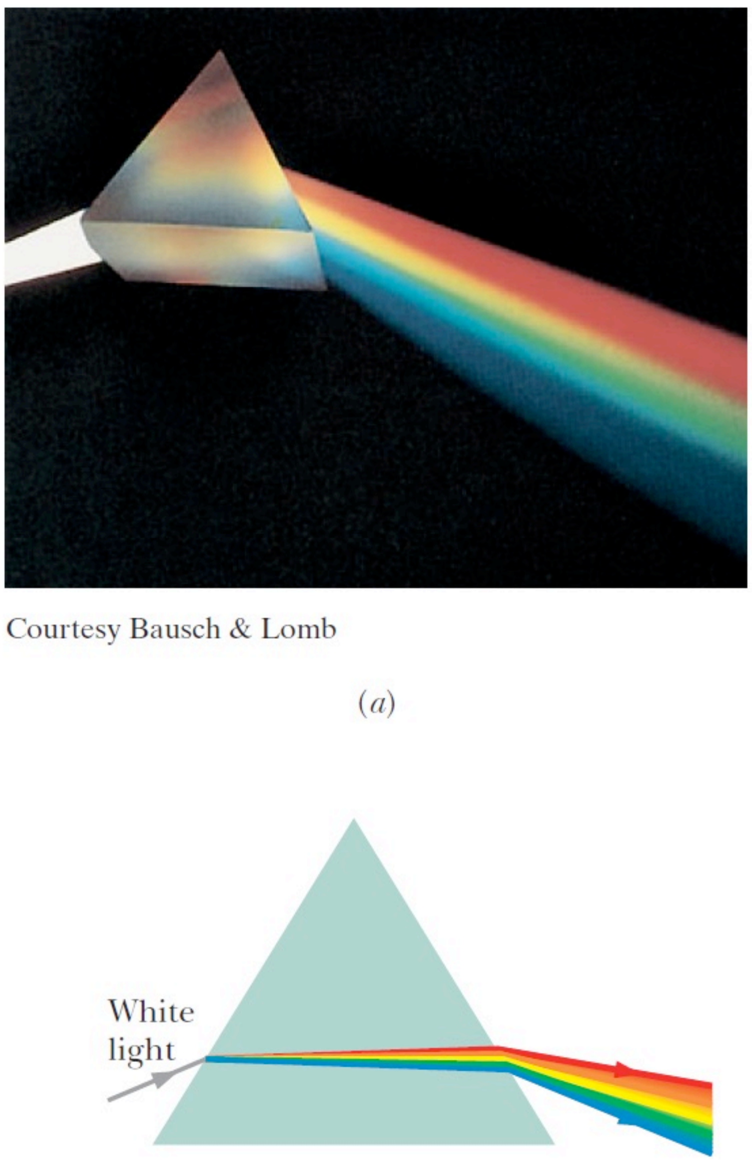


Figure 33-18 The index of refraction as a function of wavelength for fused quartz. The graph indicates that a beam of short-wavelength light, for which the index of refraction is higher, is bent more upon entering or leaving quartz than a beam of long-wavelength light.

To increase the color separation, we can use a solid glass prism with a triangular cross section, as in Fig. 33-20*a*. The dispersion at the first surface (on the left in Figs. 33-20*a*, *b*) is then enhanced by the dispersion at the second surface.



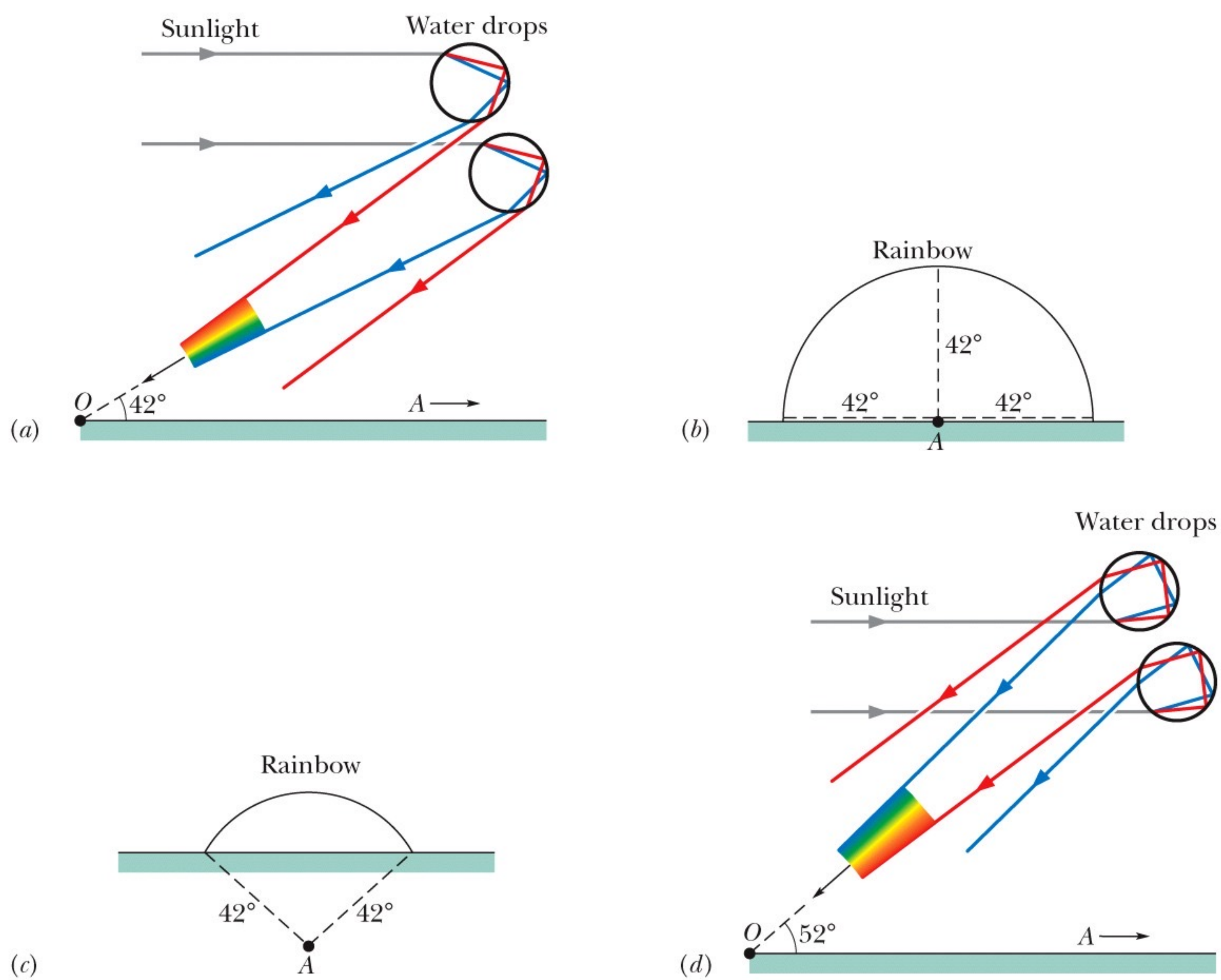
Courtesy Bausch & Lomb

(*a*)

(*b*)

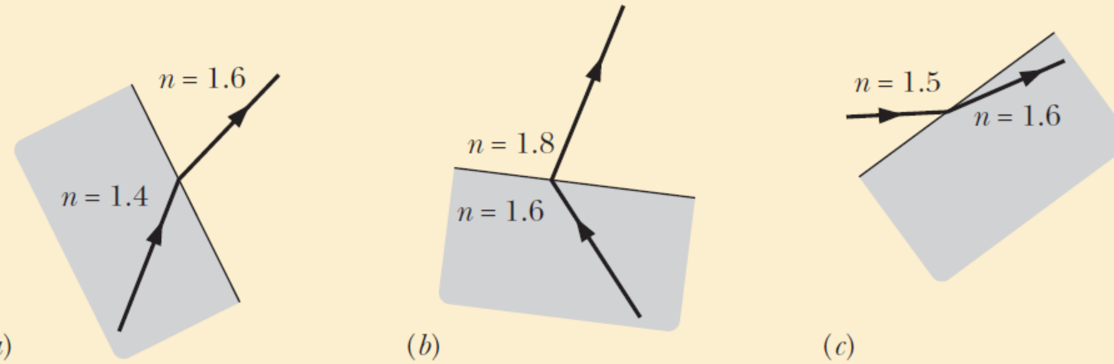
Figure 33-20 (*a*) A triangular prism separating white light into its component colors. (*b*) Chromatic dispersion occurs at the first surface and is increased at the second surface.

Rainbow: (a) The separation of colors when sunlight refracts into and out of falling raindrops leads to a primary rainbow. The *antisolar point A* is on the horizon at the right. The rainbow colors appear at an angle of 42° from the direction of A. (b) Drops at 42° from A in any direction can contribute to the rainbow. (c) The rainbow arc when the Sun is higher (and thus A is lower). (d) The separation of colors leading to a secondary rainbow.



Checkpoint 5

Which of the three drawings here (if any) show physically possible refraction?



(a) (b) (c)

a

Sample Problem 33.03 Reflection and refraction of a monochromatic beam

(a) In Fig. 33-22*a*, a beam of monochromatic light reflects and refracts at point A on the interface between material 1 with index of refraction $n_1 = 1.33$ and material 2 with index of refraction $n_2 = 1.77$. The incident beam makes an angle of 50° with the interface. What is the angle of reflection at point A? What is the angle of refraction there?

KEY IDEAS

(1) The angle of reflection is equal to the angle of incidence, and both angles are measured relative to the normal to the surface at the point of reflection. (2) When light reaches the interface between two materials with different indexes of refraction (call them n_1 and n_2), part of the light can be refracted by the interface according to Snell's law, Eq. 33-40:

$$n_2 \sin \theta_2 = n_1 \sin \theta_1, \qquad (33\text{-}42)$$

where both angles are measured relative to the normal at the point of refraction.

Calculations: In Fig. 33-22*a*, the normal at point A is drawn as a dashed line through the point. Note that the angle of incidence θ_1 is not the given 50° but is $90° - 50° = 40°$. Thus, the angle of reflection is

$$\theta_1' = \theta_1 = 40°. \qquad \text{(Answer)}$$

The light that passes from material 1 into material 2 undergoes refraction at point A on the interface between the two materials. Again we measure angles between light rays and a normal, here at the point of refraction. Thus, in Fig. 33-22*a*, the angle of refraction is the angle marked θ_2. Solving Eq. 33-42 for θ_2 gives us

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) = \sin^{-1}\left(\frac{1.33}{1.77}\sin 40°\right)$$

$$= 28.88° \approx 29°. \qquad \text{(Answer)}$$

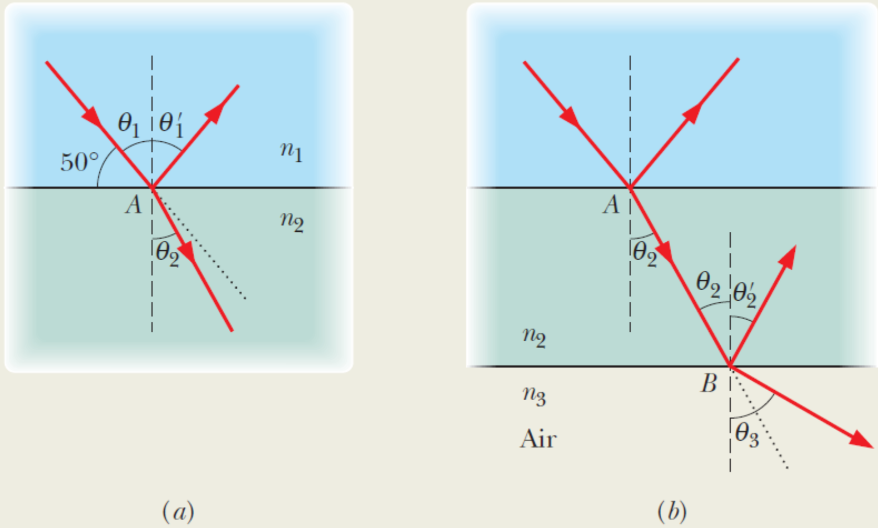


Figure 33-22 (*a*) Light reflects and refracts at point A on the interface between materials 1 and 2. (*b*) The light that passes through material 2 reflects and refracts at point B on the interface between materials 2 and 3 (air). Each dashed line is a normal. Each dotted line gives the incident direction of travel.

This result means that the beam swings toward the normal (it was at 40° to the normal and is now at 29°). The reason is that when the light travels across the interface, it moves into a material with a greater index of refraction. *Caution:* Note that the beam does *not* swing through the normal so that it appears on the left side of Fig. 33-22*a*.

(b) The light that enters material 2 at point A then reaches point B on the interface between material 2 and material 3, which is air, as shown in Fig. 33-22*b*. The interface through B is parallel to that through A. At B, some of the light reflects and the rest enters the air. What is the angle of reflection? What is the angle of refraction into the air?

Calculations: We first need to relate one of the angles at point B with a known angle at point A. Because the interface through point B is parallel to that through point A, the incident angle at B must be equal to the angle of refraction θ_2, as shown in Fig. 33-22*b*. Then for reflection, we again use the law of reflection. Thus, the angle of reflection at B is

$$\theta_2' = \theta_2 = 28.88° \approx 29°. \qquad \text{(Answer)}$$

Next, the light that passes from material 2 into the air undergoes refraction at point B, with refraction angle θ_3. Thus, we again apply Snell's law of refraction, but this time we write Eq. 33-40 as

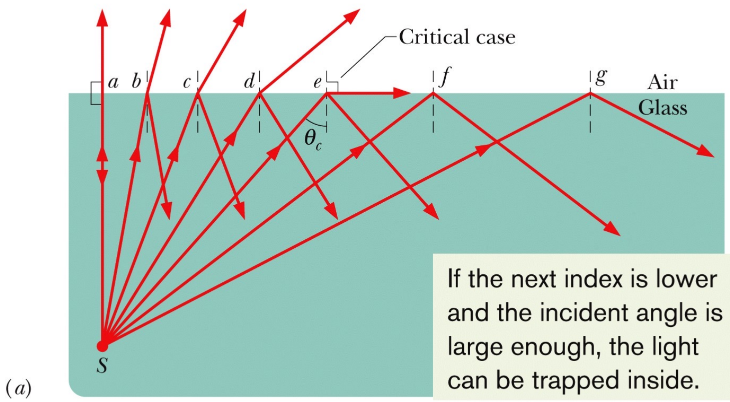
$$n_3 \sin \theta_3 = n_2 \sin \theta_2. \qquad (33\text{-}43)$$

Solving for θ_3 then leads to

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3} \sin \theta_2\right) = \sin^{-1}\left(\frac{1.77}{1.00} \sin 28.88°\right)$$
$$= 58.75° \approx 59°. \qquad \text{(Answer)}$$

Thus, the beam swings away from the normal (it was at 29° to the normal and is now at 59°) because it moves into a material (air) with a lower index of refraction.

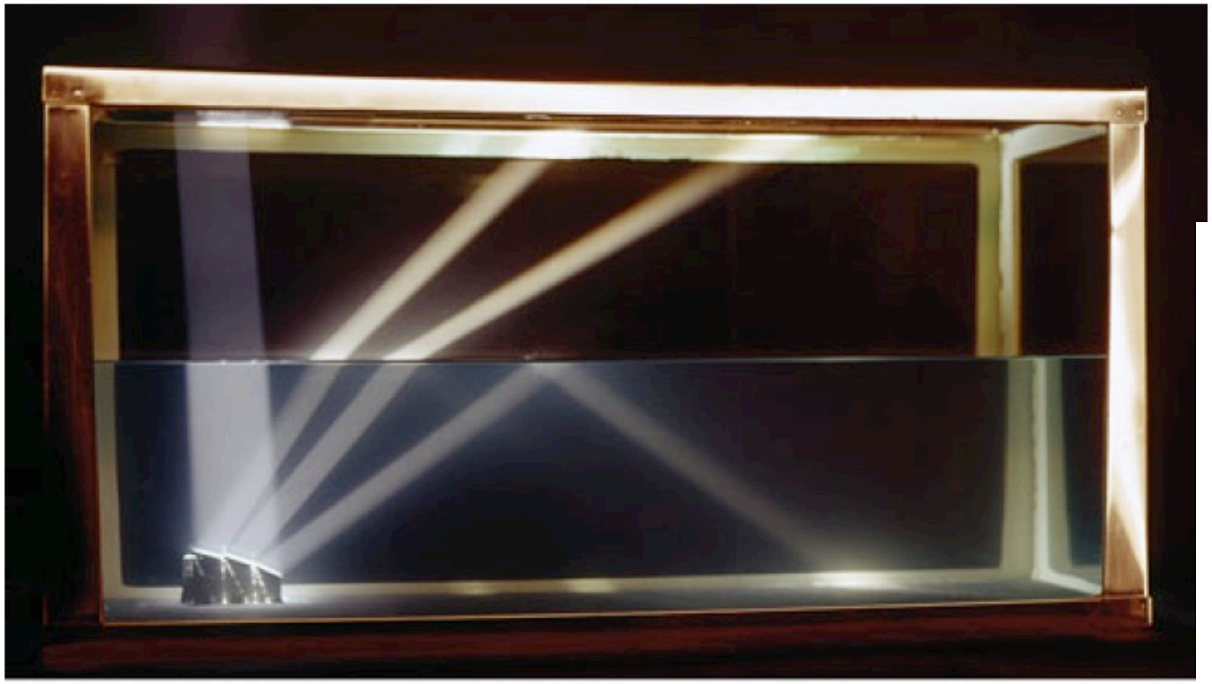
33-6 Total Internal Refraction



Ken Kay/Fundamental Photographs

(a) Total internal reflection of light from a point source S in glass occurs for all angles of incidence greater than the critical angle uc. At the critical angle, the refracted ray points along the air – glass interface. (b) A source in a tank of water.

Figure (a) shows rays of monochromatic light from a point source S in glass incident on the interface between the glass and air. For ray a, which is perpendicular to the interface, part of the light reflects at the interface and the rest travels through it with no change in direction. For rays b through e, which have progressively larger angles of incidence at the interface, there are also both reflection and refraction at the interface. As the angle of incidence increases, the angle of refraction increases; for ray e it is 90°, which means that the refracted ray points directly along the interface. The angle of incidence giving this situation is called the **critical angle θ_c**. For angles of incidence larger than θ_c, such as for rays f and g, there is no refracted ray and all the light is reflected; this effect is called **total internal reflection** because all the light remains inside the glass.



Ken Kay/Fundamental Photographs

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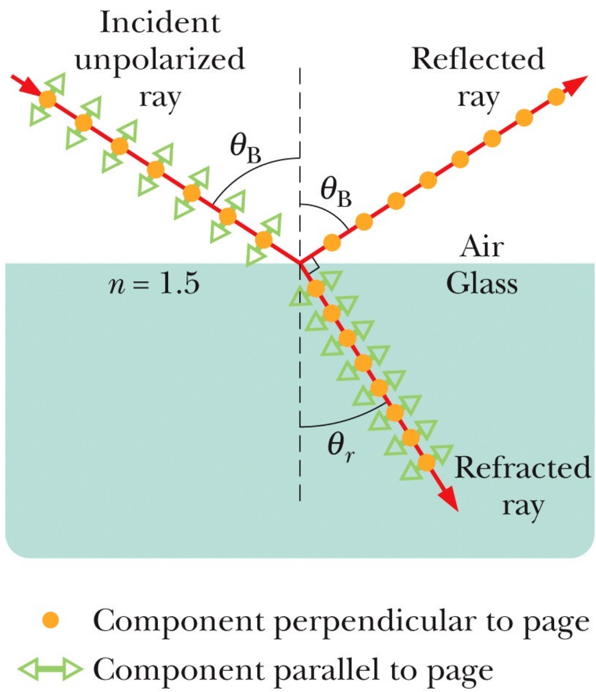
Figure 33-24 An endoscope used to inspect an artery.

33-7 Polarization by Reflection



A ray of unpolarized light in air is incident on a glass surface at the **Brewster angle** θ_B. The electric fields along that ray have been resolved into components perpendicular to the page (the plane of incidence, reflection, and refraction) and components parallel to the page. The reflected light consists only of components perpendicular to the page and is thus polarized in that direction. The refracted light consists of the original components parallel to the page and weaker components perpendicular to the page; this light is partially polarized.

As shown in the figure above a reflected wave will be fully polarized, with its ***E*** vectors perpendicular to the plane of incidence, if it strikes a boundary at the Brewster angle θ_B, where

$$\theta_B = \tan^{-1}\frac{n_2}{n_1} \quad \text{(Brewster angle).}$$

33 Summary

Electromagnetic Waves

- An electromagnetic wave consists of oscillating electric and magnetic fields as given by,

$$E = E_m \sin(kx - \omega t)$$ **Eq. 33-1**

$$B = B_m \sin(kx - \omega t),$$ **Eq. 33-2**

- The speed of any electromagnetic wave in vacuum is *c*, which can be written as

$$c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$ **Eq. 33-5&3**

Energy Flow

- The rate per unit area at which energy is trans- ported via an electromagnetic wave is given by the Poynting vector ***S***:

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}.$$ **Eq. 33-19**

- The intensity *I* of the wave is:

$$I = \frac{1}{c\mu_0} E_{rms}^2$$ **Eq. 33-26**

- The intensity of the waves at distance r from a point source of power Ps is

$$I = \frac{P_s}{4\pi r^2}.$$ **Eq. 33-27**

Radiation Pressure

- If the radiation is totally absorbed by the surface, the force is

$$F = \frac{IA}{c}$$ **Eq. 33-32**

- If the radiation is totally absorbed by the surface, the force is

$$F = \frac{2IA}{c}$$ **Eq. 33-33**

33 Summary

Radiation Pressure

- The radiation pressure p_r is the force per unit area.
- For total absorption

$$p_r = \frac{I}{c}$$ **Eq. 33-34**

- For total reflection back along path,

$$p_r = \frac{2I}{c}$$ **Eq. 33-35**

Polarization

- Electromagnetic waves are polarized if their electric field vectors are all in a single plane, called the plane of oscillation.
- If the original light is initially unpolarized, the transmitted intensity I is

$$I = \tfrac{1}{2}I_0.$$ **Eq. 33-36**

- If the original light is initially polarized, the transmitted intensity depends on the angle u between the polarization direction of the original light (the axis along which the fields oscillate) and the polarizing direction of the sheet:

$$I = I_0 \cos^2 \theta.$$ **Eq. 33-26**

Reflection and Refraction

- The angle of reflection is equal to the angle of incidence, and the angle of refraction is related to the angle of incidence by Snell's law,

$$n_2 \sin \theta_2 = n_1 \sin \theta_1$$ **Eq. 33-40**

33 Summary

Total Internal Reflection

- A wave encountering a boundary across which the index of refraction decreases will experience total internal reflection if the angle of incidence exceeds a critical angle,

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$ **Eq. 33-45**

Polarization by Reflection

- A reflected wave will be fully polarized, if the incident, unpolarized wave strikes a boundary at the Brewster angle

$$\theta_B = \tan^{-1}\frac{n_2}{n_1}$$ **Eq. 33-49**