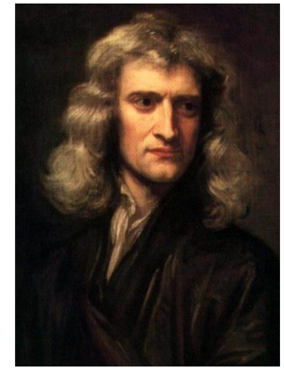
Chapter 35

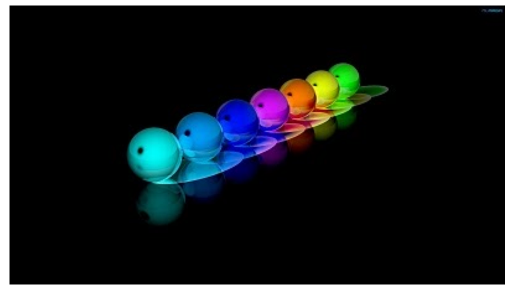
Interference

Light as a Wave diffraction
Young’s Interference
Interference and Double-Slit Intensity

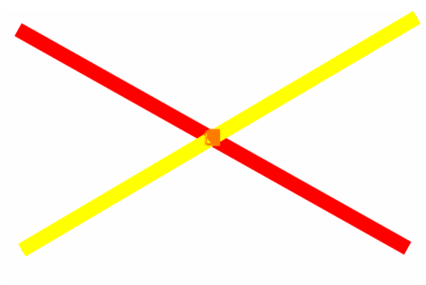
Timeline of light: particle or a wave

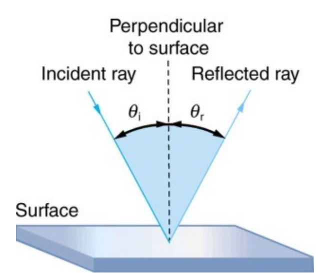


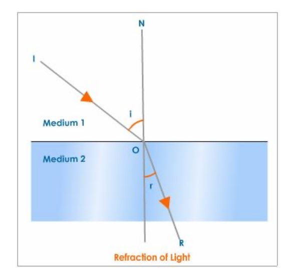
Isaac Newton (1642-1727): Light is a stream of very small particles emitted from source of light and travelling in a straight line. (PARTICLE nature of light).



Christian Huygens (1629-1695): Observed when two light beams intersect they emerge unmodified, he concluded that light is a WAVE. Also, he derived laws of reflection, refraction.







Timeline of light: Burial of light is “Particles only” opinion

Thomas Young (1773-1829): double slit exp. (WAVE nature of light)

Augustin Fresnel (1788 -1827): Light is a transverse WAVE-light polarization.

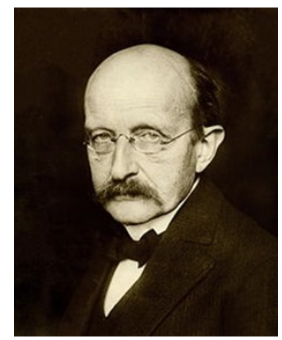
James Maxwell (1831-1879): Employed Maxwell’s electromagnetic wave equations to predict speed of light (WAVE).

Max Planck (1900): Derived correct blackbody radiation curve by assuming that atoms emit light in quanta (discrete energy chunks)

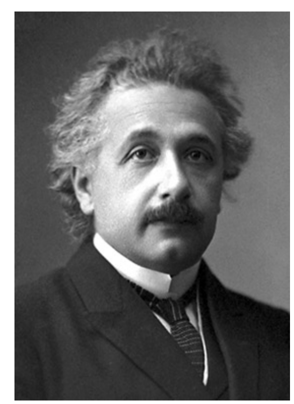
Birth of Quantum mechanics

$$E = hv$$

$h = 6.63 \text{ X } 10 - 34$ J.s (Planck’s constant)



Albert Einstein (1905): Photo-electric effect and Special theory of relativity [light is a stream of light quanta of Planck’s frequency]



35.2: Light as a Wave:

Huygen's Principle:

All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

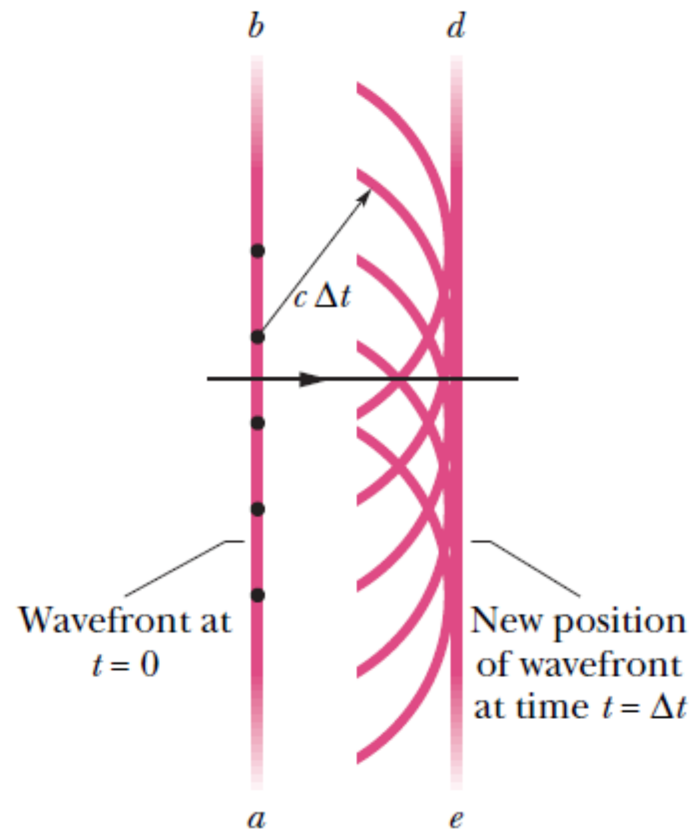


Fig. 35-2 The propagation of a plane wave in vacuum, as portrayed by Huygens' principle.

35.2: Light as a Wave, Law of Refraction:

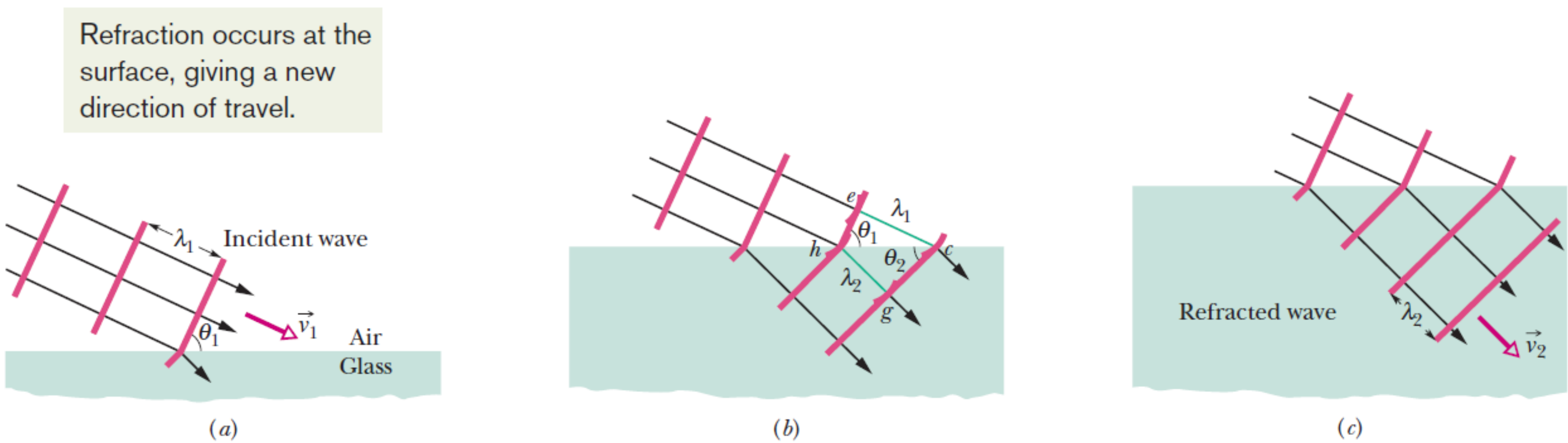


Fig. 35-3 The refraction of a plane wave at an air–glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (*a*) through (*c*) represent three successive stages of the refraction.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}, \qquad \sin\theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle } hce\text{)} \qquad \sin\theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle } hcg\text{)}.$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}. \qquad n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}. \qquad \frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{(law of refraction),}}$$

35.2: Light as a Wave, Wavelength and Law of Refraction:

$$\lambda_n = \lambda \frac{v}{c}.$$

$$\lambda_n = \frac{\lambda}{n}.$$

$$f_n = \frac{v}{\lambda_n}.$$

$$f_n = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f,$$

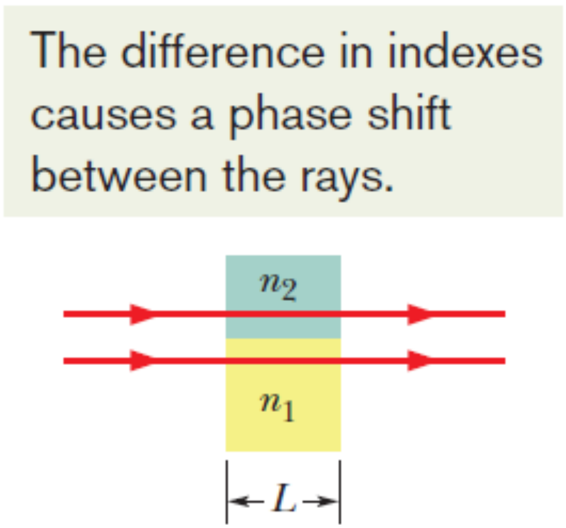


Fig. 35-4 Two light rays travel through two media having different indexes of refraction.

35.2: Light as a Wave, Rainbows and Optical Interference:

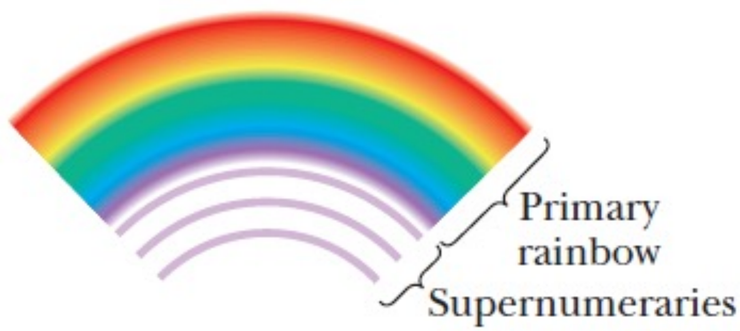


Fig. 35-5 A primary rainbow and the faint supernumeraries below it are due to optical interference.

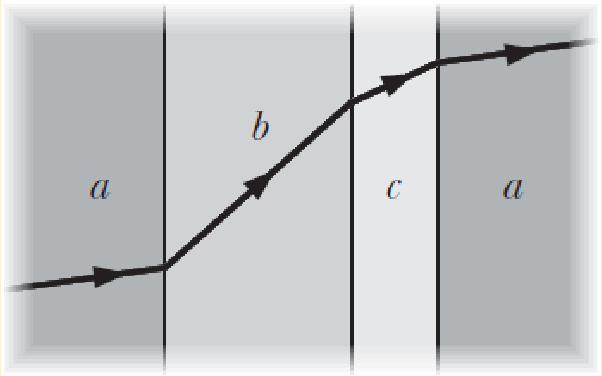
Light waves pass into a water drop along the entire side that faces the Sun. Different parts of an incoming wave will travel different paths within the drop.

That means waves will emerge from the drop with different phases. Thus, we can see that at some angles the emerging light will be in phase and give constructive interference.

The rainbow is the result of such constructive interference.

Checkpoint 1

The figure shows a monochromatic ray of light traveling across parallel interfaces, from an original material a, through layers of materials b and c, and then back into material a. Rank the materials according to the speed of light in them, greatest first.



$$b \text{ (least } n\text{)}, c, a$$

Wavelength and Index of Refraction

monochromatic light have wavelength λ and speed c in vacuum and wavelength λ_n and speed v in a medium with an index of refraction n. Now we can rewrite Eq. 35-1 as

$$\lambda_n = \lambda \frac{v}{c}. \tag{35-5}$$

Using Eq. 35-3 to substitute $1/n$ for v/c then yields

$$\lambda_n = \frac{\lambda}{n}. \tag{35-6}$$

This equation relates the wavelength of light in any medium to its wavelength in vacuum: A greater index of refraction means a smaller wavelength.

Next, let f_n represent the frequency of the light in a medium with index of refraction n. Then from the general relation of Eq. 16-13 ($v = \lambda f$), we can write

$$f_n = \frac{v}{\lambda_n}.$$

Substituting Eqs. 35-3 and 35-6 then gives us

$$f_n = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f,$$

where f is the frequency of the light in vacuum. Thus, although the speed and wavelength of light in the medium are different from what they are in vacuum, *the frequency of the light in the medium is the same as it is in vacuum.*

Phase Difference.

The phase difference between two light waves can change if the waves travel through different materials having different indexes of refraction.

The difference in indexes causes a phase shift between the rays.

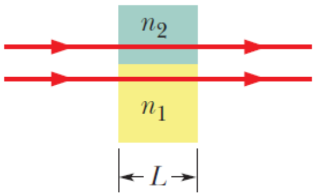


Figure 35-4 Two light rays travel through two media having different indexes of refraction.

To find their new phase difference in terms of wavelengths, we first count the number N_1 of wavelengths there are in the length L of medium 1. From Eq. 35-6, the wavelength in medium 1 is $\lambda_{n1} = \lambda/n_1$; so

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}. \tag{35-7}$$

Similarly, we count the number N_2 of wavelengths there are in the length L of medium 2, where the wavelength is $\lambda_{n2} = \lambda/n_2$:

$$N_2 = \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda}. \tag{35-8}$$

To find the new phase difference between the waves, we subtract the smaller of N_1 and N_2 from the larger. Assuming $n_2 > n_1$, we obtain

$$N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1). \tag{35-9}$$

Path Length Difference.

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \quad \text{(fully constructive interference)}, \tag{35-10}$$

and that fully destructive interference (darkness) occurs when

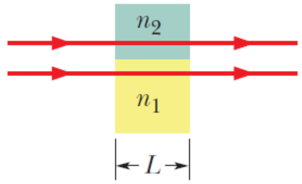
$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \quad \text{(fully destructive interference)}. \tag{35-11}$$

Intermediate values correspond to intermediate interference and thus also illumination.

Checkpoint 2

The light waves of the rays in Fig. 35-4 have the same wavelength and amplitude and are initially in phase. (a) If 7.60 wavelengths fit within the length of the top material and 5.50 wavelengths fit within that of the bottom material, which material has the greater index of refraction? (b) If the rays are angled slightly so that they meet at the same point on a distant screen, will the interference there result in the brightest possible illumination, bright intermediate illumination, dark intermediate illumination, or darkness?

The difference in indexes causes a phase shift between the rays.



(a) top; (b) bright intermediate illumination (phase difference is 2.1 wavelengths)

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}.$$

Sample Problem 35.01 Phase difference of two waves due to difference in refractive indexes

In Fig. 35-4, the two light waves that are represented by the rays have wavelength 550.0 nm before entering media 1 and 2. They also have equal amplitudes and are in phase. Medium 1 is now just air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm.

(a) What is the phase difference of the emerging waves in wavelengths, radians, and degrees? What is their effective phase difference (in wavelengths)?

KEY IDEA

The phase difference of two light waves can change if they travel through different media, with different indexes of refraction. The reason is that their wavelengths are different in the different media. We can calculate the change in phase difference by counting the number of wavelengths that fits into each medium and then subtracting those numbers.

Calculations: When the path lengths of the waves in the two media are identical, Eq. 35-9 gives the result of the subtraction. Here we have $n_1 = 1.000$ (for the air), $n_2 = 1.600$, $L = 2.600\ \mu$m, and $\lambda = 550.0$ nm. Thus, Eq. 35-9 yields

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$$

$$= \frac{2.600 \times 10^{-6}\ \text{m}}{5.500 \times 10^{-7}\ \text{m}}(1.600 - 1.000)$$

$$= 2.84. \qquad \text{(Answer)}$$

Thus, the phase difference of the emerging waves is 2.84 wave lengths. Because 1.0 wavelength is equivalent to 2π rad an 360°, you can show that this phase difference is equivalent to

$$\text{phase difference} = 17.8\ \text{rad} \approx 1020°. \qquad \text{(Answe}$$

The effective phase difference is the decimal part c the actual phase difference *expressed in wavelengths.* Thu we have

$$\text{effective phase difference} = 0.84\ \text{wavelength}. \qquad \text{(Answe}$$

You can show that this is equivalent to 5.3 rad and abou 300°. *Caution:* We do *not* find the effective phase differenc by taking the decimal part of the actual phase difference a expressed in radians or degrees. For example, we do *not* tak 0.8 rad from the actual phase difference of 17.8 rad.

(b) If the waves reached the same point on a distant screer what type of interference would they produce?

Reasoning: We need to compare the effective phase differ ence of the waves with the phase differences that give th extreme types of interference. Here the effective phase di ference of 0.84 wavelength is between 0.5 wavelength (fo fully destructive interference, or the darkest possible result and 1.0 wavelength (for fully constructive interference, c the brightest possible result), but closer to 1.0 wavelengt Thus, the waves would produce intermediate interferenc that is closer to fully constructive interference—they woul produce a relatively bright spot.

35.3: Diffraction:

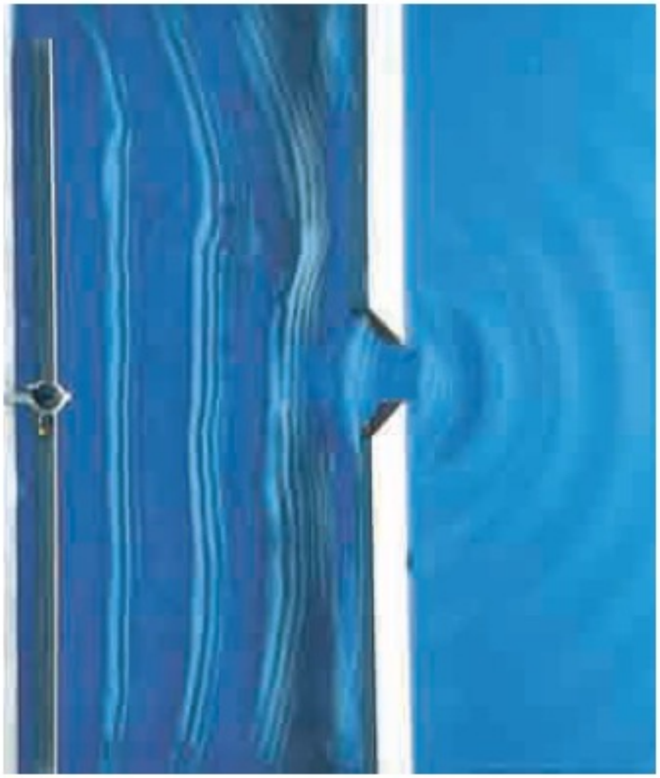


Fig. 35-6 Waves produced by an oscillating paddle at the left flare out through an opening in a barrier along the water surface. *(Runk Schoenberger/Grant Heilman Photography)*

If a wave encounters a barrier that has an opening of dimensions similar to the wavelength, the part of the wave that passes through the opening will flare (spread) out—will *diffract*—into the region beyond the barrier. The flaring is consistent with the spreading of wavelets according to Huygens principle. Diffraction occurs for waves of all types.

35-2 Young's Interference

The flaring is consistent with the spreading of wavelets in the Huygens construction. **Diffraction** occurs for waves of all types, not just light waves. Figure below shows waves passing through a slit flares.

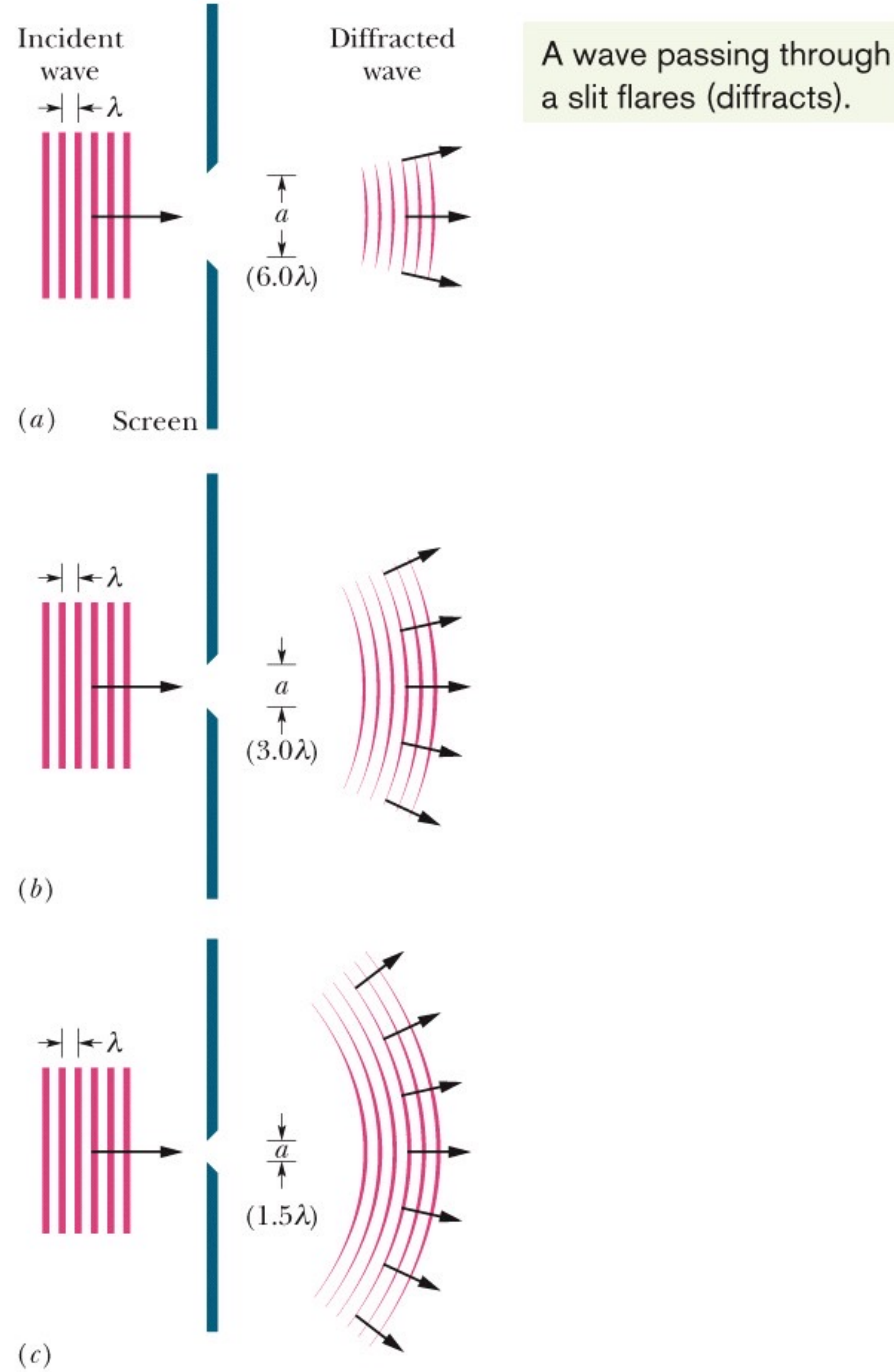
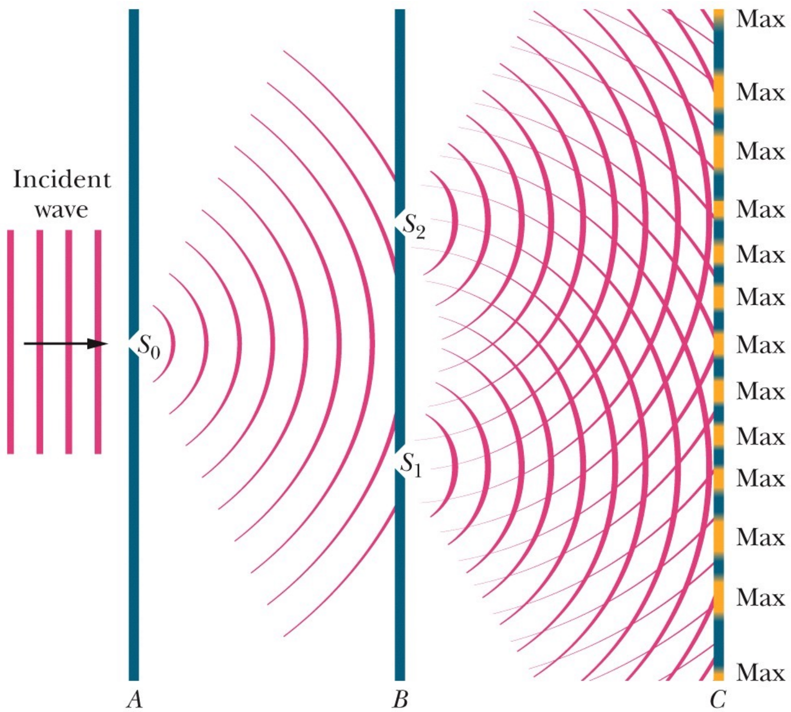


Figure (a) shows the situation schematically for an incident plane wave of wavelength λ encountering a slit that has width $a = 6.0\ \lambda$ and extends into and out of the page. The part of the wave that passes through the slit flares out on the far side. Figures (b) (with a = 3.0 λ) and (c) (a = 1.5λ) illustrate the main feature of diffraction: the narrower the slit, the greater the diffraction.

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

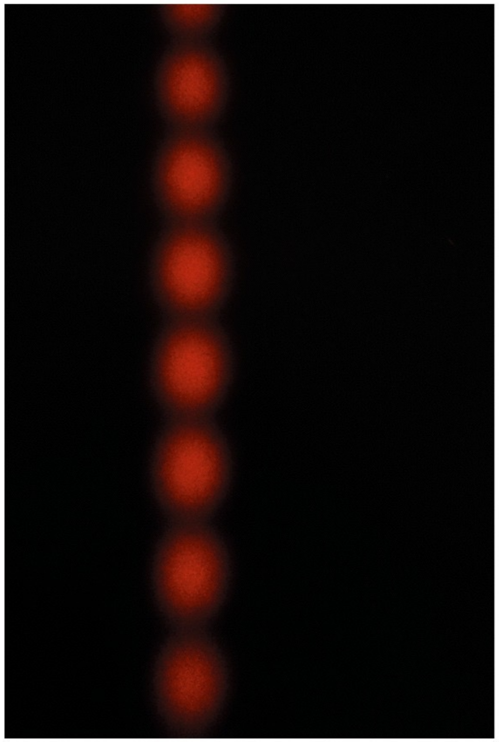
35-2 Young's Interference

Figure gives the basic arrangement of Young's experiment. Light from a distant monochromatic source illuminates slit S_0 in screen A. The emerging light then spreads via diffraction to illuminate two slits S_1 and S_2 in screen B. Diffraction of the light by these two slits sends overlapping circular waves into the region beyond screen B, where the waves from one slit interfere with the waves from the other slit.



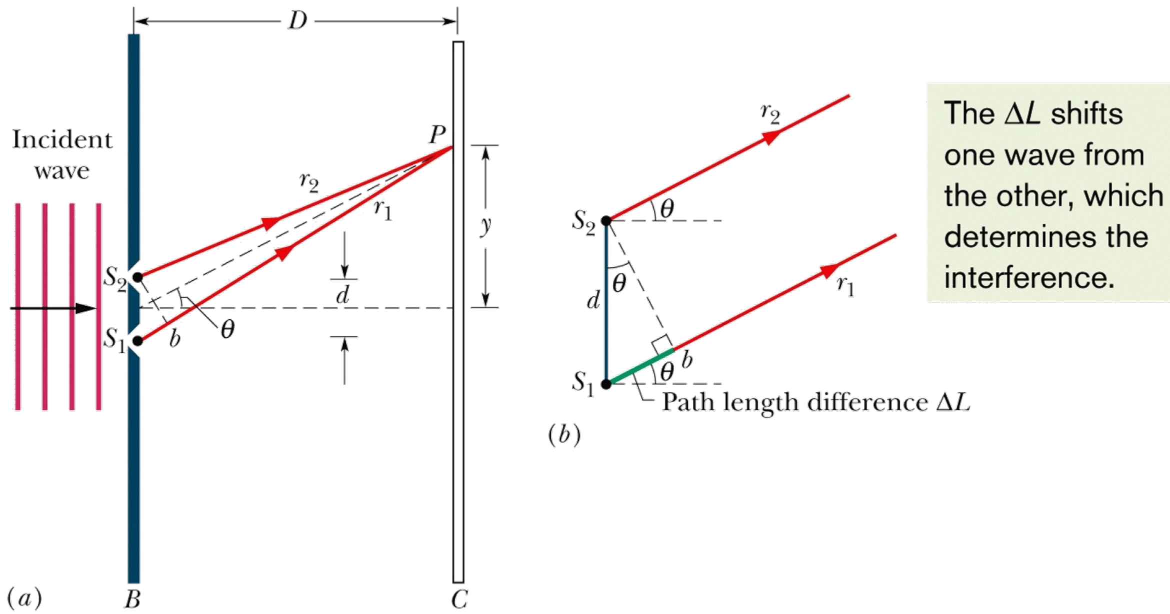
Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

A photograph of the interference pattern produced by the arrangement shown in the figure(right), but with short slits. (The photograph is a front view of part of screen C of figure on left.) The alternating maxima and minima are called interference fringes (because they resemble the decorative fringe sometimes used on clothing and rugs).



Courtesy Jearl Walker

35-2 Young's Interference



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

(a) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at P, an arbitrary point on screen C at distance *y* from the central axis. The angle θ serves as a convenient locator for *P*.

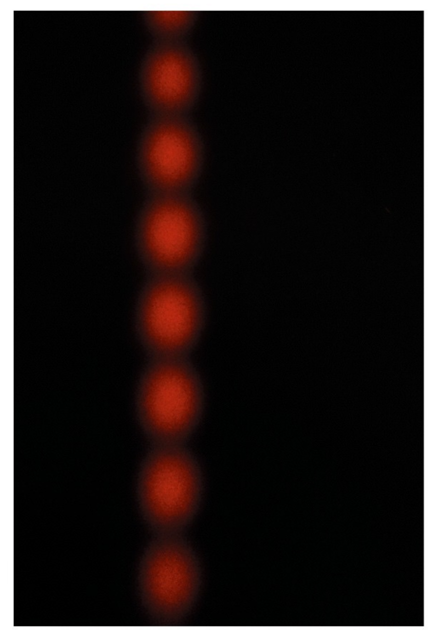
(b) For D >> d, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

The phase difference between two waves can change if the waves travel paths of different lengths.

The conditions for maximum and minimum intensity are

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$



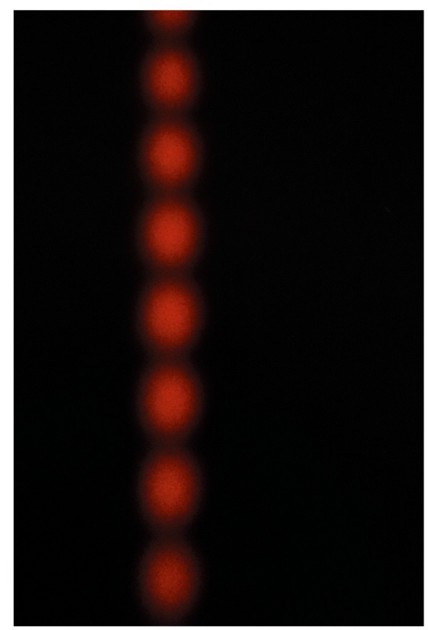
Courtesy Jearl Walker

Checkpoint 3

In Fig. 35-10a, what are ΔL (as a multiple of the wavelength) and the phase difference (in wavelengths) for the two rays if point P is (a) a third side maximum and (b) a third minimum?

$$(a)\ 3\lambda, 3; (b)\ 2.5\lambda, 2.5$$

	Path difference	Phase difference
2nd order maxima	2λ	4π
2nd order minima	1.5λ	3π
1st order maxima	λ	2π
1st order minima	0.5λ	π
Central Maxima	0	0
1st order minima	0.5λ	π
1st order maxima	λ	2π
2nd order minima	1.5λ	3π
2nd order maxima	2λ	4π



Courtesy Jearl Walker

Sample Problem 35.02 Double-slit interference pattern

What is the distance on screen C in Fig. 35-10a between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit–screen separation D is 55 cm. Assume that θ in Fig. 35-10 is small enough to permit use of the approximations $\sin\theta \approx \tan\theta \approx \theta$, in which θ is expressed in radian measure.

KEY IDEAS

(1) First, let us pick a maximum with a low value of m to ensure that it is near the center of the pattern. Then, from the geometry of Fig. 35-10a, the maximum's vertical distance y_m from the center of the pattern is related to its angle θ from the central axis by

$$\tan\theta \approx \theta = \frac{y_m}{D}.$$

(2) From Eq. 35-14, this angle θ for the mth maximum is given by

$$\sin\theta \approx \theta = \frac{m\lambda}{d}.$$

Calculations: If we equate our two expressions for angle θ and then solve for y_m, we find

$$y_m = \frac{m\lambda D}{d}. \tag{35-17}$$

For the next maximum as we move away from the pattern's center, we have

$$y_{m+1} = \frac{(m+1)\lambda D}{d}. \tag{35-18}$$

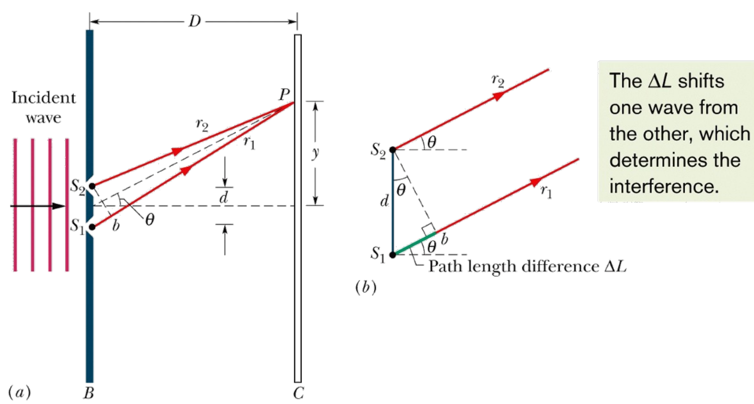
We find the distance between these adjacent maxima by subtracting Eq. 35-17 from Eq. 35-18:

$$\Delta y = y_{m+1} - y_m = \frac{\lambda D}{d}$$

$$= \frac{(546 \times 10^{-9}\text{ m})(55 \times 10^{-2}\text{ m})}{0.12 \times 10^{-3}\text{ m}}$$

$$= 2.50 \times 10^{-3}\text{ m} \approx 2.5\text{ mm.} \qquad \text{(Answer)}$$

As long as d and θ in Fig. 35-10a are small, the separation of the interference fringes is independent of m; that is, the fringes are evenly spaced.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Sample Problem 35.03 Double-slit interference pattern with plastic over one slit

A double-slit interference pattern is produced on a screen, as in Fig. 35-10; the light is monochromatic at a wavelength of 600 nm. A strip of transparent plastic with index of refraction $n = 1.50$ is to be placed over one of the slits. Its presence changes the interference between light waves from the two slits, causing the interference pattern to be shifted across the screen from the original pattern. Figure 35-11*a* shows the original locations of the central bright fringe ($m = 0$) and the first bright fringes ($m = 1$) above and below the central fringe. The purpose of the plastic is to shift the pattern upward so that the lower $m = 1$ bright fringe is shifted to the center of the pattern. Should the plastic be placed over the top slit (as arbitrarily drawn in Fig. 35-11*b*) or the bottom slit, and what thickness L should it have?

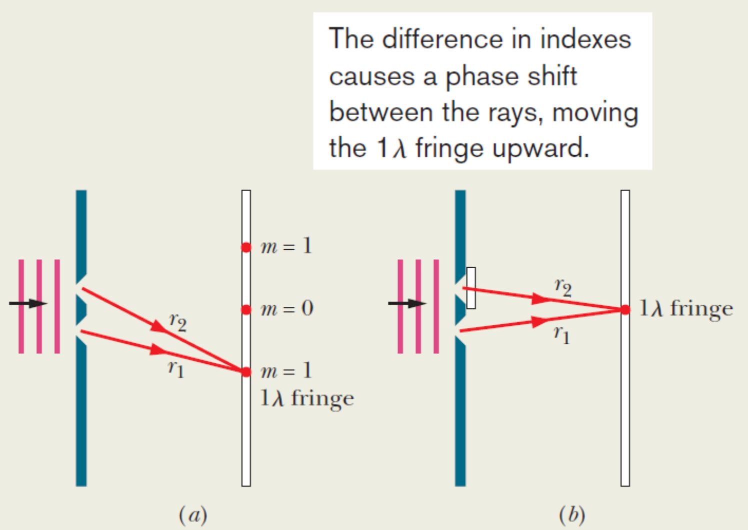


Figure 35-11 (*a*) Arrangement for two-slit interference (not to scale). The locations of three bright fringes (or maxima) are indicated. (*b*) A strip of plastic covers the top slit. We want the 1λ fringe to be at the center of the pattern.

Internal wavelength: The wavelength λ_n of light in a material with index of refraction n is smaller than the wavelength in vacuum, as given by Eq. 35-6 ($\lambda_n = \lambda/n$). Here, this means that the wavelength of the light is smaller in the plastic than in the air. Thus, the ray that passes through the plastic will have more wavelengths along it than the ray that passes through only air—so we do get the one extra wavelength we need along ray r_2 by placing the plastic over the top slit, as drawn in Fig. 35-11*b*.

Thickness: To determine the required thickness L of the plastic, we first note that the waves are initially in phase and travel equal distances L through different materials (plastic and air). Because we know the phase difference and require L, we use Eq. 35-9,

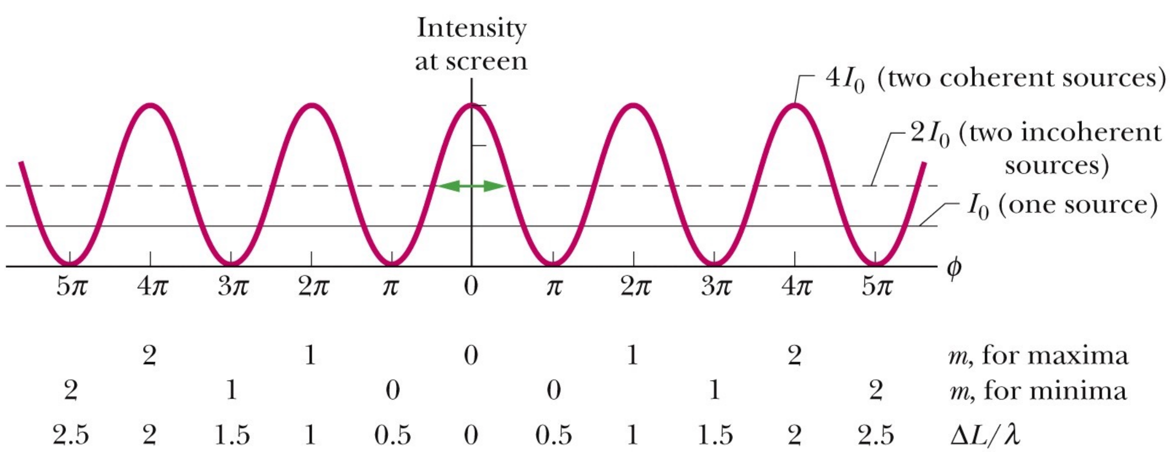
$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1). \tag{35-19}$$

We know that $N_2 - N_1$ is 1 for a phase difference of one wavelength, n_2 is 1.50 for the plastic in front of the top slit, n_1 is 1.00 for the air in front of the bottom slit, and λ is 600×10^{-9} m. Then Eq. 35-19 tells us that, to shift the lower $m = 1$ bright fringe up to the center of the interference pattern, the plastic must have the thickness

$$L = \frac{\lambda(N_2 - N_1)}{n_2 - n_1} = \frac{(600 \times 10^{-9}\text{ m})(1)}{1.50 - 1.00}$$
$$= 1.2 \times 10^{-6}\text{ m}. \qquad \text{(Answer)}$$

35-3 Interference and Double-Slit Intensity

If two light waves that meet at a point are to interfere clearly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

A plot of equation below, showing the intensity of a double-slit interference pattern as a function of the phase difference between the waves when they arrive from the two slits. I_0 is the (uniform) intensity that would appear on the screen if one slit were covered. The average intensity of the fringe pattern is $2I_0$, and the maximum intensity (for coherent light) is $4I_0$.

As shown in figure, in Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin \theta.$$

$$\begin{pmatrix}\text{phase}\\ \text{difference}\end{pmatrix} = \frac{2\pi}{\lambda}\begin{pmatrix}\text{path length}\\ \text{difference}\end{pmatrix}.$$

$$E_1 = E_0 \sin \omega t \tag{35-20}$$

and

$$E_2 = E_0 \sin(\omega t + \phi), \tag{35-21}$$

where ω is the angular frequency of the waves and ϕ is the phase constant of wave E_2. Note that the two waves have the same amplitude E_0 and a phase difference of ϕ. Because that phase difference does not vary, the waves are coherent. We shall show that these two waves will combine at P to produce an intensity I given by

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \tag{35-22}$$

and that

$$\phi = \frac{2\pi d}{\lambda} \sin \theta. \tag{35-23}$$

Maxima. Study of Eq. 35-22 shows that intensity maxima will occur when

$$\tfrac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots. \tag{35-24}$$

If we put this result into Eq. 35-23, we find

$$2m\pi = \frac{2\pi d}{\lambda} \sin \theta, \qquad \text{for } m = 0, 1, 2, \ldots$$

or

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)}, \tag{35-25}$$

which is exactly Eq. 35-14, the expression that we derived earlier for the locations of the maxima.

Minima. The minima in the fringe pattern occur when

$$\tfrac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots. \tag{35-26}$$

If we combine this relation with Eq. 35-23, we are led at once to

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima)}, \tag{35-27}$$

Ex: An interference pattern is formed on a viewing screen when light of wavelength $\lambda = 500\text{nm}$ illuminates a double-slit arrangement with slit separation $d = 1500\text{nm}$. As a multiple of I_0 (the intensity due to either of the slits alone), what is the intensity at angle $\theta = 26.0°$ in the pattern?

solution:

The pattern is set up by the interference of the rays from one slit and the rays from the other slit. At any given point, the phase difference is due to the fact that the rays travel along paths of different lengths. The phase difference ϕ in radians is given by

$$\phi = \frac{2\pi d \sin\theta}{\lambda}, \tag{35.35}$$

where $d \sin\theta$ is the path-length difference, which must be divided by λ to get the phase difference in terms of wavelengths and then multiplied by 2π to switch it to radians. The intensity is then given by

$$I = 4I_0 \cos^2\left(\frac{1}{2}\phi\right). \tag{35.36}$$

$$\begin{aligned} \phi &= \frac{2\pi(1500\text{ nm})\sin(26.0°)}{500\text{ nm}} \\ &= 8.263\text{ rad.} \end{aligned}$$

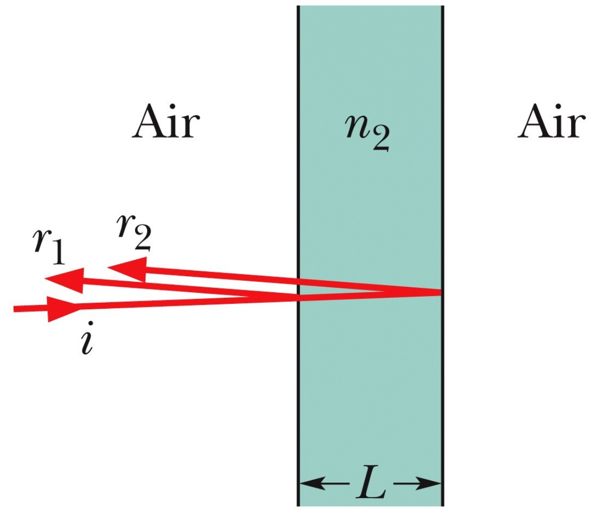
(Be careful about the calculator mode; the angle in the sine function is in degrees.) From Eq. 35.36, the intensity is then

$$\begin{aligned} I &= 4I_0 \cos^2\left(\tfrac{1}{2} 8.263\text{rad}\right) \\ &= 1.20 I_0. \end{aligned} \qquad \textbf{(Answer)}$$

(Again be careful; this angle in the cosine function is in radians.) This intensity is an intermediate one because the minima in the pattern have an intensity of $I = 0$ and the maxima have an intensity of $I = 4I_0$.

35-4 Interference from thin films

When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and minimum intensity of the light reflected from a film with air on both sides are



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Reflections from a thin film in air.

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

and

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$

where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.

If a film is sandwiched between media other than air, these equations for bright and dark films may be interchanged, depending on the relative indexes of refraction.

If the light incident at an interface between media with different indexes of refraction is initially in the medium with the smaller index of refraction, the reflection causes a phase change of π *rad*, or half a wavelength, in the reflected wave. Otherwise, there is no phase change due to the reflection. Refraction causes no phase shift.

35-4 Interference from thin films

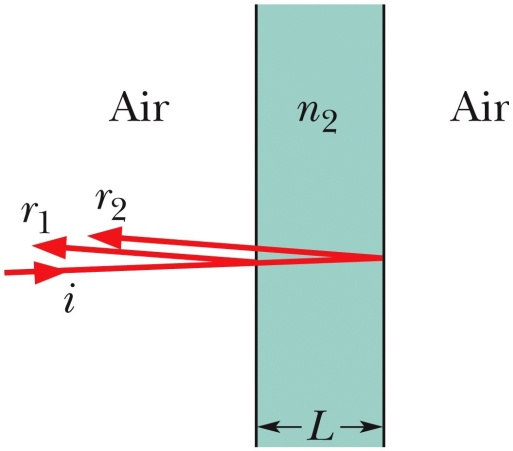
When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and minimum intensity of the light reflected from a film in air are

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

and

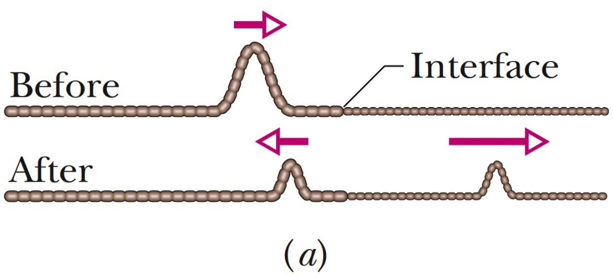
$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$

where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Reflections from a thin film in air.

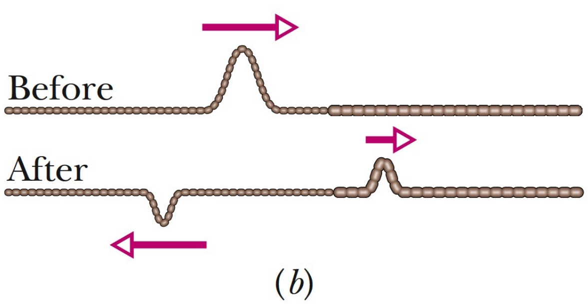


Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

The incident pulse is in the denser string.

Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

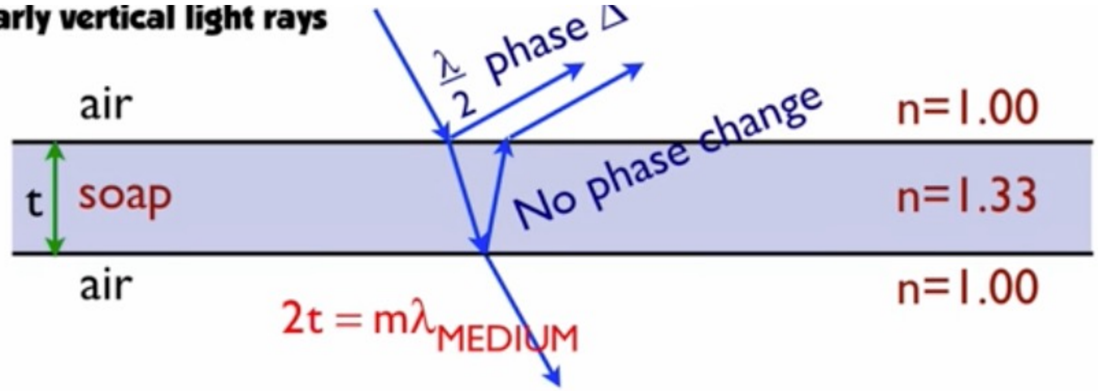


Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

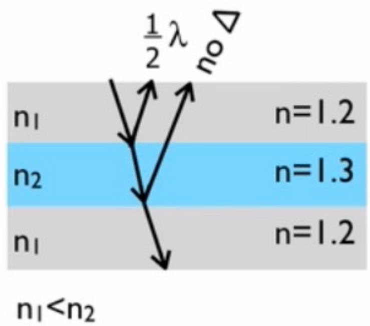
The incident pulse in the lighter string. Only here is there a phase change, and only in the reflected wave.

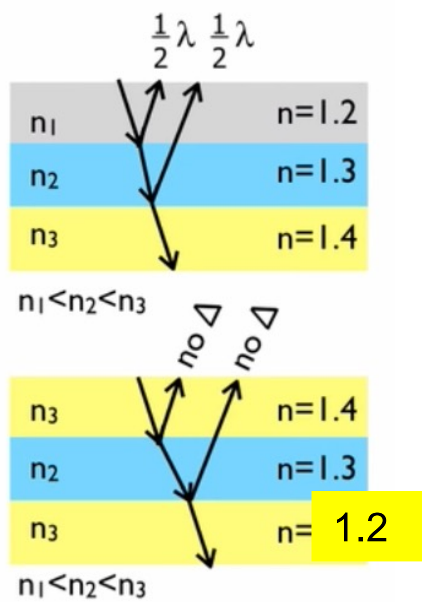
Important note:

For nearly vertical light rays



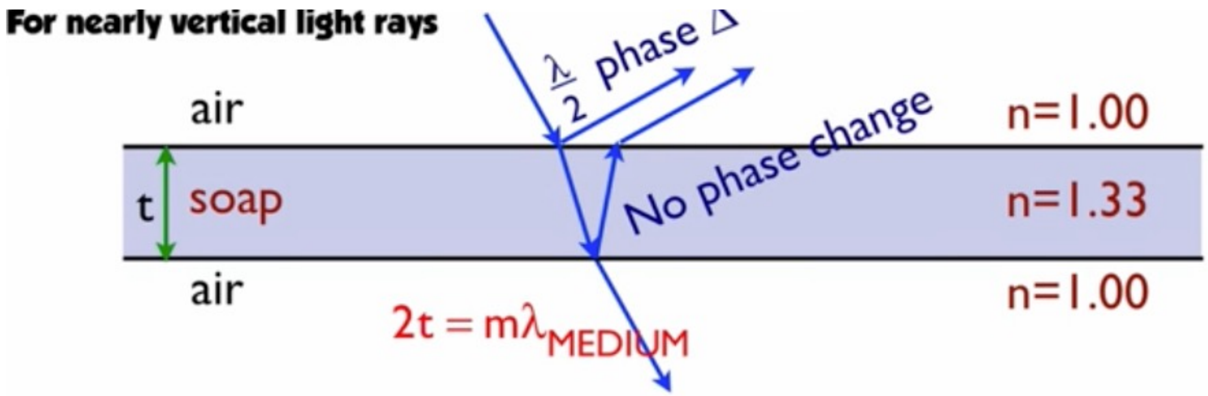
Equation m=0, 1, 2, …	1 phase shift	0 or 2 phase shifts
$2nt = \left(m + \frac{1}{2}\right)\lambda$	constructive	destructive
$2nt = m\lambda$	destructive	constructive





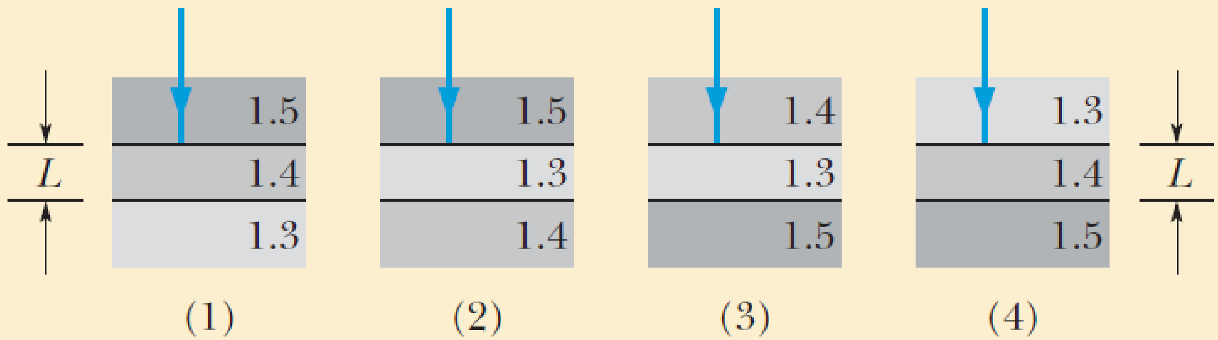
t is the thickness = L

Important note:



Checkpoint 5

The figure shows four situations in which light reflects perpendicularly from a thin film of thickness L, with indexes of refraction as given. (a) For which situations does reflection at the film interfaces cause a zero phase difference for the two reflected rays? (b) For which situations will the film be dark if the path length difference $2L$ causes a phase difference of 0.5 wavelength?



(a) 1 and 4; (b) 1 and 4

Sample Problem 35.05 Thin-film interference of a water film in air

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film, of index of refraction $n_2 = 1.33$ and thickness $L = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

KEY IDEA

The reflected light from the film is brightest at the wavelengths λ for which the reflected rays are in phase with one another. The equation relating these wavelengths λ to the given film thickness L and film index of refraction n_2 is either Eq. 35-36 or Eq. 35-37, depending on the reflection phase shifts for this particular film.

Calculations: To determine which equation is needed, we should fill out an organizing table like Table 35-1. However, because there is air on both sides of the water film, the situation here is exactly like that in Fig. 35-17, and thus the table would be exactly like Table 35-1. Then from Table 35-1, we see that the reflected rays are in phase (and thus the film is brightest) when

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2},$$

which leads to Eq. 35-36:

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}.$$

Solving for λ and substituting for L and n_2, we find

$$\lambda = \frac{2n_2L}{m + \frac{1}{2}} = \frac{(2)(1.33)(320\text{ nm})}{m + \frac{1}{2}} = \frac{851\text{ nm}}{m + \frac{1}{2}}.$$

For $m = 0$, this gives us $\lambda = 1700$ nm, which is in the infrared region. For $m = 1$, we find $\lambda = 567$ nm, which is yellow-green light, near the middle of the visible spectrum. For $m = 2$, $\lambda = 340$ nm, which is in the ultraviolet region. Thus, the wavelength at which the light seen by the observer is brightest is

$$\lambda = 567\text{ nm}. \qquad \text{(Answer)}$$

Sample Problem 35.06 Thin-film interference of a coating on a glass lens

In Fig. 35-19, a glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface. The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum ($\lambda = 550$ nm)? Assume that the light is approximately perpendicular to the lens surface.

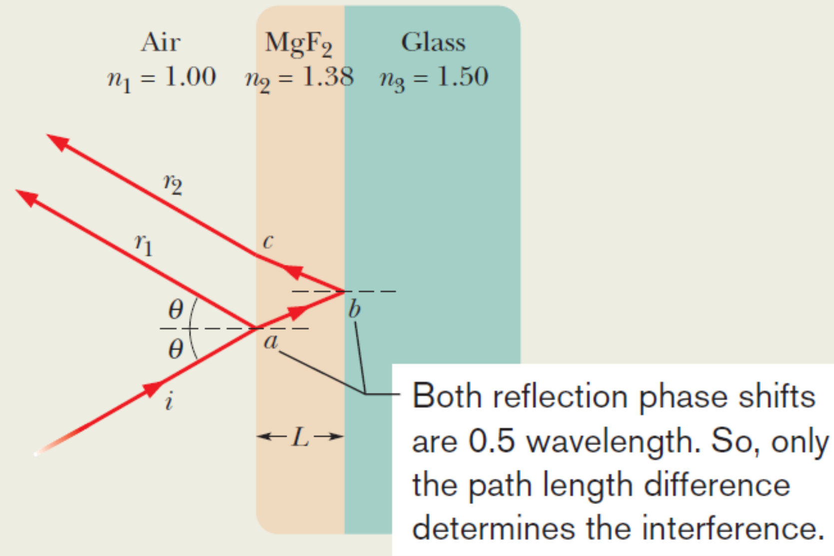


Figure 35-19 Unwanted reflections from glass can be suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of the properly chosen thickness.

KEY IDEA

Reflection is eliminated if the film thickness L is such that light waves reflected from the two film interfaces are exactly out of phase. The equation relating L to the given wavelength λ and the index of refraction n_2 of the thin film is either Eq. 35-36 or Eq. 35-37, depending on the reflection phase shifts at the interfaces.

Calculations: To determine which equation is needed, we fill out an organizing table like Table 35-1. At the first interface, the incident light is in air, which has a lesser index of refraction than the MgF_2 (the thin film). Thus, we fill in 0.5 wavelength under r_1 in our organizing table (meaning that the waves of ray r_1 are shifted by 0.5λ at the first interface). At the second interface, the incident light is in the MgF_2, which has a lesser index of refraction than the glass on the other side of the interface. Thus, we fill in 0.5 wavelength under r_2 in our table.

Because both reflections cause the same phase shift, they tend to put the waves of r_1 and r_2 in phase. Since we want those waves to be *out of phase,* their path length difference $2L$ must be an odd number of half-wavelengths:

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2}.$$

This leads to Eq. 35-36 (for a bright film sandwiched in air but for a dark film in the arrangement here). Solving that equation for L then gives us the film thicknesses that will eliminate reflection from the lens and coating:

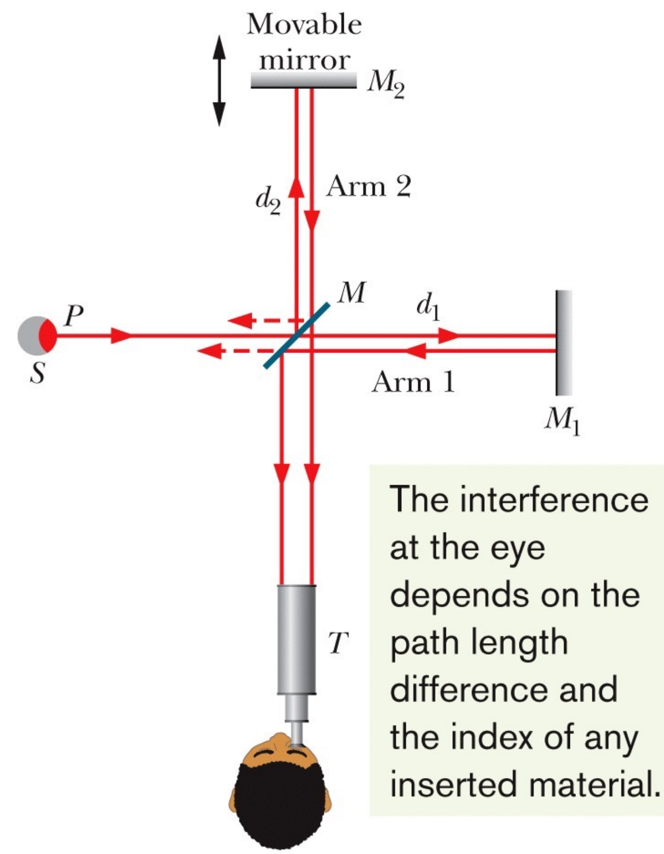
$$L = (m + \tfrac{1}{2})\frac{\lambda}{2n_2}, \qquad \text{for } m = 0, 1, 2, \ldots. \qquad (35\text{-}38)$$

We want the least thickness for the coating—that is, the smallest value of L. Thus, we choose $m = 0$, the smallest possible value of m. Substituting it and the given data in Eq. 35-38, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550 \text{ nm}}{(4)(1.38)} = 99.6 \text{ nm}. \qquad \text{(Answer)}$$

35-5 Michelson's Interferometer

An interferometer is a device that can be used to measure lengths or changes in length with great accuracy by means of interference fringes.
In Michelson's interferometer, a light wave is split into two beams that then recombine after traveling along different paths.
The interference pattern they produce depends on the difference in the lengths of those paths and the indexes of refraction along the paths.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

If a transparent material of index *n* and thickness *L* is in one path, the phase difference (in terms of wavelength) in the recombining beams is equal to

$$N_m = \frac{2L}{\lambda_n} = \frac{2Ln}{\lambda}. \tag{35-41}$$

The number of wavelengths in the same thickness $2L$ of air before the insertion of the material is

$$N_a = \frac{2L}{\lambda}. \tag{35-42}$$

When the material is inserted, the light returned from mirror M_1 undergoes a phase change (in terms of wavelengths) of

$$N_m - N_a = \frac{2Ln}{\lambda} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n - 1). \tag{35-43}$$

where λ is the wavelength of the light.

Michelson's interferometer, showing the path of light originating at point P of an extended source S. Mirror *M* splits the light into two beams, which reflect from mirrors M_1 and M_2 back to *M* and then to telescope *T*. In the telescope an observer sees a pattern of interference fringes.

35 Summary

Huygen's Principle

- The three-dimensional transmission of waves, including light, may often be predicted by Huygens' principle, which states that all points on a wavefront serve as point sources of spherical secondary wavelets.

Wavelength and Index of Refraction

- The wavelength λ_n of light in a medium depends on the index of refraction n of the medium:

$$\lambda_n = \frac{\lambda}{n}, \qquad \textbf{Eq. 35-6}$$

in which λ is the wavelength in vacuum.

Young's Experiment

- In Young's interference experiment, light passing through a single slit falls on two slits in a screen. The light leaving these slits flares out (by diffraction), and interference occurs in the region beyond the screen. A fringe pattern, due to the interference, forms on a viewing screen.
- The conditions for maximum and minimum intensity are

$$d \sin\theta = m\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright fringes)}, \qquad \textbf{Eq. 35-14}$$

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \quad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark fringes)}, \qquad \textbf{Eq. 35-16}$$

35 Summary

Coherence

- If two light waves that meet at a point are to interfere perceptibly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.

Intensity in Two-Slit Interference

- In Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \qquad \text{where } \phi = \frac{2\pi d}{\lambda} \sin \theta.$$

Eqs. 35-22 & 23

Thin-Film Interference

- When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and minimum intensity of the light reflected from a *film of index n_2 in air* are

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots$$

Eq. 35-36

(maxima—bright film in air),

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots$$

Eq. 35-37

(minima—dark film in air),

Michelson's Interferometer

- In Michelson's interferometer a light wave is split into two beams that, after traversing paths of different lengths, are recombined so they interfere and form a fringe pattern.