# Chapter 40

**Properties of Atoms,
Pauli Exclusion Principle, Lasers**

All About Atoms

40.1: Some Properties of Atoms:

Atoms are stable. Essentially all the atoms that form our tangible world have existed without change for billions of years.

Atoms combine with each other. They stick together to form stable molecules and stack up to form rigid solids.

Useful link **https://ptable.com/?lang=en#Electrons**
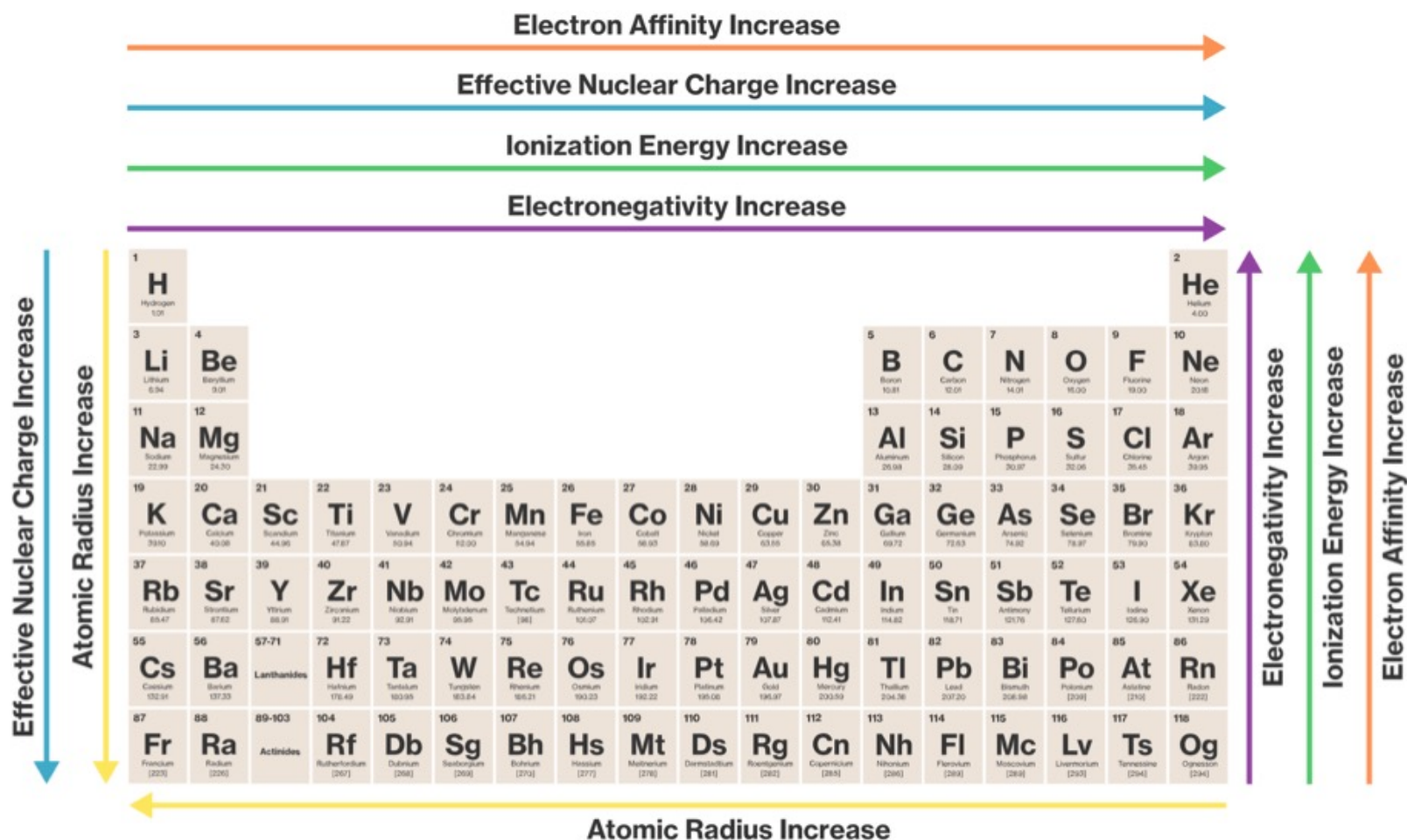
40.1: Some Properties of Atoms: Atoms Are Put Together Systematically
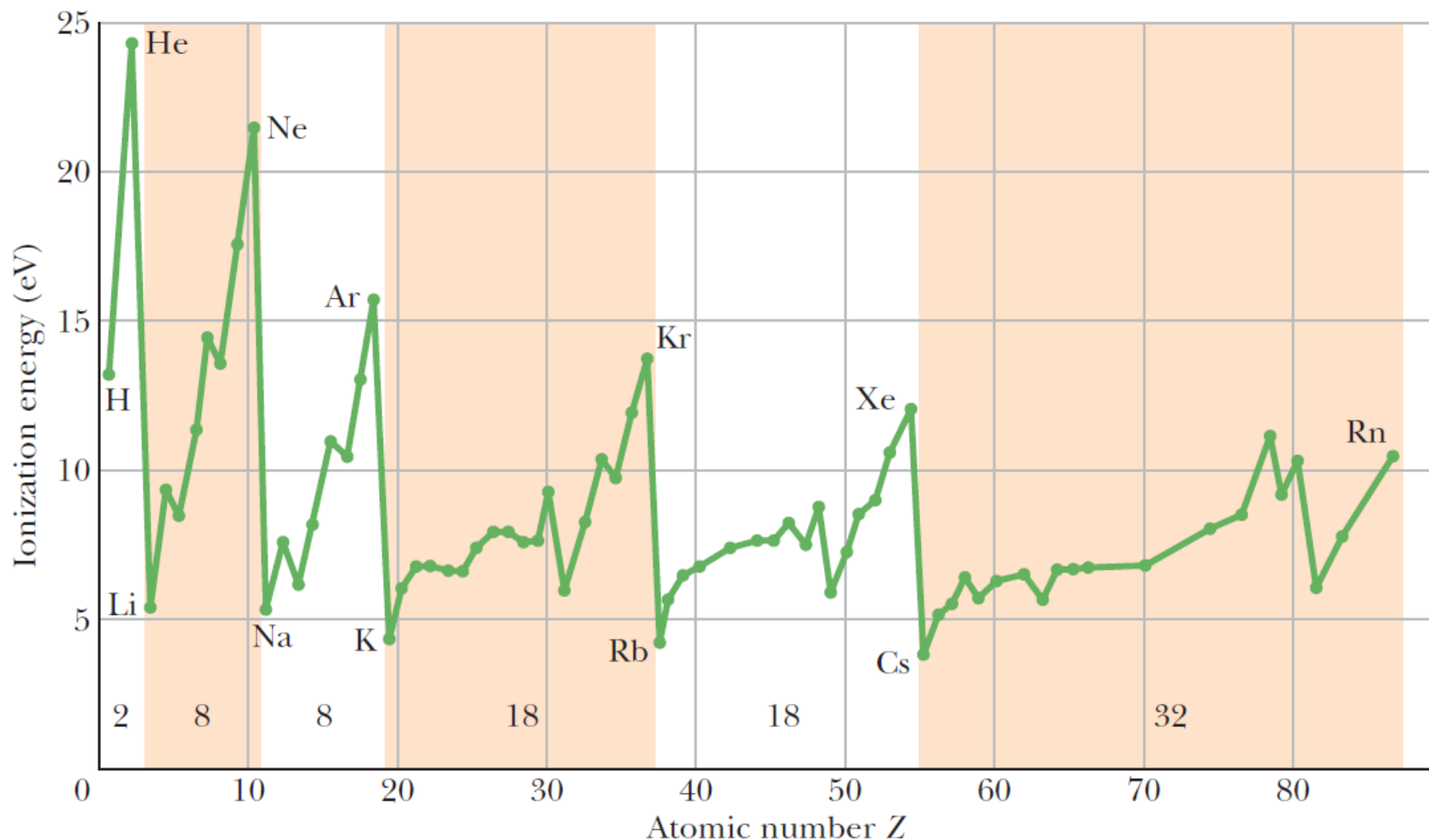
Fig. 40-2 A plot of the ionization energies of the elements as a function of atomic number, showing the periodic repetition of properties through the six complete horizontal periods of the periodic table. The number of elements in each of these periods is indicated.

The elements are arranged in the periodic table in six complete horizontal **periods** (and a seventh incomplete period): except for the first, each period starts at the left with a highly reactive alkali metal (lithium, sodium, potassium, and so on) and ends at the right with a chemically inert noble gas (neon, argon, krypton, and so on).

Quantum physics accounts for the chemical properties of these elements.

The numbers of elements in the six periods are 2, 8, 8, 18, 18, and 32.

Quantum physics predicts these numbers.

40.1: Some Properties of Atoms: Atoms Emit and Absorb Light:

An atom can make a transition from one state to another by emitting light (to jump to a lower energy level E_{low}) or by absorbing light (to jump to a higher energy level E_{high}).

The light is emitted or absorbed as a photon with energy

$$hf = E_{high} - E_{low}.$$

40.1: Some Properties of Atoms:
Atoms have Angular Momentum and Magnetism
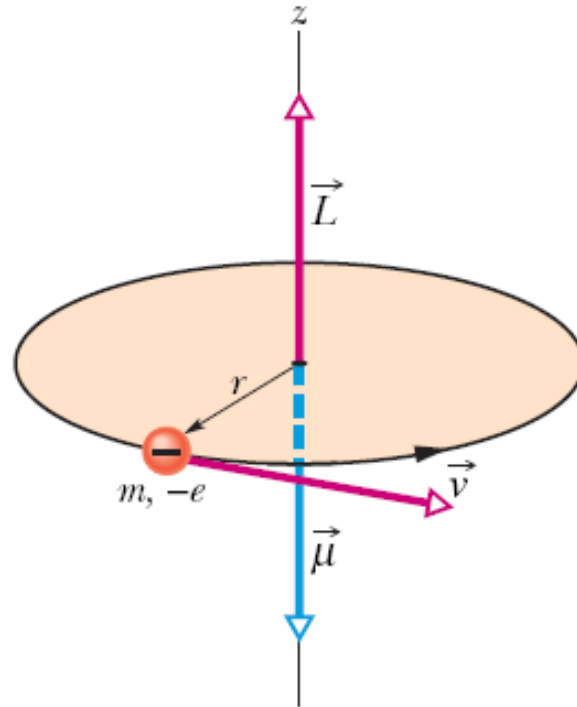
Fig. 40-3 A classical model showing a particle of mass m and charge $-e$ moving with speed v in a circle of radius r. The moving particle has an angular momentum $\vec{L}$ given by $\vec{r} \times \vec{p}$, where $\vec{p}$ is its linear momentum $m\vec{v}$. The particle's motion is equivalent to a current loop that has an associated magnetic moment $\vec{\mu}$ that is directed opposite $\vec{L}$.

Angular momentum and magnetic moment of individual atoms are coupled.
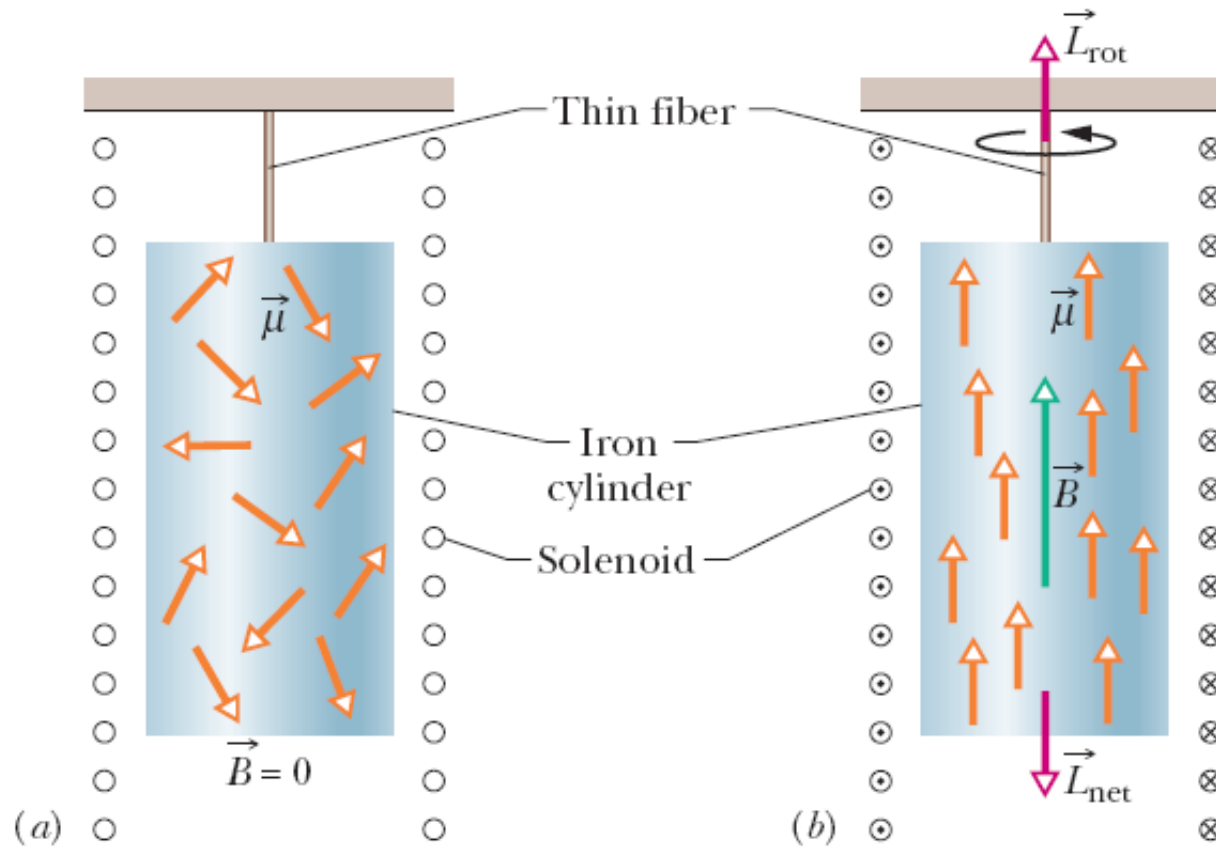
Aligning the magnetic moment vectors rotates the cylinder.

Fig. 40-4 The Einstein–de Haas experimental setup. (a) Initially, the magnetic field in the iron cylinder is zero and the magnetic dipole moment vectors of its atoms are randomly oriented. (b)When a magnetic field is set up along the cylinder's axis, the magnetic dipole moment vectors μ line up parallel to the magnetic field and the cylinder begins to rotate.

Electron Spin:

The electron has an intrinsic **spin angular momentum S**, often called simply **spin.**

The magnitude of **S** is quantized and depends on a **spin quantum number s,** which is always ½ for electrons (and for protons and neutrons).

The component of **S** is measured along any axis, is quantized, and depends on a **spin magnetic quantum number m_s,** which can have only the value +½ or -½

Table 40-1

Electron States for an Atom

Quantum Number	Symbol	Allowed Values	Related to
Principal	n	$1, 2, 3, \ldots$	Distance from the nucleus
Orbital	ℓ	$0, 1, 2, \ldots, (n-1)$	Orbital angular momentum
Orbital magnetic	m_ℓ	$0, \pm 1, \pm 2, \ldots, \pm \ell$	Orbital angular momentum (z component)
Spin	s	$\frac{1}{2}$	Spin angular momentum
Spin magnetic	m_s	$\pm \frac{1}{2}$	Spin angular momentum (z component)

Angular Momenta and Magnetic Dipole Moments:

The magnitude L of the **orbital angular momentum L** of an electron in an atom is quantized; that is, it can have only certain values

$$L = \sqrt{\ell(\ell + 1)}\hbar,$$

The magnetic dipole has an **orbital magnetic dipole moment μ_{orb},** is related to the angular momentum by

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}. = \frac{e}{2m}\sqrt{\ell(\ell + 1)}\hbar.$$

If the atom is located in a magnetic field **B**, with a z axis extending in the direction of the field lines at the atom's location, we can measure the z components of μ_{orb} and L along that axis.
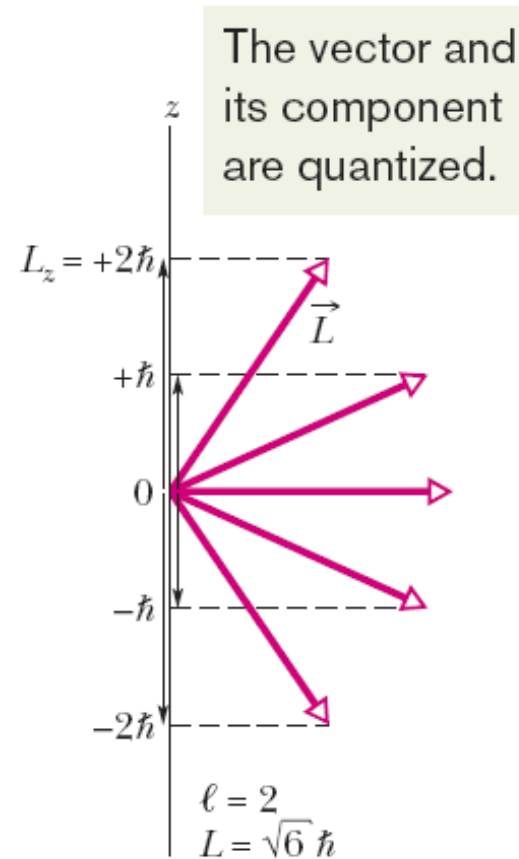
$$\mu_{orb,z} = -m_\ell \mu_B.$$

Here μ_B is the **Bohr Magneton**
$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T}$$

Angular Momenta and Magnetic Dipole Moments:

The components L_z of the angular momentum are also quantized, and they are given by

$$L_z = m_\ell \hbar.$$

The vector and its component are quantized.

$L_z = +2\hbar$

$\vec{L}$

$+\hbar$

0

$-\hbar$

$-2\hbar$

$\ell = 2$
$L = \sqrt{6}\,\hbar$

Fig. 40-5   The allowed values of L_z for an electron in a quantum state with $\ell = 2$. For every orbital angular momentum vector $\vec{L}$ in the figure, there is a vector pointing in the opposite direction, representing the magnitude and direction of the orbital magnetic dipole moment $\vec{\mu}_{orb}$.

Summary:

Orbital Angular Momentum and Magnetic Dipole Moments The magnitude of the orbital angular momentum of an electron trapped in an atom has quantized values given by

$$L = \sqrt{\ell(\ell + 1)}\, \hbar, \qquad \text{for } \ell = 0, 1, 2, \ldots, (n - 1), \qquad (40\text{-}2)$$

where $\hbar$ is $h/2\pi$, ℓ is the orbital magnetic quantum number, and n is the electron's principal quantum number. The component L_z of the orbital angular momentum on a z axis is quantized and given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell, \qquad (40\text{-}3)$$

where m_ℓ is the orbital magnetic quantum number. The magnitude μ_{orb} of the orbital magnetic moment of the electron is quantized with the values given by

$$\mu_{orb} = \frac{e}{2m} \sqrt{\ell(\ell + 1)}\, \hbar, \qquad (40\text{-}6)$$

where m is the electron mass. The component $\mu_{orb,z}$ on a z axis is also quantized according to

$$\mu_{orb,z} = -\frac{e}{2m} m_\ell \hbar = -m_\ell \mu_B, \qquad (40\text{-}7)$$

where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \, \text{J/T}. \qquad (40\text{-}8)$$

Spin Angular Momentum and Magnetic Dipole Moment Every electron, whether trapped or free, has an intrinsic spin angular momentum $\vec{S}$ with a magnitude that is quantized as

$$S = \sqrt{s(s + 1)}\, \hbar, \quad \text{for } s = \tfrac{1}{2}, \qquad (40\text{-}9)$$

spin-$\tfrac{1}{2}$ particle. The component S_z on a z axis is also quantized according to

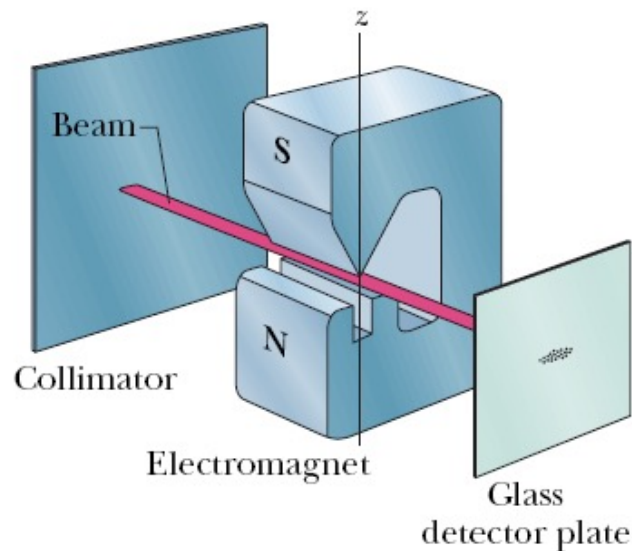$$S_z = m_s \hbar, \quad \text{for } m_s = \pm s = \pm\tfrac{1}{2}, \qquad (40\text{-}10)$$

where m_s is the spin magnetic quantum number. Every electron, whether trapped or free, has an intrinsic spin magnetic dipole moment $\vec{\mu}_s$ with a magnitude that is quantized as

$$\mu_s = \frac{e}{m} \sqrt{s(s + 1)}\, \hbar, \quad \text{for } s = \tfrac{1}{2}. \qquad (40\text{-}12)$$

The component $\mu_{s,z}$ on a z axis is also quantized according to

$$\mu_{s,z} = -2m_s \mu_B, \quad \text{for } m_s = \pm\tfrac{1}{2}. \qquad (40\text{-}13)$$

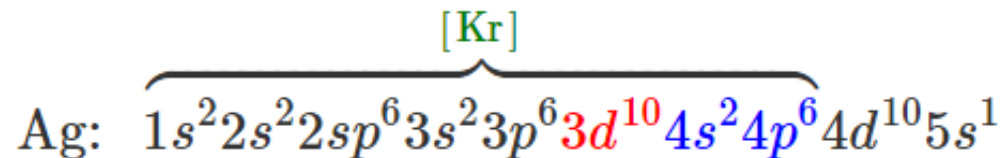# 40.2: The Stern-Gerlach Experiment: *Quantization of magnetic moment*

Fig. 40-8 Apparatus used by Stern and Gerlach.

In 1922, Otto Stern and Walther Gerlach showed experimentally that the magnetic moment of silver atoms is quantized. In the experiment silver is vaporized in an oven, and some of the atoms in that vapor escape through a narrow slit in the oven wall and pass into an evacuated tube. The beam passes between the poles of an electromagnet and then lands on a glass detector plate where it forms a silver deposit. By analyzing the silver deposit on the plate, one can determine what deflections the atoms underwent in the magnetic field.

Why silver (Ag)?

$$\text{Ag:} \quad 1s^2 2s^2 2sp^6 3s^2 3p^6 \overbrace{3d^{10} 4s^2 4p^6 4d^{10}}^{[\text{Kr}]} 5s^1$$
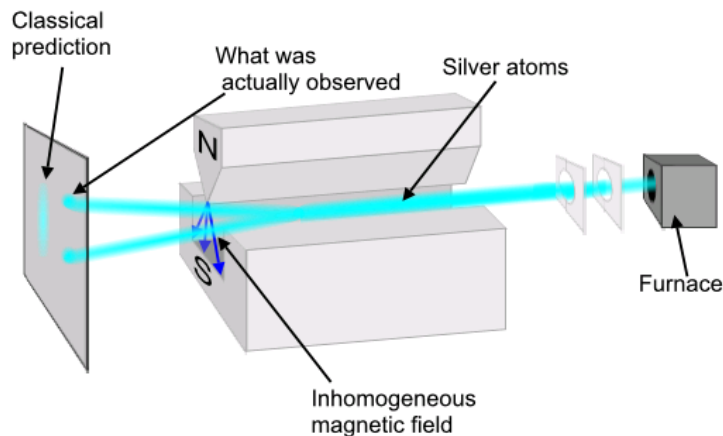
40.2: The Stern-Gerlach Experiment:

The type of magnetic force the silver atom experiences in the Stern-Gerlach experiment is due to an interaction between the magnetic field of the electromagnet and the magnetic dipole of the individual silver atom.

The energy U of the dipole in the magnetic field is $U = -\vec{\mu} \cdot \vec{B}$, is the magnetic dipole moment of the silver atom.
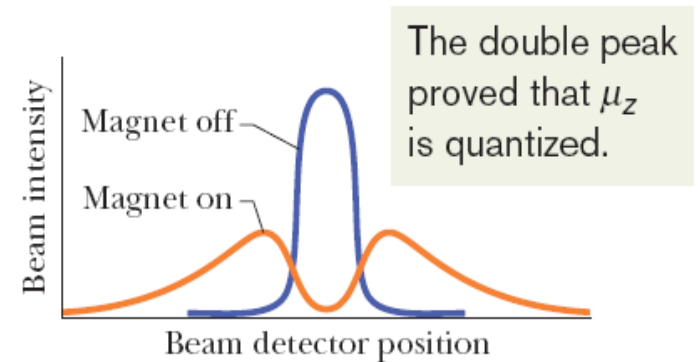
Therefore $U = -\mu_z B$.

And $F_z = -\dfrac{dU}{dz} = \mu_z \dfrac{dB}{dz}$.

$$\mu_{s,z} = -2m_s\mu_B,$$

$\mu_{s,z} = -2(+\tfrac{1}{2})\mu_B = -\mu_B$ and $\mu_{s,z} = -2(-\tfrac{1}{2})\mu_B = +\mu_B$.

$$F_z = -\mu_B\left(\dfrac{dB}{dz}\right) \quad \text{and} \quad F_z = +\mu_B\left(\dfrac{dB}{dz}\right),$$

The double peak proved that μ_z is quantized.

Magnet off

Magnet on

Beam intensity

Beam detector position

Fig. 40-9 Results of a modern repetition of the Stern–Gerlach experiment. With the electromagnet turned off, there is only a single beam; with the electromagnet turned on, the original beam splits into two sub-beams. The two subbeams correspond to parallel and antiparallel alignment of the magnetic moments of cesium atoms with the external magnetic field.

Example, Beam Separation in a Stern-Gerlach experiment:

In the Stern–Gerlach experiment of Fig. 40-8, a beam of silver atoms passes through a magnetic field gradient dB/dz of magnitude 1.4 T/mm that is set up along the z axis. This region has a length w of 3.5 cm in the direction of the original beam. The speed of the atoms is 750 m/s. By what distance d have the atoms been deflected when they leave the region of the field gradient? The mass M of a silver atom is 1.8×10^{-25} kg.

KEY IDEAS

(1) The deflection of a silver atom in the beam is due to an interaction between the magnetic dipole of the atom and the magnetic field, because of the gradient dB/dz. The deflecting force is directed along the field gradient (along the z axis) and is given by Eqs. 40-19. Let us consider only deflection in the positive direction of z; thus, we shall use $F_z = \mu_B(dB/dz)$ from Eqs. 40-19.

(2) We assume the field gradient dB/dz has the same value throughout the region through which the silver atoms travel. Thus, force component F_z is constant in that region, and from Newton's second law, the acceleration a_z of an atom along the z axis due to F_z is also constant.

Calculations: Putting these ideas together, we write the acceleration as

$$a_z = \frac{F_z}{M} = \frac{\mu_B(dB/dz)}{M}.$$

Because this acceleration is constant, we can use Eq. 2-15 (from Table 2-1) to write the deflection d parallel to the z axis as

$$d = v_{0z}t + \tfrac{1}{2}a_z t^2 = 0t + \tfrac{1}{2}\left(\frac{\mu_B(dB/dz)}{M}\right)t^2. \qquad (40\text{-}20)$$
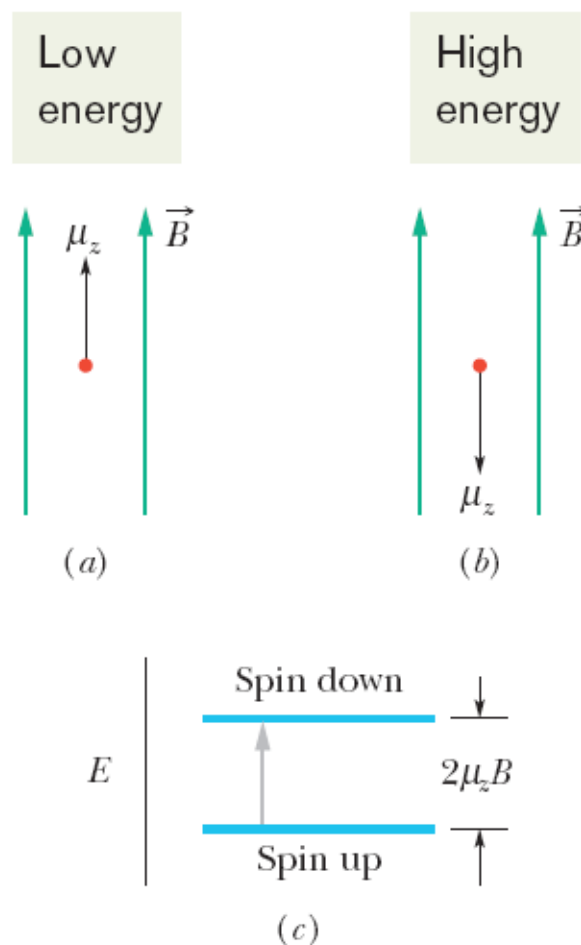
Because the deflecting force on the atom acts perpendicular to the atom's original direction of travel, the component v of the atom's velocity along the original direction of travel is not changed by the force. Thus, the atom requires time $t = w/v$ to travel through length w in that direction. Substituting w/v for t into Eq. 40-20, we find

$$d = \tfrac{1}{2}\left(\frac{\mu_B(dB/dz)}{M}\right)\left(\frac{w}{v}\right)^2 = \frac{\mu_B(dB/dz)w^2}{2Mv^2}$$

$$= (9.27 \times 10^{-24}\ \text{J/T})(1.4 \times 10^3\ \text{T/m})$$

$$\times \frac{(3.5 \times 10^{-2}\ \text{m})^2}{(2)(1.8 \times 10^{-25}\ \text{kg})(750\ \text{m/s})^2}$$

$$= 7.85 \times 10^{-5}\ \text{m} \approx 0.08\ \text{mm}. \qquad \text{(Answer)}$$

The separation between the two subbeams is twice this, or 0.16 mm. This separation is not large but is easily measured.

40.3: Magnetic Resonance:

$$\Delta E = \mu_z B - (-\mu_z B) = 2\mu_z B.$$

$$hf = 2\mu_z B. \quad \text{(Energy of absorbing photon)}$$

Such absorption is called **magnetic resonance** or, **nuclear magnetic resonance (NMR),** and the consequent reversal of S_z is called **spin-flipping**.
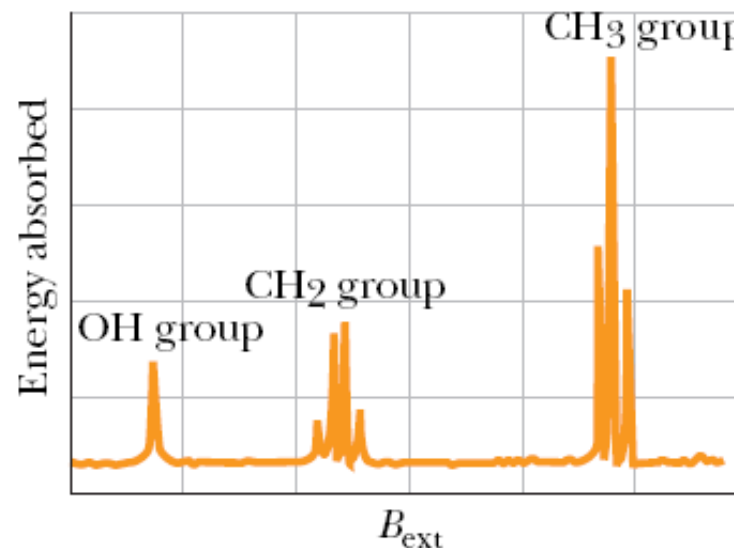

Fig. 40-10 The z component of $\vec{\mu}$ for a proton in the (a) lower-energy (spin-up) and (b) higher-energy (spin-down) state. (c) An energy-level diagram for the states, showing the upward quantum jump the proton makes when its spin flips from up to down.

Fig. 40-11 A nuclear magnetic resonance spectrum for ethanol, CH_3CH_2OH.

40.4: The Pauli Exclusion Principle and Multiple Electrons in a Trap:

The Pauli Exclusion Principle

→ No two electrons confined to the same trap can have the same set of values for their quantum numbers.

As we shall discuss in Module 40-5, this principle means that no two electrons in an atom can have the same four values for the quantum numbers n, ℓ, m_ℓ, and m_s. All electrons have the same quantum number $s = \frac{1}{2}$. Thus, any two electrons in an atom must differ in at least one of these other quantum numbers. Were this not true, atoms would collapse, and thus you and the world could not exist.

This principle applies not only to electrons but also to protons and neutrons, all of which have s=½. The principle is known as the **Pauli exclusion principle** after Wolfgang Pauli, who formulated it in 1925

Multiple Electrons in Rectangular Traps:

❑The Pauli exclusion principle disallows any more electrons from occupying that lowest energy level, and the next electron must occupy the next higher level.

❑When an energy level cannot be occupied by more electrons because of the Pauli exclusion principle, we say that level is *full or fully occupied*.

❑In contrast, a level that is not occupied by any electrons is *empty or unoccupied*.

❑For intermediate situations, the level is *partially occupied*.

❑The *electron configuration of a system of trapped* electrons is a listing or drawing either of the energy levels the electrons occupy or of the set of the quantum numbers of the electrons.

40.5: Building The Periodic Table:

For the purpose of labeling subshells, the values of ℓ are represented by letters:

$$\ell = 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \ldots$$
$$s \quad p \quad d \quad f \quad g \quad h \ldots$$

For example, the $n = 3, \ell = 2$ subshell would be labeled the $3d$ subshell.

Neon: The atom has 10 electrons. It has three closed subshells (*1s, 2s,* and *2p)* and, like the other **noble gases** that form the right-hand column of the periodic table, is almost chemically inert.

Sodium: The atom has 11 electrons. Ten of them form a closed neon-like core, and has zero angular momentum. The remaining electron is largely outside this inert core, in the $3s$ subshell. This is the **valence electron** of the atom, and the atom's angular momentum and magnetic dipole moment must be due entirely to the spin of this single electron.

Chlorine: This, with 17 electrons, has the outermost 7 electrons in $3p$ subshell, leaving a "hole" in this state. It is receptive to interacting with other atoms that have a valence electron that might fill this hole. Chlorine, like the other **halogens** that form column VIIA of the periodic table, is chemically active.

Iron: $1s^2 \quad 2s^2\,2p^6 \quad 3s^2\,3p^6\,3d^6 \quad 4s^2.$ This atom of 26 electrons, has the first 18 electrons form the five filled subshells that are marked off by the bracket. 6 of the remaining 8 electrons go into the $3d$ subshell, and the remaining two go into the $4s$ subshell. The configuration $3d^6 4s^2$ is of lower energy than $3d^8$.

Angular Momenta and Magnetic Dipole Moments:

$$S = \sqrt{s(s+1)}\hbar$$
$$= \sqrt{\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}+1\right)}\hbar = 0.866\hbar,$$

$$\vec{\mu}_s = -\frac{e}{m}\,\vec{S}. \qquad \text{(Spin magnetic dipole moment)}$$
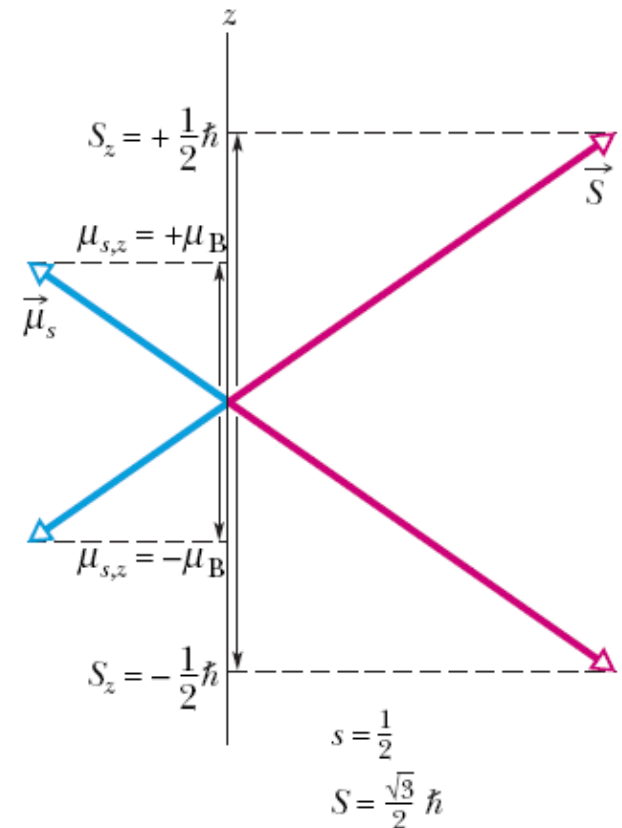
$$= \frac{e}{m}\sqrt{s(s+1)}\hbar.$$

$S_z = m_s\hbar,$ (components of S can be measured along a preferred axis).

$m_s = +\tfrac{1}{2}$ (Spin angular quantum number can only have two values)

$m_s = -\tfrac{1}{2}$

$\mu_{s,z} = -2m_s\mu_B.$ (The components of $\mu_{s,z}$ are also quantized)

Fig. 40-6 The allowed values of S_z and μ_z for an electron.

Angular Momenta and Magnetic Dipole Moments:

For an atom containing more than one electron, we define a total angular momentum, **J**, which is the vector sum of orbital and their spin angular momenta of the individual electrons.

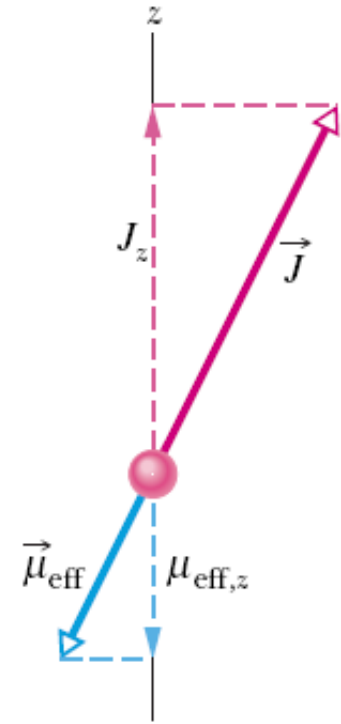This number of protons is defined as being the **atomic number** Z of the element.

$$\vec{J} = (\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_Z) + (\vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \cdots + \vec{S}_Z).$$

The **effective magnetic dipole moment, μ$_{eff}$,** for the atom is the component of the vector sum of the individual magnetic dipole moments in the direction of –**J**.

Fig. 40-7 A classical model showing the total angular momentum vector $\vec{J}$ and the effective magnetic moment vector $\vec{\mu}_{eff}$.

☑ **Checkpoint 1**

An electron is in a quantum state for which the magnitude of the electron's orbital angular momentum $\vec{L}$ is $2\sqrt{3}\hbar$. How many projections of the electron's orbital magnetic dipole moment on a z axis are allowed?
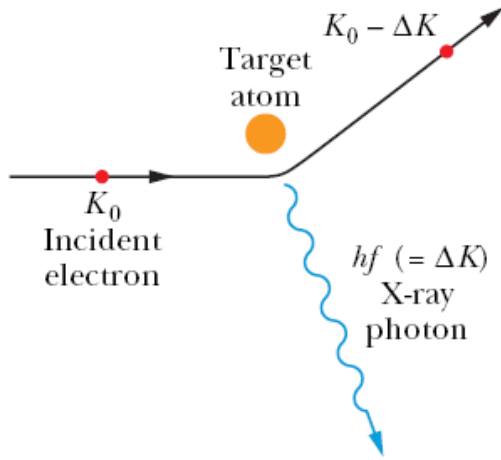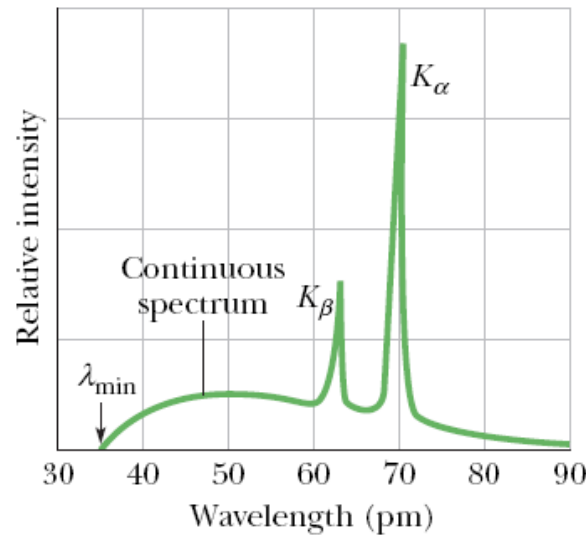
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40.6: X-Rays and the Ordering of the Elements:

The minimum possible x-ray wavelength:
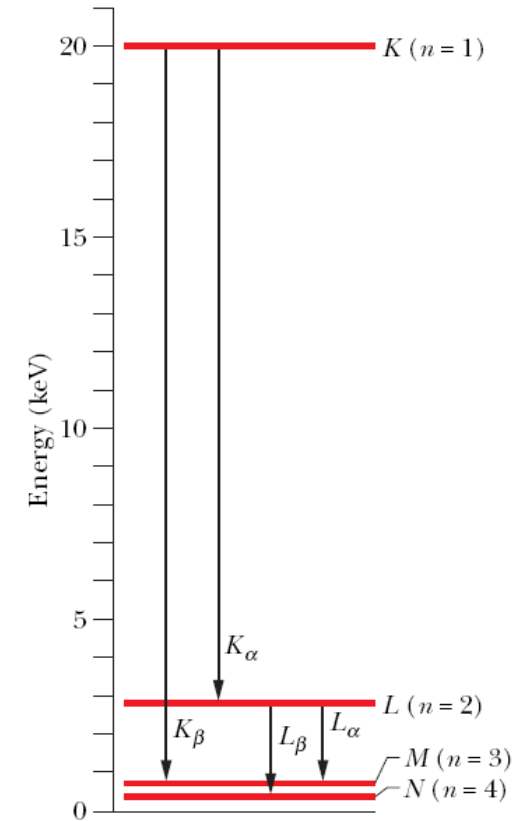
$$K_0 = hf = \frac{hc}{\lambda_{min}},$$

$$\lambda_{min} = \frac{hc}{K_0} \qquad \text{(cutoff wavelength)}.$$

Fig. 40-14 An electron of kinetic energy K_0 passing near an atom in the target may generate an x-ray photon, the electron losing part of its energy in the process. The continuous x-ray spectrum arises in this way.

Fig. 40-13 The distribution by wavelength of the x rays produced when 35 keV electrons strike a molybdenum target

Fig. 40-15 A simplified energy-level diagram for a molybdenum atom, showing the transitions (of holes rather than electrons) that give rise to some of the characteristic x rays of that element. Each horizontal line represents the energy of the atom with a hole (a missing electron) in the shell indicated.
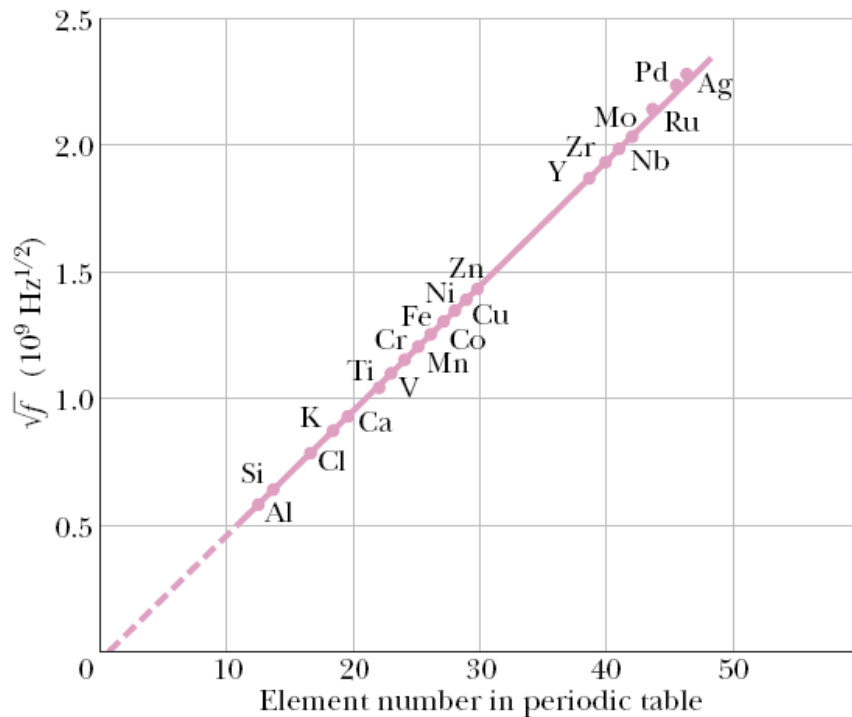
✓ Checkpoint 2

Does the cutoff wavelength λ_{min} of the continuous x-ray spectrum increase, decrease, or remain the same if you (a) increase the kinetic energy of the electrons that strike the x-ray target, (b) allow the electrons to strike a thin foil rather than a thick block of the target material, (c) change the target to an element of higher atomic number?

(a) decrease; (b)–(c) remain the same

$$K_0 = hf = \frac{hc}{\lambda_{min}},$$

$$\lambda_{min} = \frac{hc}{K_0} \qquad \text{(cutoff wavelength).}$$

Accounting for Moseley's Plot:

For hydrogen atom:

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13.60\ \text{eV}}{n^2}, \qquad \text{for } n = 1, 2, 3, \ldots$$

Fig. 40-16 A Moseley plot of the K_α line of the characteristic x-ray spectra of 21 elements. The frequency is calculated from the measured wavelength.

For one of the two innermost electrons in the K shell of a multi-electron atom, because of the presence of the other K-shell electron, it "sees" an effective nuclear charge of approximately $(Z - 1)e$, where e is the electronic charge. Therefore the effective energy of the atom is:

$$E_n = -\frac{(13.60\ \text{eV})(Z - 1)^2}{n^2}.$$

Therefore,

$$\Delta E = E_2 - E_1$$

$$= \frac{-(13.60\ \text{eV})(Z - 1)^2}{2^2} - \frac{-(13.60\ \text{eV})(Z - 1)^2}{1^2}$$

$$= (10.2\ \text{eV})(Z - 1)^2.$$

And,

$$f = \frac{\Delta E}{h} = \frac{(10.2\ \text{eV})(Z - 1)^2}{(4.14 \times 10^{-15}\ \text{eV} \cdot \text{s})}$$

$$= (2.46 \times 10^{15}\ \text{Hz})(Z - 1)^2.$$

➡ $\sqrt{f} = CZ - C,$

C is a constant $(= 4.96 \times 10^7\ \text{Hz}^{1/2})$.

Sample Problem 40.03 Characteristic spectrum in x-ray production

A cobalt target is bombarded with electrons, and the wavelengths of its characteristic x-ray spectrum are measured. There is also a second, fainter characteristic spectrum, which is due to an impurity in the cobalt. The wavelengths of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity), and the proton number for cobalt is $Z_{Co} = 27$. Determine the impurity using only these data.

KEY IDEA

The wavelengths of the K_α lines for both the cobalt (Co) and the impurity (X) fall on a K_α Moseley plot, and Eq. 40-27 is the equation for that plot.

Calculations: Substituting c/λ for f in Eq. 40-27, we obtain

$$\sqrt{\frac{c}{\lambda_{Co}}} = CZ_{Co} - C \quad \text{and} \quad \sqrt{\frac{c}{\lambda_X}} = CZ_X - C.$$

Dividing the second equation by the first neatly eliminates C, yielding

$$\sqrt{\frac{\lambda_{Co}}{\lambda_X}} = \frac{Z_X - 1}{Z_{Co} - 1}.$$

Substituting the given data yields

$$\sqrt{\frac{178.9 \text{ pm}}{143.5 \text{ pm}}} = \frac{Z_X - 1}{27 - 1}.$$

Solving for the unknown, we find that

$$Z_X = 30.0. \qquad \text{(Answer)}$$

Thus, the number of protons in the impurity nucleus is 30, and a glance at the periodic table identifies the impurity as zinc. Note that with a larger value of Z than cobalt, zinc has a smaller value of the K_α line. This means that the energy associated with that jump must be greater in zinc than cobalt.

40.7: Lasers and Laser Light:

1. *Laser light is highly monochromatic.*
2. *Laser light is highly coherent.*
3. *Laser light is highly directional.*
4. *Laser light can be sharply focused.*

Fig. 40-17 A patient's head is scanned and mapped by (red) laser light in preparation for brain surgery. During the surgery, the laser-derived image of the head will be superimposed on the model of the brain shown on the monitor, to guide the surgical team into the region shown in green on the model. *(Sam Ogden/Photo Researchers)*
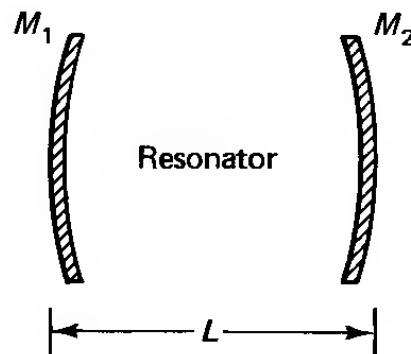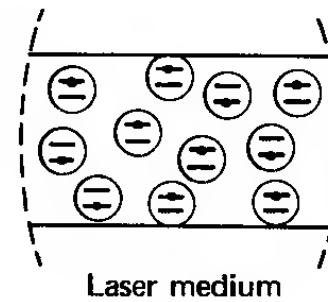
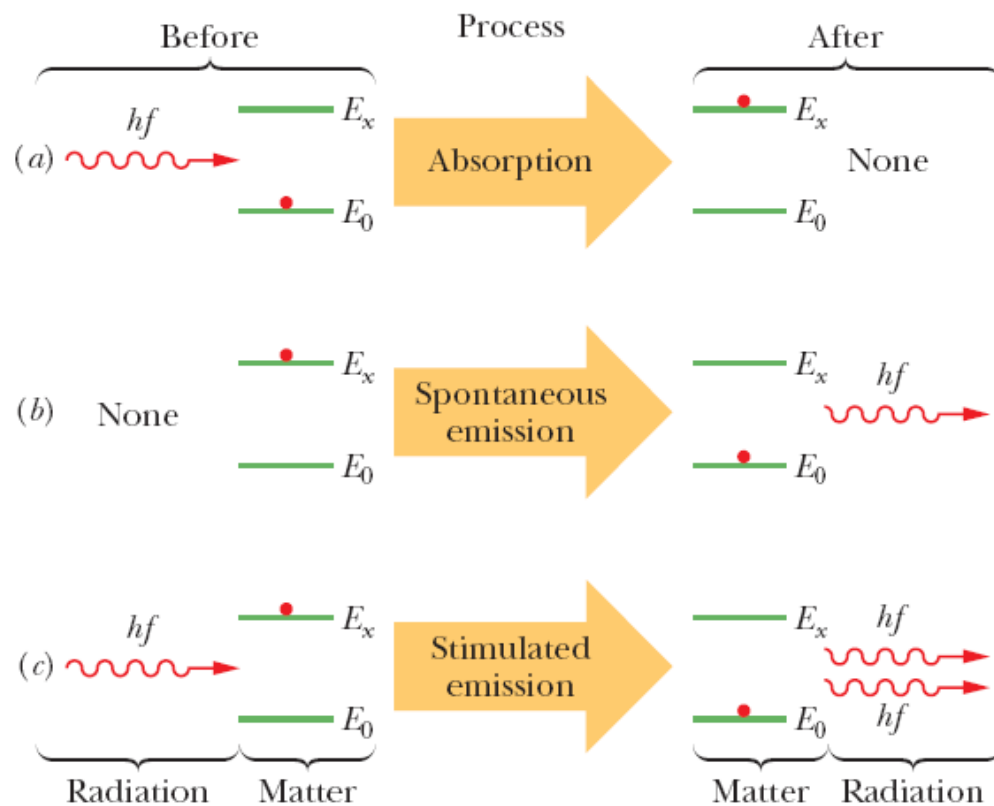# Essential Elements of LASER
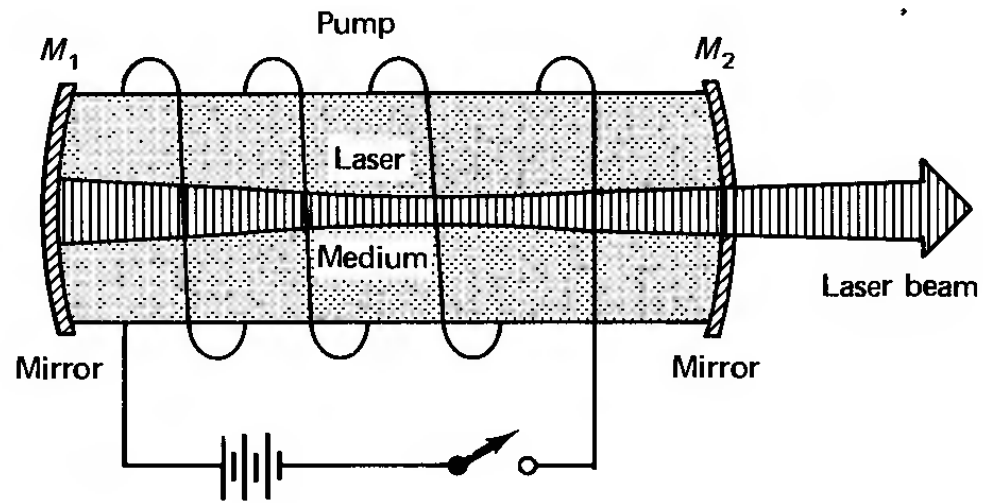
(a) Laser
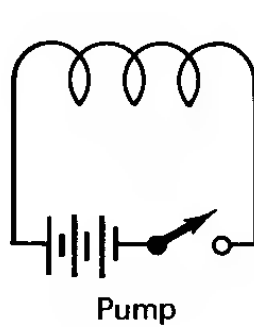
(b) Pump

(c) Resonator

(d) Medium

How Lasers Work:

Fig. 40-18 The interaction of radiation and matter in the processes of (a) absorption, (b) spontaneous emission, and (c) stimulated emission. An atom (matter) is represented by the red dot; the atom is in either a lower quantum state with energy E_0 or a higher quantum state with energy E_x. In (a) the atom absorbs a photon of energy hf from a passing light wave. In (b) it emits a light wave by emitting a photon of energy hf. In (c) a passing light wave with photon energy hf causes the atom to emit a photon of the same energy, increasing the energy of the light wave.

How Lasers Work:
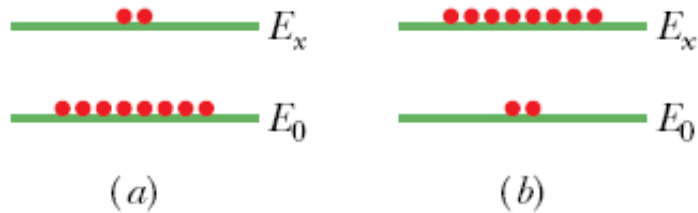
Fig. 40-19 (a) The equilibrium distribution of atoms between the ground state E_0 and excited state E_x accounted for by thermal agitation. (b) An inverted population, obtained by special methods. Such a population inversion is essential for laser action.
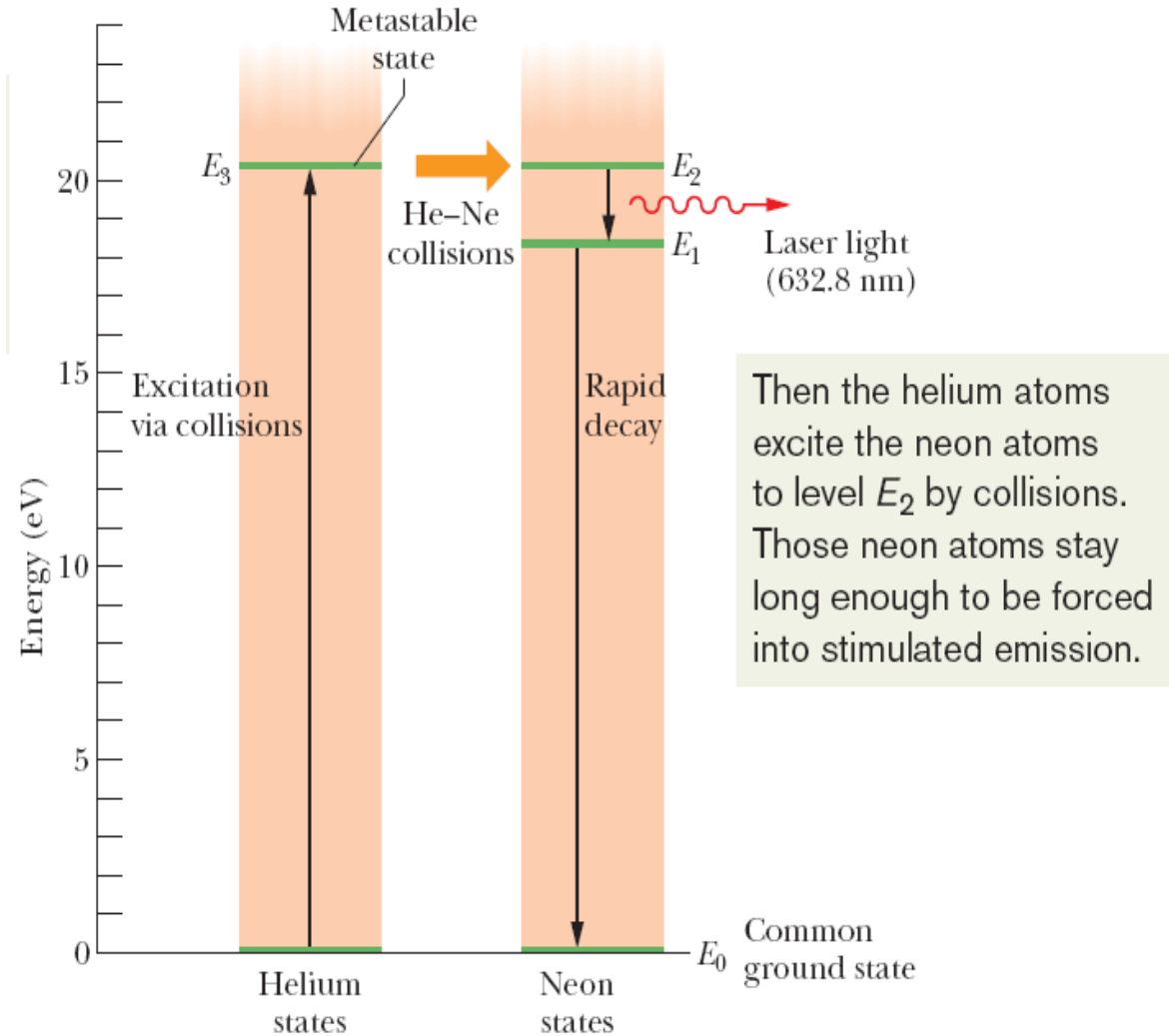
If the atoms of Fig. 40-19a are flooded with photons of energy $(E_x - E_0)$, photons will disappear via absorption by ground-state atoms and photons will be generated largely via stimulated emission of excited-state atoms. Thus, because there are more atoms in the ground state, the *net effect* will be the absorption of photons.

To produce laser light, one must have more photons emitted than absorbed; that is, one must have a situation in which stimulated emission dominates. Thus, one needs more atoms in the excited state than in the ground state, as in Fig. 40-19b.

The Helium-Neon Gas Laser:
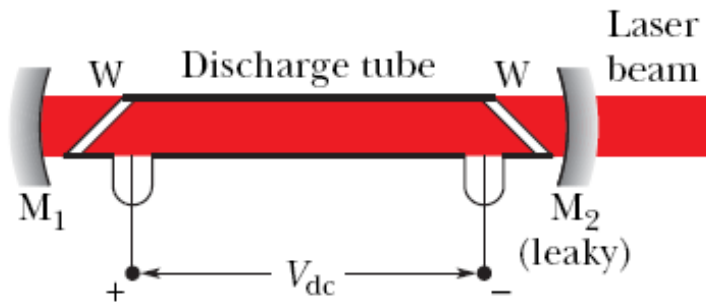
Fig. 40-20 The elements of a helium–neon gas laser. An applied potential V_{dc} sends electrons through a discharge tube containing a mixture of helium gas and neon gas. Electrons collide with helium atoms, which then collide with neon atoms, which emit light along the length of the tube. The light passes through transparent windows W and reflects back and forth through the tube from mirrors M_1 and M_2 to cause more neon atom emissions. Some of the light leaks through mirror M_2 to form the laser beam.

Then the helium atoms excite the neon atoms to level E_2 by collisions. Those neon atoms stay long enough to be forced into stimulated emission.

Fig. 40-21 Five essential energy levels for helium and neon atoms in a helium–neon gas laser. Laser action occurs between levels E_2 and E_1 of neon when more atoms are at the E_2 level than at the E_1 level.

Checkpoint 3

The wavelength of light from laser A (a helium–neon gas laser) is 632.8 nm; that from laser B (a carbon dioxide gas laser) is 10.6 μm; that from laser C (a gallium arsenide semiconductor laser) is 840 nm. Rank these lasers according to the energy interval between the two quantum states responsible for laser action, greatest first.

$$A, C, B$$

Example, Population inversion in a laser:

In the helium–neon laser of Fig. 40-20, laser action occurs between two excited states of the neon atom. However, in many lasers, laser action (*lasing*) occurs between the ground state and an excited state, as suggested in Fig. 40-19*b*.

(a) Consider such a laser that emits at wavelength $\lambda = 550$ nm. If a population inversion is not generated, what is the ratio of the population of atoms in state E_x to the population in the ground state E_0, with the atoms at room temperature?

KEY IDEAS

(1) The naturally occurring population ratio N_x/N_0 of the two states is due to thermal agitation of the gas atoms (Eq. 40-29):

$$N_x/N_0 = e^{-(E_x - E_0)/kT}. \qquad (40\text{-}30)$$

To find N_x/N_0 with Eq. 40-30, we need to find the energy separation $E_x - E_0$ between the two states. (2) We can obtain $E_x - E_0$ from the given wavelength of 550 nm for the lasing between those two states.

Calculation: The lasing wavelength gives us

$$E_x - E_0 = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34}\,\text{J·s})(3.00 \times 10^8\,\text{m/s})}{(550 \times 10^{-9}\,\text{m})(1.60 \times 10^{-19}\,\text{J/eV})}$$

$$= 2.26\,\text{eV}.$$

To solve Eq. 40-30, we also need the mean energy of thermal agitation kT for an atom at room temperature (assumed to be 300 K), which is

$$kT = (8.62 \times 10^{-5}\,\text{eV/K})(300\,\text{K}) = 0.0259\,\text{eV},$$

in which k is Boltzmann's constant.

Substituting the last two results into Eq. 40-30 gives us the population ratio at room temperature:

$$N_x/N_0 = e^{-(2.26\,\text{eV})/(0.0259\,\text{eV})}$$

$$\approx 1.3 \times 10^{-38}. \qquad \text{(Answer)}$$

This is an extremely small number. It is not unreasonable, however. Atoms with a mean thermal agitation energy of only 0.0259 eV will not often impart an energy of 2.26 eV to another atom in a collision.

(b) For the conditions of (a), at what temperature would the ratio N_x/N_0 be 1/2?

Calculation: Now we want the temperature T such that thermal agitation has bumped enough neon atoms up to the higher-energy state to give $N_x/N_0 = 1/2$. Substituting that ratio into Eq. 40-30, taking the natural logarithm of both sides, and solving for T yield

$$T = \frac{E_x - E_0}{k(\ln 2)} = \frac{2.26\,\text{eV}}{(8.62 \times 10^{-5}\,\text{eV/K})(\ln 2)}$$

$$= 38\,000\,\text{K}. \qquad \text{(Answer)}$$