Chapter 35

Interference

35-1 Light as a Wave

The three-dimensional transmission of waves, including light, may often be predicted by Huygens' principle, which states that

> All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.

Figure 1 shows the propagation of a plane wave in vacuum, as portrayed by Huygens' principle.

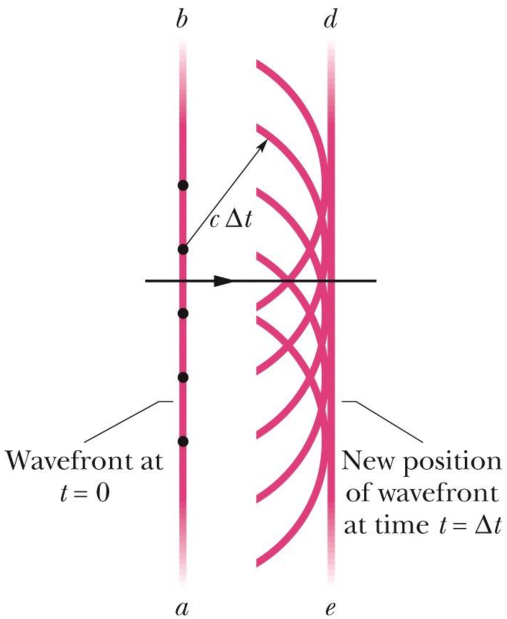
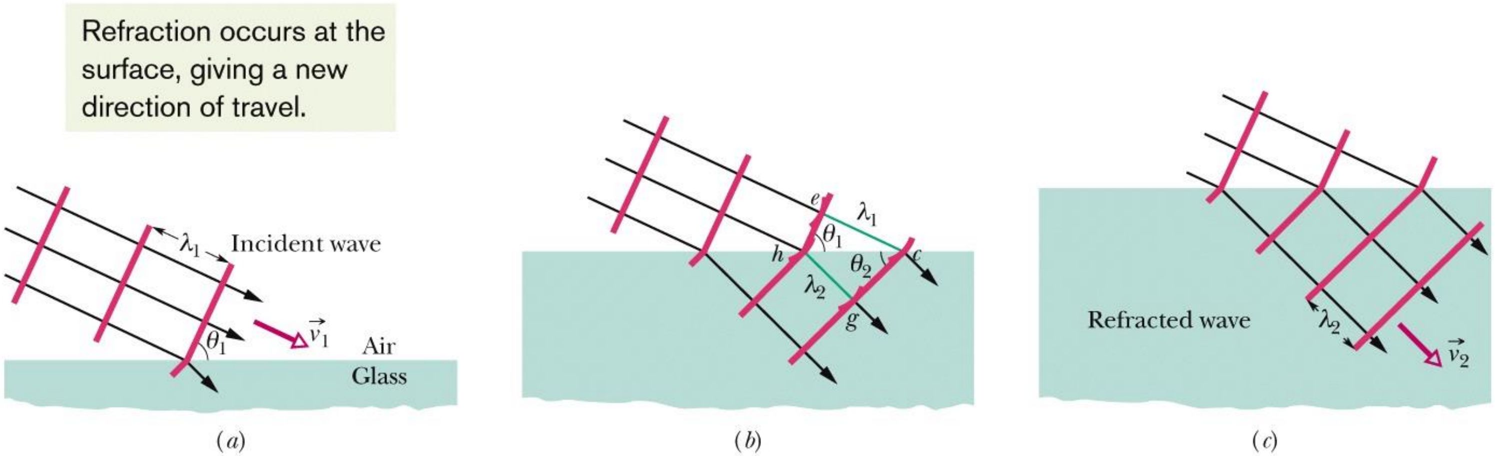
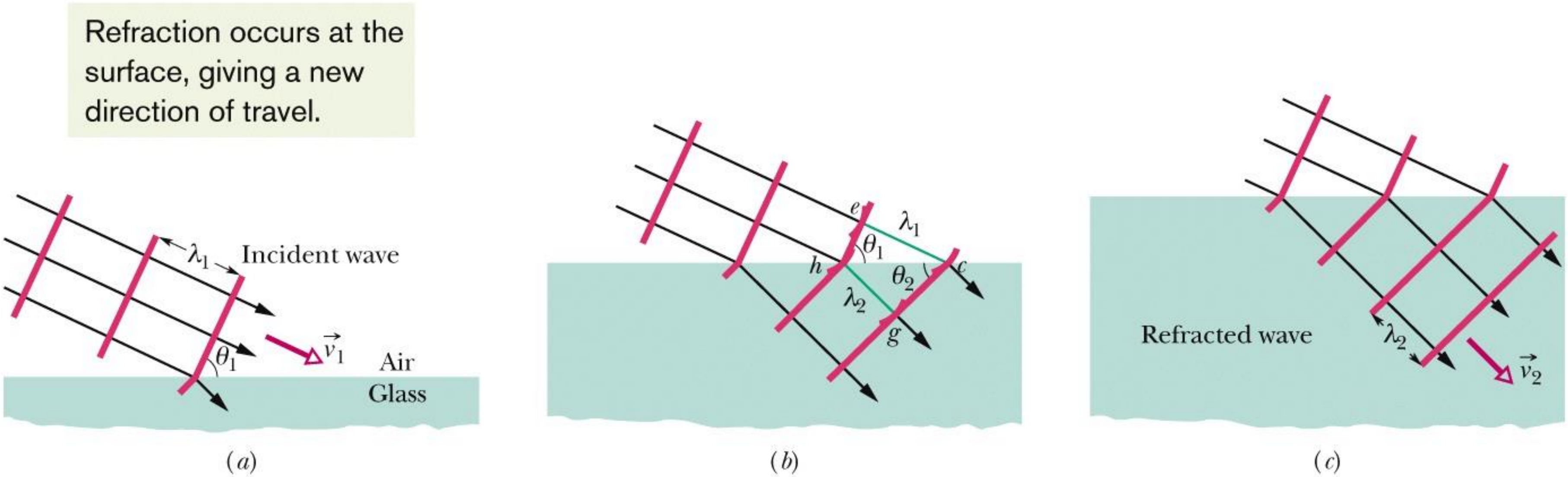


Figure 1



The refraction of a plane wave at an air–glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.



The refraction of a plane wave at an air – glass interface, as portrayed by Huygens' principle. The wavelength in glass is smaller than that in air. For simplicity, the reflected wave is not shown. Parts (a) through (c) represent three successive stages of the refraction.

The law of refraction can be derived from Huygens' principle by assuming that the index of refraction of any medium is

$$n = c/v,$$

in which v is the speed of light in the medium and c is the speed of light in vacuum. The wavelength λ_n of light in a medium depends on the index of refraction n of the medium:

$$\lambda_n = \frac{\lambda}{n},$$

where λ is the wavelength of vacuum

Because of this dependency, the phase difference between two waves can change if they pass through different materials with different indexes of refraction.

For the right triangles *hce* and *hcg* in Fig. 35-3*b* we may write

$$\sin\theta_1 = \frac{\lambda_1}{hc} \quad \text{(for triangle } hce\text{)}$$

and

$$\sin\theta_2 = \frac{\lambda_2}{hc} \quad \text{(for triangle } hcg\text{).}$$

Dividing the first of these two equations by the second and using Eq. 35-1, we find

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}. \tag{35-2}$$

We can define the **index of refraction** n for each medium as the ratio of the speed of light in vacuum to the speed of light v in the medium. Thus,

$$n = \frac{c}{v} \quad \text{(index of refraction).} \tag{35-3}$$

In particular, for our two media, we have

$$n_1 = \frac{c}{v_1} \quad \text{and} \quad n_2 = \frac{c}{v_2}.$$

We can now rewrite Eq. 35-2 as

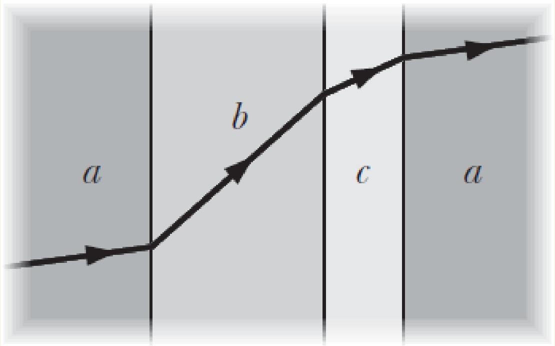
$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

or

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \quad \text{(law of refraction),} \tag{35-4}$$

Checkpoint 1

The figure shows a monochromatic ray of light traveling across parallel interfaces, from an original material a, through layers of materials b and c, and then back into material a. Rank the materials according to the speed of light in them, greatest first.



b (least n), c, a

Wavelength and Index of Refraction

monochromatic light have wavelength λ and speed c in vacuum and wavelength λ_n and speed v in a medium with an index of refraction n. Now we can rewrite Eq. 35-1 as

$$\lambda_n = \lambda \frac{v}{c}. \tag{35-5}$$

Using Eq. 35-3 to substitute $1/n$ for v/c then yields

$$\lambda_n = \frac{\lambda}{n}. \tag{35-6}$$

This equation relates the wavelength of light in any medium to its wavelength in vacuum: A greater index of refraction means a smaller wavelength.

Next, let f_n represent the frequency of the light in a medium with index of refraction n. Then from the general relation of Eq. 16-13 ($v = \lambda f$), we can write

$$f_n = \frac{v}{\lambda_n}.$$

Substituting Eqs. 35-3 and 35-6 then gives us

$$f_n = \frac{c/n}{\lambda/n} = \frac{c}{\lambda} = f,$$

where f is the frequency of the light in vacuum. Thus, although the speed and wavelength of light in the medium are different from what they are in vacuum, *the frequency of the light in the medium is the same as it is in vacuum.*

Phase Difference.

The phase difference between two light waves can change if the waves travel through different materials having different indexes of refraction.

The difference in indexes causes a phase shift between the rays.

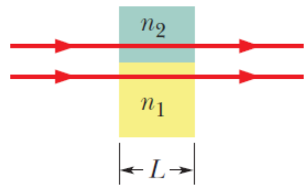


Figure 35-4 Two light rays travel through two media having different indexes of refraction.

To find their new phase difference in terms of wavelengths, we first count the number N_1 of wavelengths there are in the length L of medium 1. From Eq. 35-6, the wavelength in medium 1 is $\lambda_{n1} = \lambda/n_1$; so

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}. \qquad (35\text{-}7)$$

Similarly, we count the number N_2 of wavelengths there are in the length L of medium 2, where the wavelength is $\lambda_{n2} = \lambda/n_2$:

$$N_2 = \frac{L}{\lambda_{n2}} = \frac{Ln_2}{\lambda}. \qquad (35\text{-}8)$$

To find the new phase difference between the waves, we subtract the smaller of N_1 and N_2 from the larger. Assuming $n_2 > n_1$, we obtain

$$N_2 - N_1 = \frac{Ln_2}{\lambda} - \frac{Ln_1}{\lambda} = \frac{L}{\lambda}(n_2 - n_1). \qquad (35\text{-}9)$$

Path Length Difference.

$$\frac{\Delta L}{\lambda} = 0, 1, 2, \ldots \quad \text{(fully constructive interference)}, \qquad (35\text{-}10)$$

and that fully destructive interference (darkness) occurs when

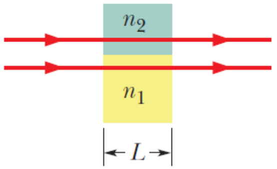
$$\frac{\Delta L}{\lambda} = 0.5, 1.5, 2.5, \ldots \quad \text{(fully destructive interference)}. \qquad (35\text{-}11)$$

Intermediate values correspond to intermediate interference and thus also illumination.

Checkpoint 2

The light waves of the rays in Fig. 35-4 have the same wavelength and amplitude and are initially in phase. (a) If 7.60 wavelengths fit within the length of the top material and 5.50 wavelengths fit within that of the bottom material, which material has the greater index of refraction? (b) If the rays are angled slightly so that they meet at the same point on a distant screen, will the interference there result in the brightest possible illumination, bright intermediate illumination, dark intermediate illumination, or darkness?

The difference in indexes causes a phase shift between the rays.



(a) top; (b) bright intermediate illumination (phase difference is 2.1 wavelengths)

$$N_1 = \frac{L}{\lambda_{n1}} = \frac{Ln_1}{\lambda}.$$

Sample Problem 35.01 Phase difference of two waves due to difference in refractive indexes

In Fig. 35-4, the two light waves that are represented by the rays have wavelength 550.0 nm before entering media 1 and 2. They also have equal amplitudes and are in phase. Medium 1 is now just air, and medium 2 is a transparent plastic layer of index of refraction 1.600 and thickness 2.600 μm.

(a) What is the phase difference of the emerging waves in wavelengths, radians, and degrees? What is their effective phase difference (in wavelengths)?

KEY IDEA

The phase difference of two light waves can change if they travel through different media, with different indexes of refraction. The reason is that their wavelengths are different in the different media. We can calculate the change in phase difference by counting the number of wavelengths that fits into each medium and then subtracting those numbers.

Calculations: When the path lengths of the waves in the two media are identical, Eq. 35-9 gives the result of the subtraction. Here we have $n_1 = 1.000$ (for the air), $n_2 = 1.600$, $L = 2.600\ \mu\text{m}$, and $\lambda = 550.0$ nm. Thus, Eq. 35-9 yields

$$
\begin{aligned}
N_2 - N_1 &= \frac{L}{\lambda}(n_2 - n_1) \\
&= \frac{2.600 \times 10^{-6}\ \text{m}}{5.500 \times 10^{-7}\ \text{m}}(1.600 - 1.000) \\
&= 2.84. \qquad \text{(Answer)}
\end{aligned}
$$

Thus, the phase difference of the emerging waves is 2.84 wavelengths. Because 1.0 wavelength is equivalent to 2π rad and 360°, you can show that this phase difference is equivalent to

$$\text{phase difference} = 17.8\ \text{rad} \approx 1020°. \qquad \text{(Answer)}$$

The effective phase difference is the decimal part of the actual phase difference *expressed in wavelengths.* Thus, we have

$$\text{effective phase difference} = 0.84\ \text{wavelength}. \qquad \text{(Answer)}$$

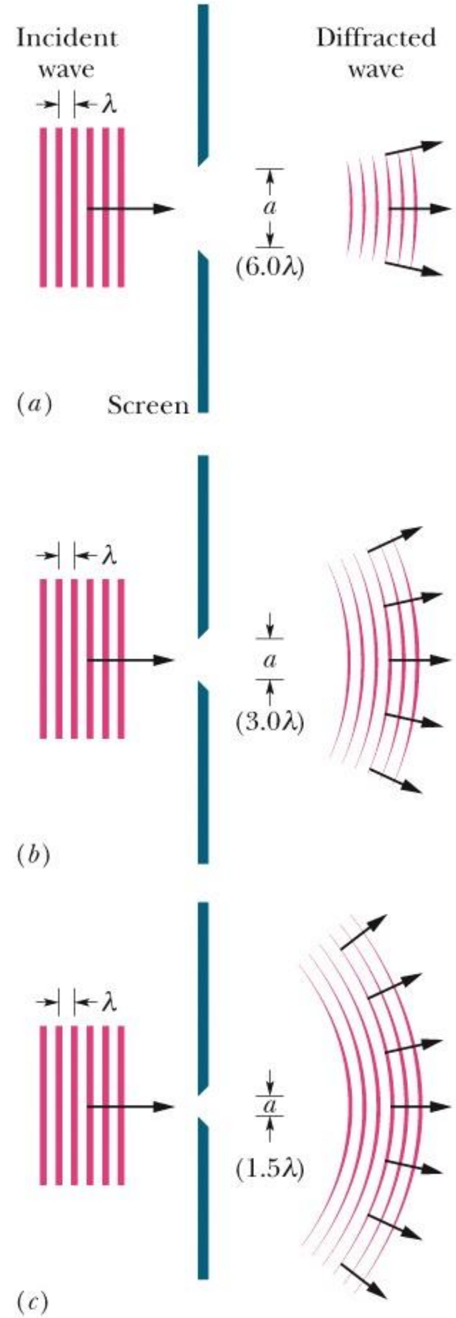
You can show that this is equivalent to 5.3 rad and about 300°. *Caution:* We do *not* find the effective phase difference by taking the decimal part of the actual phase difference as expressed in radians or degrees. For example, we do *not* take 0.8 rad from the actual phase difference of 17.8 rad.

(b) If the waves reached the same point on a distant screen, what type of interference would they produce?

Reasoning: We need to compare the effective phase difference of the waves with the phase differences that give the extreme types of interference. Here the effective phase difference of 0.84 wavelength is between 0.5 wavelength (for fully destructive interference, or the darkest possible result) and 1.0 wavelength (for fully constructive interference, or the brightest possible result), but closer to 1.0 wavelength. Thus, the waves would produce intermediate interference that is closer to fully constructive interference—they would produce a relatively bright spot.

35-2 Young's Interference

The flaring is consistent with the spreading of wavelets in the Huygens construction. **Diffraction** occurs for waves of all types, not just light waves. Figure below shows waves passing through a slit flares.



A wave passing through a slit flares (diffracts).

Figure (a) shows the situation schematically for an incident plane wave of wavelength λ encountering a slit that has width $a = 6.0\ \lambda$ and extends into and out of the page. The part of the wave that passes through the slit flares out on the far side. Figures (b) (with a = 3.0 λ) and (c) (a = 1.5λ) illustrate the main feature of diffraction: the narrower the slit, the greater the diffraction.