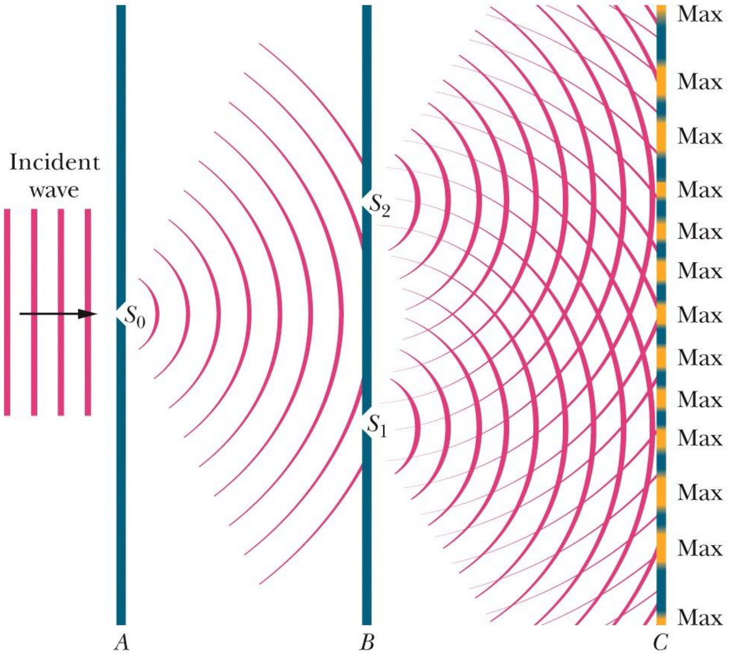
Chapter 35

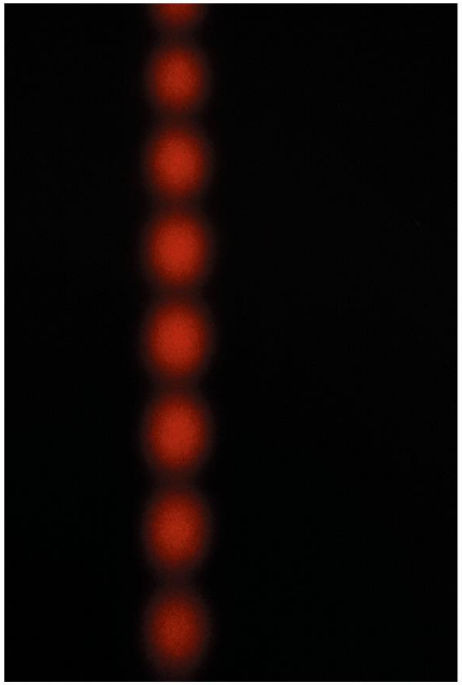
Interference

35-2 Young's Interference

Figure gives the basic arrangement of Young's experiment. Light from a distant monochromatic source illuminates slit S_0 in screen A. The emerging light then spreads via diffraction to illuminate two slits S_1 and S_2 in screen B. Diffraction of the light by these two slits sends overlapping circular waves into the region beyond screen B, where the waves from one slit interfere with the waves from the other slit.

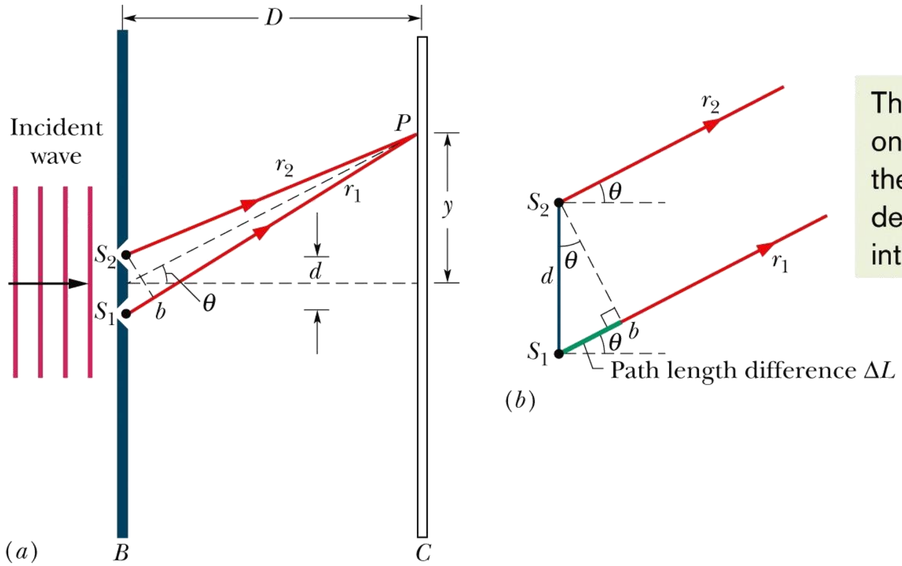


A photograph of the interference pattern produced by the arrangement shown in the figure(right), but with short slits. (The photograph is a front view of part of screen C of figure on left.) The alternating maxima and minima are called interference fringes (because they resemble the decorative fringe sometimes used on clothing and rugs).



Courtesy Jearl Walker

35-2 Young's Interference



The ΔL shifts one wave from the other, which determines the interference.

(a) Waves from slits S_1 and S_2 (which extend into and out of the page) combine at P, an arbitrary point on screen C at distance *y* from the central axis. The angle θ serves as a convenient locator for *P*.

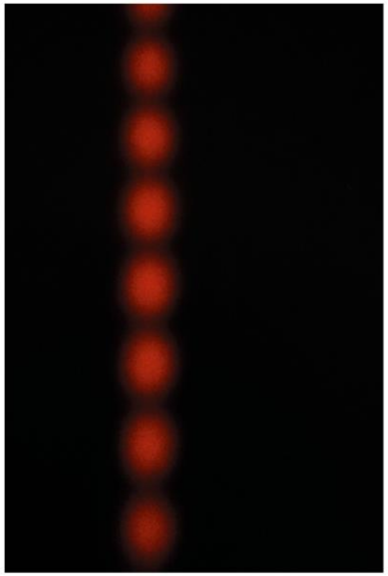
(b) For D >> d, we can approximate rays r_1 and r_2 as being parallel, at angle θ to the central axis.

The phase difference between two waves can change if the waves travel paths of different lengths.

The conditions for maximum and minimum intensity are

$$d \sin \theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$

$$d \sin \theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots$$



Courtesy Jearl Walker

Checkpoint 3

In Fig. 35-10*a*, what are ΔL (as a multiple of the wavelength) and the phase difference (in wavelengths) for the two rays if point P is (a) a third side maximum and (b) a third minimum?

(a) 3λ, 3; (b) 2.5λ, 2.5

Sample Problem 35.02 Double-slit interference pattern

What is the distance on screen C in Fig. 35-10a between adjacent maxima near the center of the interference pattern? The wavelength λ of the light is 546 nm, the slit separation d is 0.12 mm, and the slit–screen separation D is 55 cm. Assume that θ in Fig. 35-10 is small enough to permit use of the approximations $\sin\theta \approx \tan\theta \approx \theta$, in which θ is expressed in radian measure.

KEY IDEAS

(1) First, let us pick a maximum with a low value of m to ensure that it is near the center of the pattern. Then, from the geometry of Fig. 35-10a, the maximum's vertical distance y_m from the center of the pattern is related to its angle θ from the central axis by

$$\tan\theta \approx \theta = \frac{y_m}{D}.$$

(2) From Eq. 35-14, this angle θ for the mth maximum is given by

$$\sin\theta \approx \theta = \frac{m\lambda}{d}.$$

Calculations: If we equate our two expressions for angle θ and then solve for y_m, we find

$$y_m = \frac{m\lambda D}{d}. \tag{35-17}$$

For the next maximum as we move away from the pattern's center, we have

$$y_{m+1} = \frac{(m+1)\lambda D}{d}. \tag{35-18}$$

We find the distance between these adjacent maxima by subtracting Eq. 35-17 from Eq. 35-18:

$$\begin{aligned}\Delta y &= y_{m+1} - y_m = \frac{\lambda D}{d}\\ &= \frac{(546 \times 10^{-9}\ \text{m})(55 \times 10^{-2}\ \text{m})}{0.12 \times 10^{-3}\ \text{m}}\\ &= 2.50 \times 10^{-3}\ \text{m} \approx 2.5\ \text{mm}. \qquad \text{(Answer)}\end{aligned}$$

As long as d and θ in Fig. 35-10a are small, the separation of the interference fringes is independent of m; that is, the fringes are evenly spaced.

Sample Problem 35.03 Double-slit interference pattern with plastic over one slit

A double-slit interference pattern is produced on a screen, as in Fig. 35-10; the light is monochromatic at a wavelength of 600 nm. A strip of transparent plastic with index of refraction $n = 1.50$ is to be placed over one of the slits. Its presence changes the interference between light waves from the two slits, causing the interference pattern to be shifted across the screen from the original pattern. Figure 35-11*a* shows the original locations of the central bright fringe ($m = 0$) and the first bright fringes ($m = 1$) above and below the central fringe. The purpose of the plastic is to shift the pattern upward so that the lower $m = 1$ bright fringe is shifted to the center of the pattern. Should the plastic be placed over the top slit (as arbitrarily drawn in Fig. 35-11*b*) or the bottom slit, and what thickness L should it have?

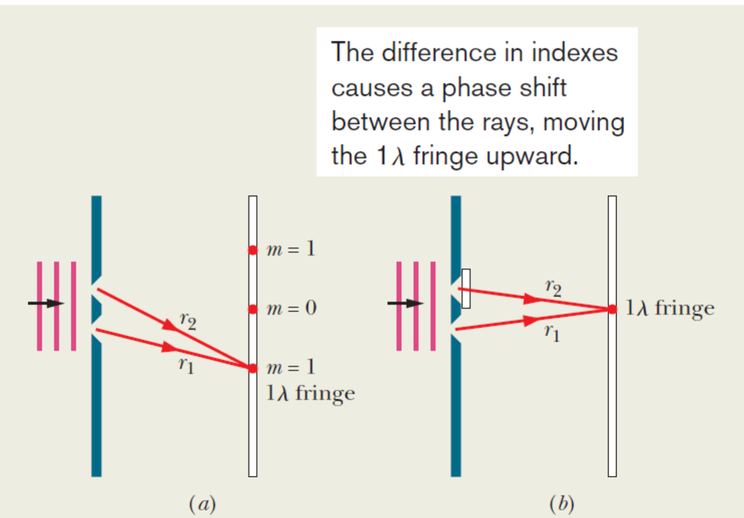


Figure 35-11 (*a*) Arrangement for two-slit interference (not to scale). The locations of three bright fringes (or maxima) are indicated. (*b*) A strip of plastic covers the top slit. We want the 1λ fringe to be at the center of the pattern.

Internal wavelength: The wavelength λ_n of light in a material with index of refraction n is smaller than the wavelength in vacuum, as given by Eq. 35-6 ($\lambda_n = \lambda/n$). Here, this means that the wavelength of the light is smaller in the plastic than in the air. Thus, the ray that passes through the plastic will have more wavelengths along it than the ray that passes through only air—so we do get the one extra wavelength we need along ray r_2 by placing the plastic over the top slit, as drawn in Fig. 35-11*b*.

Thickness: To determine the required thickness L of the plastic, we first note that the waves are initially in phase and travel equal distances L through different materials (plastic and air). Because we know the phase difference and require L, we use Eq. 35-9,

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1). \tag{35-19}$$

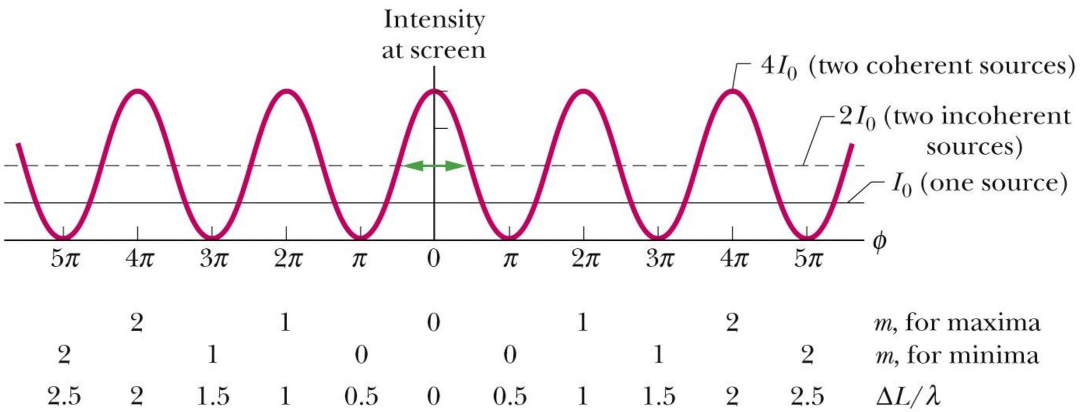
We know that $N_2 - N_1$ is 1 for a phase difference of one wavelength, n_2 is 1.50 for the plastic in front of the top slit, n_1 is 1.00 for the air in front of the bottom slit, and λ is 600×10^{-9} m. Then Eq. 35-19 tells us that, to shift the lower $m = 1$ bright fringe up to the center of the interference pattern, the plastic must have the thickness

$$L = \frac{\lambda(N_2 - N_1)}{n_2 - n_1} = \frac{(600 \times 10^{-9}\ \text{m})(1)}{1.50 - 1.00}$$

$$= 1.2 \times 10^{-6}\ \text{m}. \quad \text{(Answer)}$$

35-3 Interference and Double-Slit Intensity

If two light waves that meet at a point are to interfere perceptibly, both must have the same wavelength and the phase difference between them must remain constant with time; that is, the waves must be coherent.



A plot of equation below, showing the intensity of a double-slit interference pattern as a function of the phase difference between the waves when they arrive from the two slits. I_0 is the (uniform) intensity that would appear on the screen if one slit were covered. The average intensity of the fringe pattern is $2I_0$, and the maximum intensity (for coherent light) is $4I_0$.

As shown in figure, in Young's interference experiment, two waves, each with intensity I_0, yield a resultant wave of intensity I at the viewing screen, with

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi,$$

where

$$\phi = \frac{2\pi d}{\lambda} \sin\theta.$$

$$E_1 = E_0 \sin \omega t \tag{35-20}$$

and

$$E_2 = E_0 \sin(\omega t + \phi), \tag{35-21}$$

where ω is the angular frequency of the waves and ϕ is the phase constant of wave E_2. Note that the two waves have the same amplitude E_0 and a phase difference of ϕ. Because that phase difference does not vary, the waves are coherent. We shall show that these two waves will combine at P to produce an intensity I given by

$$I = 4I_0 \cos^2 \tfrac{1}{2}\phi, \tag{35-22}$$

and that

$$\phi = \frac{2\pi d}{\lambda} \sin\theta. \tag{35-23}$$

Maxima. Study of Eq. 35-22 shows that intensity maxima will occur when

$$\tfrac{1}{2}\phi = m\pi, \qquad \text{for } m = 0, 1, 2, \ldots. \tag{35-24}$$

If we put this result into Eq. 35-23, we find

$$2m\pi = \frac{2\pi d}{\lambda} \sin\theta, \qquad \text{for } m = 0, 1, 2, \ldots$$

or

$$d \sin\theta = m\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima)}, \tag{35-25}$$

which is exactly Eq. 35-14, the expression that we derived earlier for the locations of the maxima.

Minima. The minima in the fringe pattern occur when

$$\tfrac{1}{2}\phi = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 0, 1, 2, \ldots. \tag{35-26}$$

If we combine this relation with Eq. 35-23, we are led at once to

$$d \sin\theta = (m + \tfrac{1}{2})\lambda, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima)}, \tag{35-27}$$

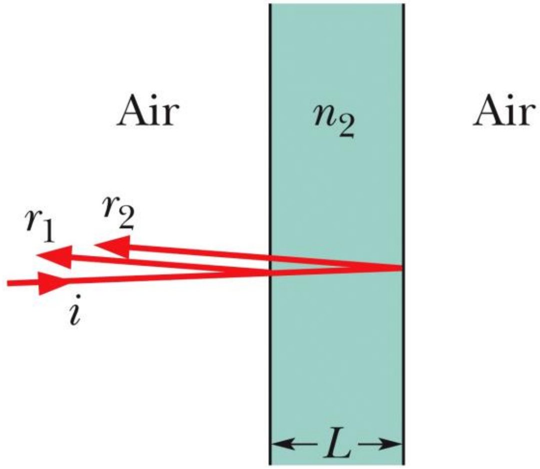
35-4 Interference from thin films

When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and mini- mum intensity of the light reflected from a film with air on both sides are

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

and

$$2L = m\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$



Reflections from a thin film in air.

where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.

If a film is sandwiched between media other than air, these equations for bright and dark films may be interchanged, depending on the relative indexes of refraction.

If the light incident at an interface between media with different indexes of refraction is initially in the medium with the smaller index of refraction, the reflection causes a phase change of π *rad*, or half a wavelength, in the reflected wave. Otherwise, there is no phase change due to the reflection. Refraction causes no phase shift.

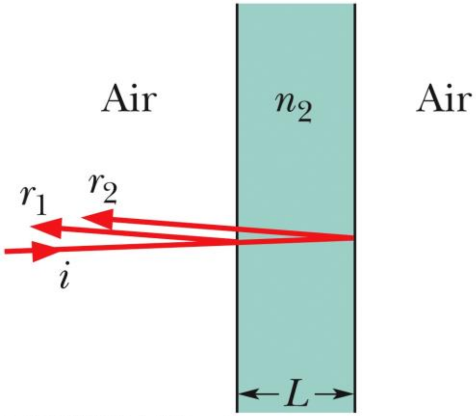
35-4 Interference from thin films

When light is incident on a thin transparent film, the light waves reflected from the front and back surfaces interfere. For near-normal incidence, the wavelength conditions for maximum and mini- mum intensity of the light reflected from a film in air are

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(maxima—bright film in air).}$$

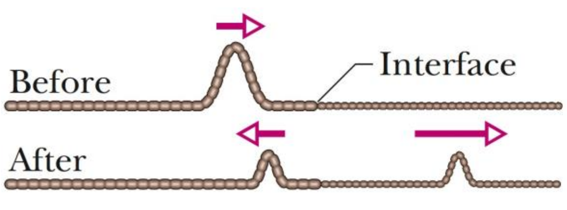
and

$$2L = m\,\frac{\lambda}{n_2}, \qquad \text{for } m = 0, 1, 2, \ldots \quad \text{(minima—dark film in air).}$$



Reflections from a thin film in air.

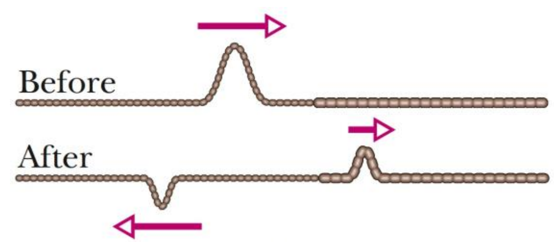
where n_2 is the index of refraction of the film, L is its thickness, and λ is the wavelength of the light in air.



(a)

The incident pulse is in the denser string.

Reflection	Reflection phase shift
Off lower index	0
Off higher index	0.5 wavelength

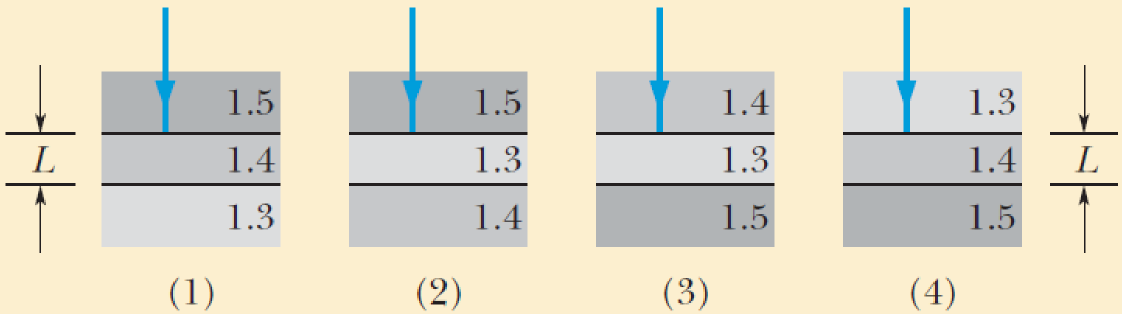


(b)

The incident pulse in the lighter string. Only here is there a phase change, and only in the reflected wave.

Checkpoint 5

The figure shows four situations in which light reflects perpendicularly from a thin film of thickness L, with indexes of refraction as given. (a) For which situations does reflection at the film interfaces cause a zero phase difference for the two reflected rays? (b) For which situations will the film be dark if the path length difference $2L$ causes a phase difference of 0.5 wavelength?



(a) 1 and 4; (b) 1 and 4

Sample Problem 35.05 Thin-film interference of a water film in air

White light, with a uniform intensity across the visible wavelength range of 400 to 690 nm, is perpendicularly incident on a water film, of index of refraction $n_2 = 1.33$ and thickness $L = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

KEY IDEA

The reflected light from the film is brightest at the wavelengths λ for which the reflected rays are in phase with one another. The equation relating these wavelengths λ to the given film thickness L and film index of refraction n_2 is either Eq. 35-36 or Eq. 35-37, depending on the reflection phase shifts for this particular film.

Calculations: To determine which equation is needed, we should fill out an organizing table like Table 35-1. However, because there is air on both sides of the water film, the situation here is exactly like that in Fig. 35-17, and thus the table would be exactly like Table 35-1. Then from Table 35-1, we see that the reflected rays are in phase (and thus the film is brightest) when

$$2L = \frac{\text{odd number}}{2} \times \frac{\lambda}{n_2},$$

which leads to Eq. 35-36:

$$2L = (m + \tfrac{1}{2})\frac{\lambda}{n_2}.$$

Solving for λ and substituting for L and n_2, we find

$$\lambda = \frac{2n_2L}{m + \frac{1}{2}} = \frac{(2)(1.33)(320\text{ nm})}{m + \frac{1}{2}} = \frac{851\text{ nm}}{m + \frac{1}{2}}.$$

For $m = 0$, this gives us $\lambda = 1700$ nm, which is in the infrared region. For $m = 1$, we find $\lambda = 567$ nm, which is yellow-green light, near the middle of the visible spectrum. For $m = 2$, $\lambda = 340$ nm, which is in the ultraviolet region. Thus, the wavelength at which the light seen by the observer is brightest is

$$\lambda = 567\text{ nm}. \qquad \text{(Answer)}$$