Chapter 36

Diffraction

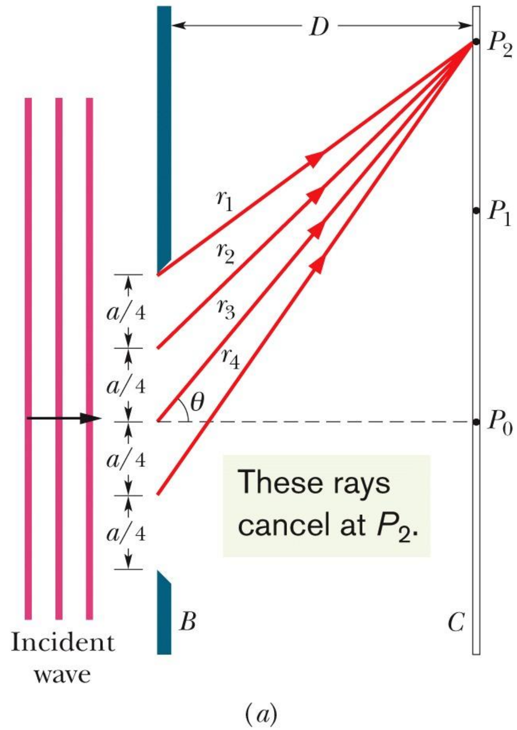
36-1 Single-Slit Diffraction

When waves encounter an edge, an obstacle, or an aperture the size of which is comparable to the wavelength of the waves, those waves spread out as they travel and, as a result, undergo interference. This type of interference is called diffraction.

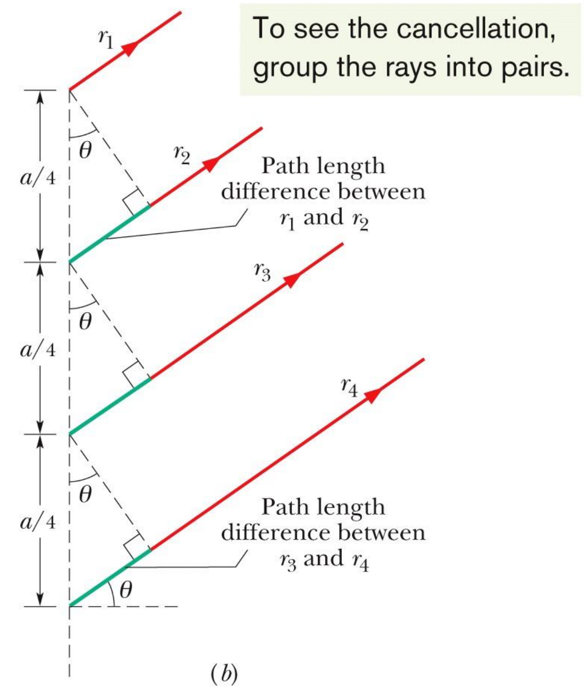
Waves passing through a long narrow slit of width *a* produce, on a viewing screen, a single-slit diffraction pattern that includes a central maximum (bright fringe) and other maxima. They are separated by minima that are located relative to the central axis by angles θ:

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots$$

The maxima are located approximately halfway between minima.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

(a) Waves from the top points of four zones of width a/4 undergo fully destructive interference at point P_2.
(b) For $D >> a$, we can approximate rays r_1, r_2, r_3, and r_4 as being parallel, at angle θ to the central axis.

Checkpoint 1

We produce a diffraction pattern on a viewing screen by means of a long narrow slit illuminated by blue light. Does the pattern expand away from the bright center (the maxima and minima shift away from the center) or contract toward it if we (a) switch to yellow light or (b) decrease the slit width?

(a) expand; (b) expand

$$a \sin\theta = m\lambda, \qquad \text{for } m = 1, 2, 3, \ldots$$

Sample Problem 36.01 Single-slit diffraction pattern with white light

A slit of width a is illuminated by white light.

(a) For what value of a will the first minimum for red light of wavelength $\lambda = 650$ nm appear at $\theta = 15°$?

KEY IDEA

Diffraction occurs separately for each wavelength in the range of wavelengths passing through the slit, with the locations of the minima for each wavelength given by Eq. 36-3 ($a \sin \theta = m\lambda$).

Calculation: When we set $m = 1$ (for the first minimum) and substitute the given values of θ and λ, Eq. 36-3 yields

$$a = \frac{m\lambda}{\sin \theta} = \frac{(1)(650 \text{ nm})}{\sin 15°}$$

$$= 2511 \text{ nm} \approx 2.5\ \mu\text{m}. \qquad \text{(Answer)}$$

For the incident light to flare out that much ($\pm 15°$ to the first minima) the slit has to be very fine indeed—in this case, a mere four times the wavelength. For comparison, note that a fine human hair may be about 100 μm in diameter.

(b) What is the wavelength λ' of the light whose first side diffraction maximum is at 15°, thus coinciding with the first minimum for the red light?

KEY IDEA

The first side maximum for any wavelength is about halfway between the first and second minima for that wavelength.

Calculations: Those first and second minima can be located with Eq. 36-3 by setting $m = 1$ and $m = 2$, respectively. Thus, the first side maximum can be located *approximately* by setting $m = 1.5$. Then Eq. 36-3 becomes

$$a \sin \theta = 1.5\lambda'.$$

Solving for λ' and substituting known data yield

$$\lambda' = \frac{a \sin \theta}{1.5} = \frac{(2511 \text{ nm})(\sin 15°)}{1.5}$$

$$= 430 \text{ nm}. \qquad \text{(Answer)}$$

Light of this wavelength is violet (far blue, near the short-wavelength limit of the human range of visible light). From the two equations we used, can you see that the first side maximum for light of wavelength 430 nm will always coincide with the first minimum for light of wavelength 650 nm, no matter what the slit width is? However, the angle θ at which this overlap occurs does depend on slit width. If the slit is relatively narrow, the angle will be relatively large, and conversely.

36-2 Intensity in Single-Slit Diffraction

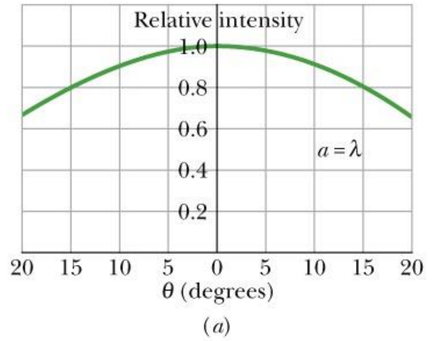
The intensity of the diffraction pattern at any given angle θ is

$$I(\theta) = I_m \left(\frac{\sin \alpha}{\alpha} \right)^2,$$

where, I_m is the intensity at the center of the pattern and

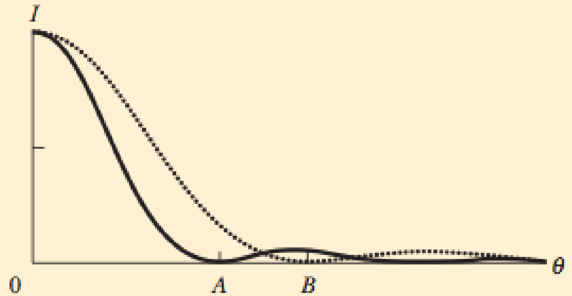
$$\alpha = \tfrac{1}{2}\phi = \frac{\pi a}{\lambda} \sin \theta.$$

The plots show the relative intensity in single-slit diffraction for three values of the ratio a/λ. The wider the slit is, the narrower is the central diffraction maximum.



Checkpoint 3

Two wavelengths, 650 and 430 nm, are used separately in a single-slit diffraction experiment. The figure shows the results as graphs of intensity I versus angle θ for the two diffraction patterns. If both wavelengths are then used simultaneously, what color will be seen in the combined diffraction pattern at (a) angle A and (b) angle B?



Answer
(a) 650 nm
(b) 430 nm

Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

Sample Problem 36.02 Intensities of the maxima in a single-slit interference pattern

Find the intensities of the first three secondary maxima (side maxima) in the single-slit diffraction pattern of Fig. 36-1, measured as a percentage of the intensity of the central maximum.

KEY IDEAS

The secondary maxima lie approximately halfway between the minima, whose angular locations are given by Eq. 36-7 ($\alpha = m\pi$). The locations of the secondary maxima are then given (approximately) by

$$a = (m + \tfrac{1}{2})\pi, \qquad \text{for } m = 1, 2, 3, \ldots,$$

with α in radian measure. We can relate the intensity I at any point in the diffraction pattern to the intensity I_m of the central maximum via Eq. 36-5.

Calculations: Substituting the approximate values of α for the secondary maxima into Eq. 36-5 to obtain the relative intensities at those maxima, we get

$$\frac{I}{I_m} = \left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin(m + \frac{1}{2})\pi}{(m + \frac{1}{2})\pi}\right)^2, \quad \text{for } m = 1, 2, 3, \ldots.$$

The first of the secondary maxima occurs for $m = 1$, and its relative intensity is

$$\frac{I_1}{I_m} = \left(\frac{\sin(1 + \frac{1}{2})\pi}{(1 + \frac{1}{2})\pi}\right)^2 = \left(\frac{\sin 1.5\pi}{1.5\pi}\right)^2$$

$$= 4.50 \times 10^{-2} \approx 4.5\%. \qquad \text{(Answer)}$$

For $m = 2$ and $m = 3$ we find that

$$\frac{I_2}{I_m} = 1.6\% \quad \text{and} \quad \frac{I_3}{I_m} = 0.83\%. \qquad \text{(Answer)}$$

As you can see from these results, successive secondary maxima decrease rapidly in intensity. Figure 36-1 was deliberately overexposed to reveal them.

36-3 Diffraction by a Circular Aperture

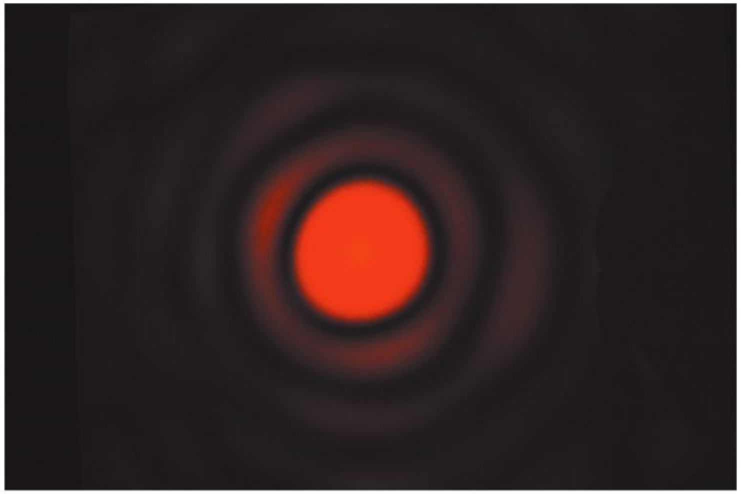
Diffraction by a circular aperture or a lens with diameter *d* produces a central maximum and concentric maxima and minima, given by

$$\sin\theta = 1.22\frac{\lambda}{d} \quad \text{(first minimum—circular aperture).}$$

The angle *θ* here is the angle from the central axis to any point on that (circular) minimum.

$$\sin\theta = \frac{\lambda}{a} \quad \text{(first minimum—single slit),}$$

which locates the first minimum for a long narrow slit of width a. The main difference is the factor 1.22, which enters because of the circular shape of the aperture.

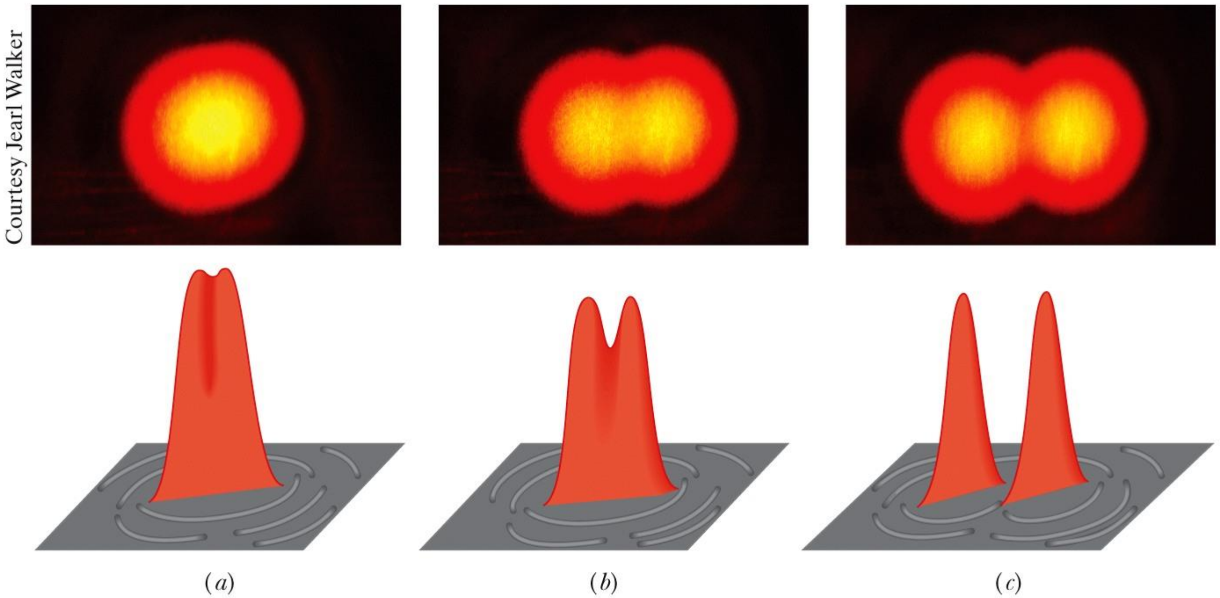


Courtesy Jearl Walker

The diffraction pattern of a circular aperture. Note the central maximum and the circular secondary maxima. The figure has been overexposed to bring out these secondary maxima, which are much less intense than the central maximum.

36-3 Diffraction by a Circular Aperture

Resolvability



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

The images of two point sources (stars) formed by a converging lens. At the bottom, representations of the image intensities. In (a) the angular separation of the sources is too small for them to be distinguished, in (b) they can be marginally distinguished, and in (c) they are clearly distinguished. Rayleigh's criterion is satisfied in (b), with the central maximum of one diffraction pattern coinciding with the first minimum of the other.

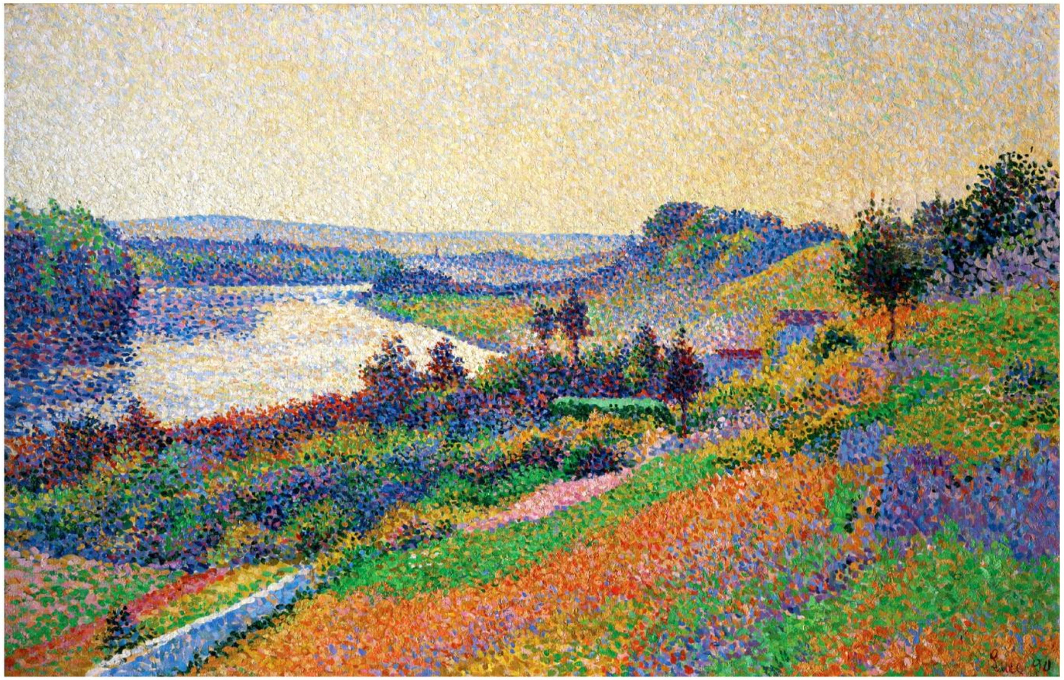
Rayleigh's criterion suggests that two objects are on the verge of resolvability if the central diffraction maximum of one is at the first minimum of the other. Their angular separation can then be no less than

$$\theta_R = 1.22\frac{\lambda}{d} \quad \text{(Rayleigh's criterion).}$$

in which *d* is the diameter of the aperture through which the light passes.

36-3 Diffraction by a Circular Aperture

Pointillism



Maximilien Luce, *The Seine at Herblay*, 1890. Musée d'Orsay, Paris, France. Photo by Erich Lessing/Art Resource

Rayleigh's criterion can explain the arresting illusions of color in the style of painting known as pointillism. In this style, a painting is made not with brush strokes in the usual sense but rather with a myriad of small colored dots. One fascinating aspect of a pointillistic painting is that when you change your distance from it, the colors shift in subtle, almost subconscious ways. This color shifting has to do with whether you can resolve the colored dots.

Checkpoint 4

Suppose that you can barely resolve two red dots because of diffraction by the pupil of your eye. If we increase the general illumination around you so that the pupil decreases in diameter, does the resolvability of the dots improve or diminish? Consider only diffraction. (You might experiment to check your answer.)

$$\theta_R = 1.22 \frac{\lambda}{d} \quad \text{(Rayleigh's criterion)}.$$

diminish

Sample Problem 36.04 Rayleigh's criterion for resolving two distant objects

A circular converging lens, with diameter $d = 32$ mm and focal length $f = 24$ cm, forms images of distant point objects in the focal plane of the lens. The wavelength is $\lambda = 550$ nm.

(a) Considering diffraction by the lens, what angular separation must two distant point objects have to satisfy Rayleigh's criterion?

KEY IDEA

Figure 36-14 shows two distant point objects P_1 and P_2, the lens, and a viewing screen in the focal plane of the lens. It also shows, on the right, plots of light intensity I versus position on the screen for the central maxima of the images formed by the lens. Note that the angular separation θ_o of the objects equals the angular separation θ_i of the images. Thus, if the images are to satisfy Rayleigh's criterion, these separations must be given by Eq. 36-14 (for small angles).

Calculations: From Eq. 36-14, we obtain

$$\theta_o = \theta_i = \theta_R = 1.22\,\frac{\lambda}{d}$$

$$= \frac{(1.22)(550 \times 10^{-9}\text{ m})}{32 \times 10^{-3}\text{ m}} = 2.1 \times 10^{-5}\text{ rad.} \quad \text{(Answer)}$$

Each central maximum in the two intensity curves of Fig. 36-14 is centered on the first minimum of the other curve.

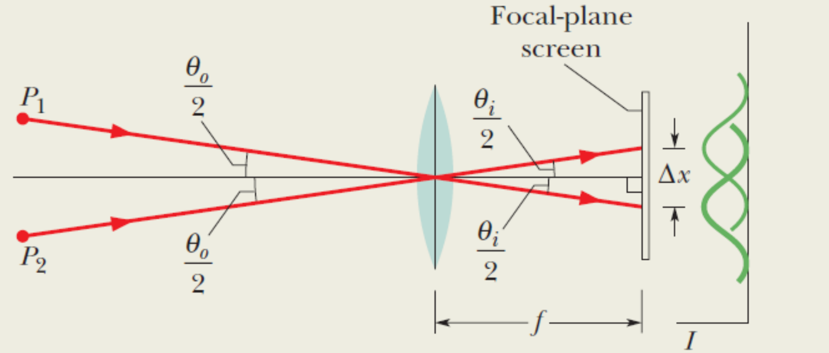


Figure 36-14 Light from two distant point objects P_1 and P_2 passes through a converging lens and forms images on a viewing screen in the focal plane of the lens. Only one representative ray from each object is shown. The images are not points but diffraction patterns, with intensities approximately as plotted at the right.

(b) What is the separation Δx of the centers of the *images* in the focal plane? (That is, what is the separation of the *central* peaks in the two intensity-versus-position curves?)

Calculations: From either triangle between the lens and the screen in Fig. 36-14, we see that $\tan \theta_i/2 = \Delta x/2f$. Rearranging this equation and making the approximation $\tan\theta \approx \theta$, we find

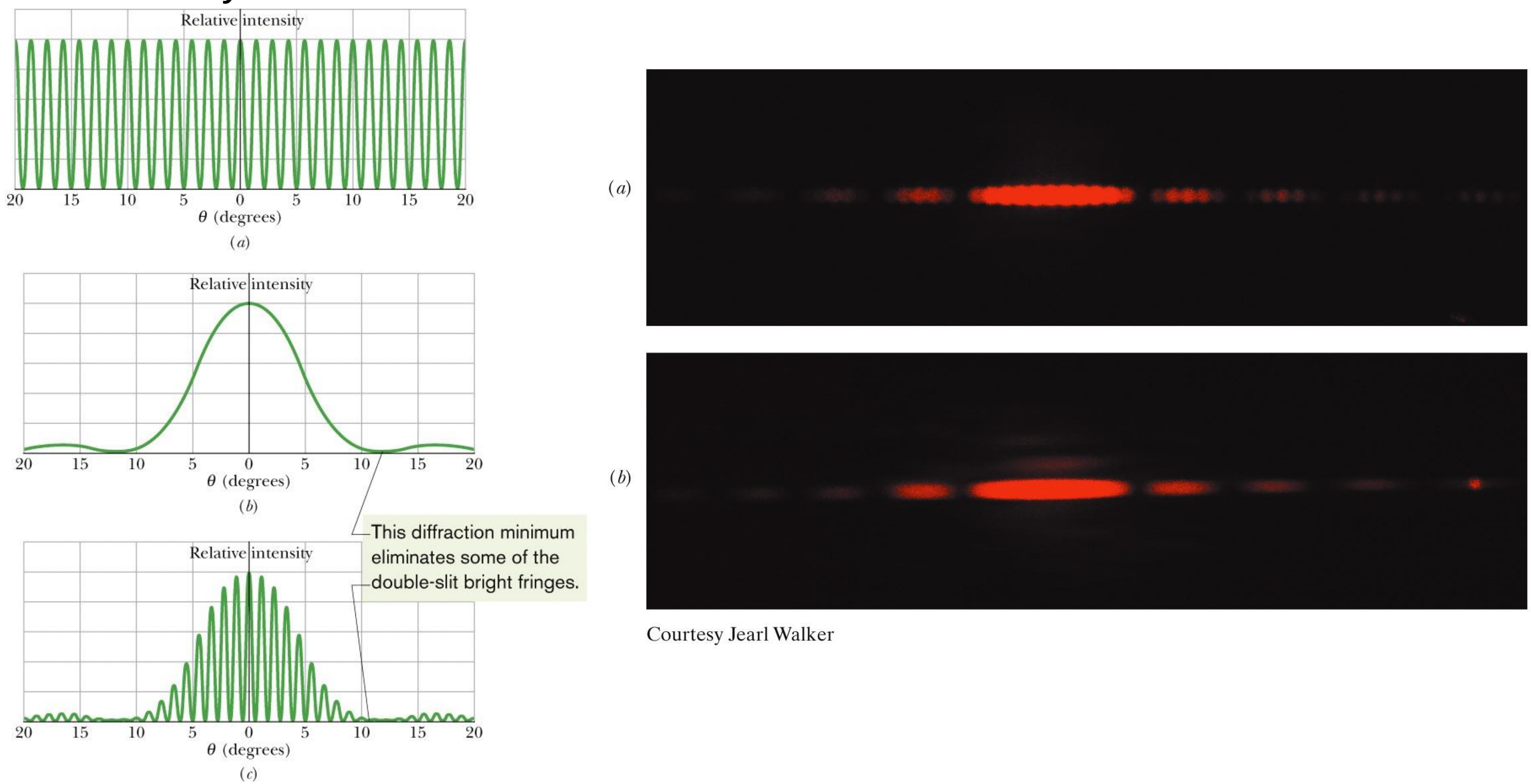
$$\Delta x = f\theta_i, \tag{36-18}$$

where θ_i is in radian measure. We then find

$$\Delta x = (0.24\text{ m})(2.1 \times 10^{-5}\text{ rad}) = 5.0\ \mu\text{m}. \quad \text{(Answer)}$$

36-4 Diffraction by a Double Slit

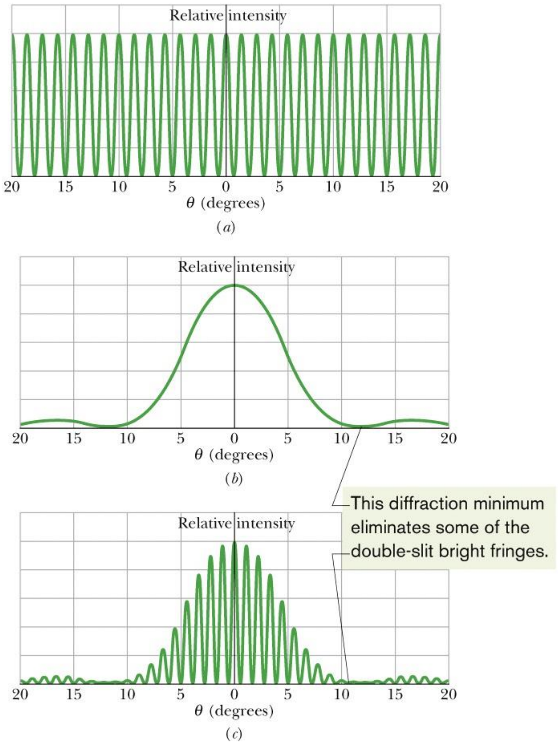
Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

(a) The intensity plot to be expected in a double-slit interference experiment with vanishingly narrow slits. (b) The intensity plot for diffraction by a typical slit of width a (not vanishingly narrow). (c) The intensity plot to be expected for two slits of width a. The curve of (b) acts as an envelope, limiting the intensity of the double-slit fringes in (a). Note that the first minima of the diffraction pattern of (b) eliminate the double-slit fringes that would occur near 12° in (c).

Waves passing through two slits produce a combination of double-slit interference and diffraction by each slit.



Copyright © 2014 John Wiley & Sons, Inc. All rights reserved.

For identical slits with width a and center-to-center separation d, the intensity in the pattern varies with the angle θ from the central axis as

$$I(\theta) = I_m(\cos^2 \beta)\left(\frac{\sin \alpha}{\alpha}\right)^2 \quad \text{(double slit)},$$

in which

$$\beta = \frac{\pi d}{\lambda} \sin \theta$$

and

$$\alpha = \frac{\pi a}{\lambda} \sin \theta.$$

Note carefully that the right side of double slit equation is the product of I_m and two factors. (1) The interference factor $\cos^2\beta$ is due to the interference between two slits with slit separation d. (2) The diffraction factor [(sin a)/a]^2 is due to diffraction by a single slit of width a.

Sample Problem 36.05 Double-slit experiment with diffraction of each slit included

In a double-slit experiment, the wavelength λ of the light source is 405 nm, the slit separation d is 19.44 μm, and the slit width a is 4.050 μm. Consider the interference of the light from the two slits and also the diffraction of the light through each slit.

(a) How many bright interference fringes are within the central peak of the diffraction envelope?

Calculations: We can locate the first diffraction minimum within the double-slit fringe pattern by dividing Eq. 36-23 by Eq. 36-22 and solving for m_2. By doing so and then substituting the given data, we obtain

$$m_2 = \frac{d}{a} = \frac{19.44\ \mu\text{m}}{4.050\ \mu\text{m}} = 4.8.$$

This tells us that the bright interference fringe for $m_2 = 4$ fits into the central peak of the one-slit diffraction pattern, but the fringe for $m_2 = 5$ does not fit. Within the central diffraction peak we have the central bright fringe ($m_2 = 0$), and four bright fringes (up to $m_2 = 4$) on each side of it. Thus, a total of nine bright fringes of the double-slit interference pattern are within the central peak of the diffraction

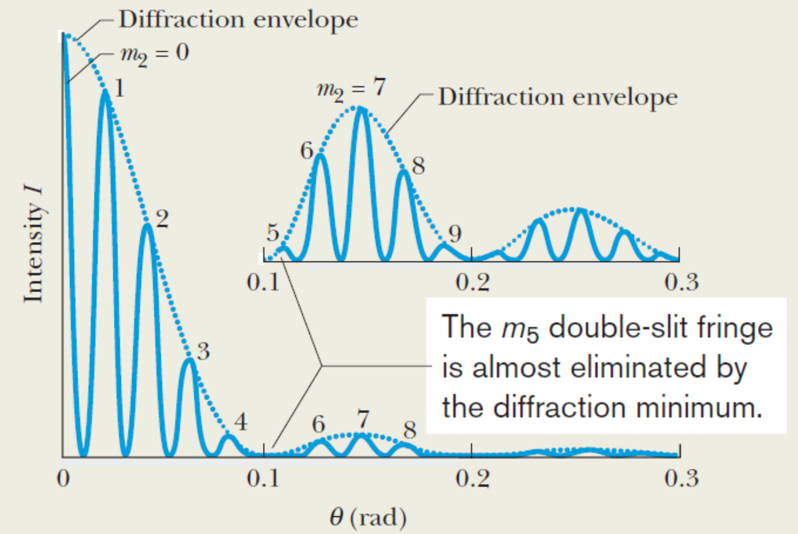


Figure 36-17 One side of the intensity plot for a two-slit interference experiment. The inset shows (vertically expanded) the plot within the first and second side peaks of the diffraction envelope.

(b) How many bright fringes are within either of the first side peaks of the diffraction envelope?

Calculation: Dividing Eq. 36-23 by Eq. 36-24, we find

$$m_2 = \frac{2d}{a} = \frac{(2)(19.44\ \mu\text{m})}{4.050\ \mu\text{m}} = 9.6.$$

This tells us that the second diffraction minimum occurs just before the bright interference fringe for $m_2 = 10$ in Eq. 36-23. Within either first side diffraction peak we have the fringes from $m_2 = 5$ to $m_2 = 9$, for a total of five bright fringes of the double-slit interference pattern (shown in the inset of Fig. 36-17). However, if the $m_2 = 5$ bright fringe, which is almost eliminated by the first diffraction minimum, is considered too dim to count, then only four bright fringes are in the first side diffraction peak.