# Chapter 38

## Photons and Matter Waves

# 38.1: The Photon, the Quantum of Light:

In 1905, Einstein proposed that electromagnetic radiation (or simply *light)* is quantized and exists in elementary amounts (quanta) that we now call **photons.**

According to that proposal, the quantum of a light wave of frequency $f$ has the energy

$$E = hf \qquad \text{(photon energy)}.$$

Here $h$ is the ***Planck constant,*** which has the value

$$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}.$$

The smallest amount of energy a light wave of frequency $f$ can have is $hf$, the energy of a single photon. If the wave has more energy, its total energy must be an integer multiple of $hf$. The light cannot have an energy of, say, $0.6hf$ or $75.5hf$.

## Checkpoint 1

Rank the following radiations according to their associated photon energies, greatest first: (a) yellow light from a sodium vapor lamp, (b) a gamma ray emitted by a radioactive nucleus, (c) a radio wave emitted by the antenna of a commercial radio station, (d) a microwave beam emitted by airport traffic control radar.

b, a, d, c

# Example, Emission and absorption of light as photons:

## Sample Problem 38.01 Emission and absorption of light as photons

A sodium vapor lamp is placed at the center of a large sphere that absorbs all the light reaching it. The rate at which the lamp emits energy is 100 W; assume that the emission is entirely at a wavelength of 590 nm. At what rate are photons absorbed by the sphere?

### KEY IDEAS

The light is emitted and absorbed as photons. We assume that all the light emitted by the lamp reaches (and thus is absorbed by) the sphere. So, the rate $R$ at which photons are absorbed by the sphere is equal to the rate $R_{\text{emit}}$ at which photons are emitted by the lamp.

***Calculations:*** That rate is

$$R_{\text{emit}} = \frac{\text{rate of energy emission}}{\text{energy per emitted photon}} = \frac{P_{\text{emit}}}{E}.$$

Next, into this we can substitute from Eq. 38-2 ($E = hf$), Einstein's proposal about the energy $E$ of each quantum of light (which we here call a photon in modern language). We can then write the absorption rate as

$$R = R_{\text{emit}} = \frac{P_{\text{emit}}}{hf}.$$

Using Eq. 38-1 ($f = c/\lambda$) to substitute for $f$ and then entering known data, we obtain

$$\begin{aligned} R &= \frac{P_{\text{emit}}\lambda}{hc} \\ &= \frac{(100\ \text{W})(590 \times 10^{-9}\ \text{m})}{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^{8}\ \text{m/s})} \\ &= 2.97 \times 10^{20}\ \text{photons/s}. \qquad \text{(Answer)} \end{aligned}$$

# 38.2: The Photoelectric Effect:

Let us analyze two basic photoelectric experiments, each using the apparatus of Fig. 38-1, in which light of frequency $f$ is directed onto target T and ejects electrons from it. A potential difference $V$ is maintained between target T and collector cup C to sweep up these electrons, said to be **photoelectrons.** This collection produces a **photoelectric current** $i$ that is measured with meter A.

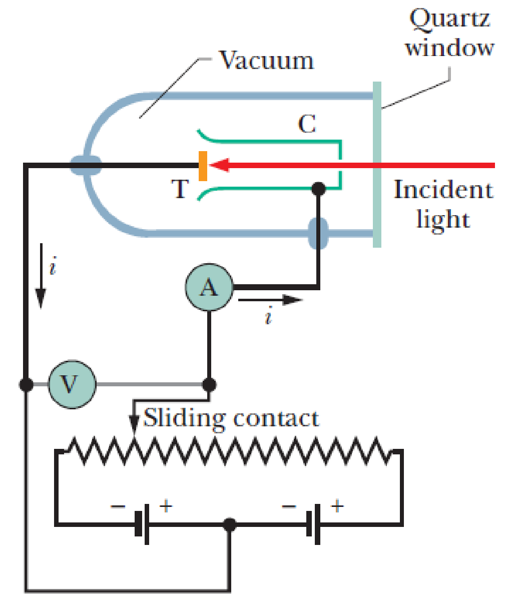



**Fig. 38-1** An apparatus used to study the photoelectric effect. The incident light shines on target T, ejecting electrons, which are collected by collector cup C. The electrons move in the circuit in a direction opposite the conventional current arrows. The batteries and the variable resistor are used to produce and adjust the electric potential difference between T and C.

# I- First Photoelectric Experiment:

We then vary $V$ until it reaches a certain value, called the **stopping potential** $V_{stop}$, at which point the reading of meter A has just dropped to zero. When $V = V_{stop}$, the most energetic ejected electrons are turned back just before reaching the collector. Then $K_{max}$, *the* kinetic energy of these most energetic electrons, is

$$K_{\max} = eV_{\text{stop}},$$

Measurements show that for light of a given frequency, $K_{\max}$ *does not depend on the intensity of the light source.* Whether the source is dazzling bright or so feeble that you can scarcely detect it (or has some intermediate brightness), the maximum kinetic energy of the ejected electrons always has the same value.

# II- Second Photoelectric Experiment:

If the frequency $f$ of the incident light is varied and the associated stopping potential $V_{stop}$ is measured, then the plot of $V_{stop}$ versus $f$ as shown in the figure is obtained. The photoelectric effect does not occur if the frequency is below a certain **cutoff frequency** $f_0$. This is so no matter how intense the incident light is.

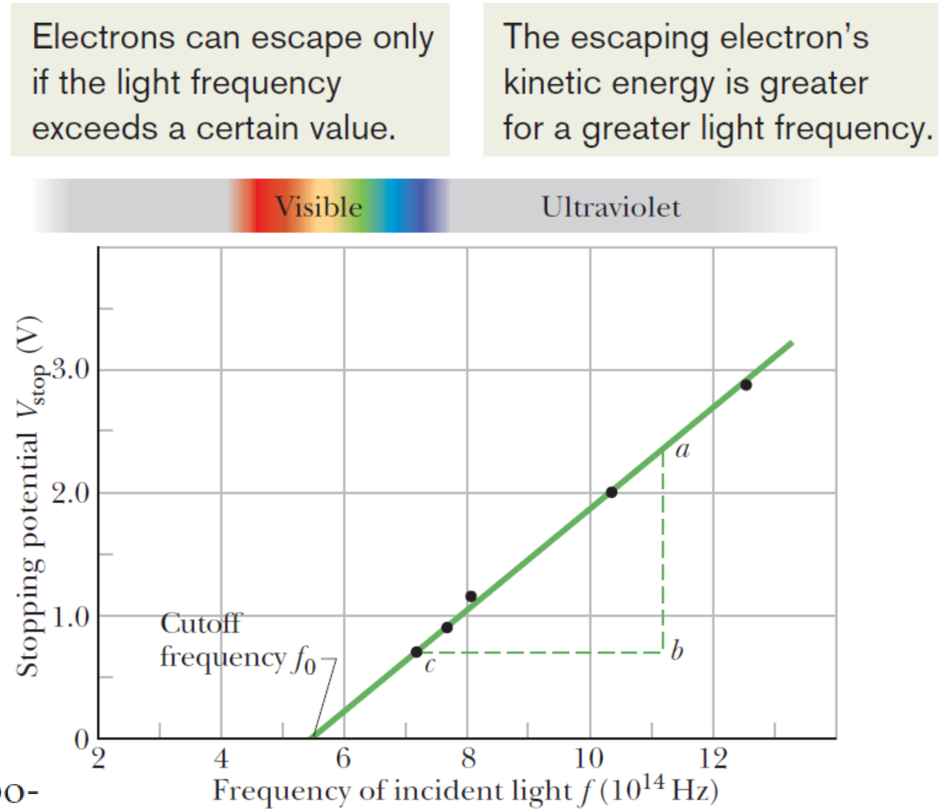



**Figure 38-2** The stopping potential $V_{stop}$ as a function of the frequency $f$ of the incident light for a sodium target T in the apparatus of Fig. 38-1. (Data reported by R. A. Millikan in 1916.)

The electrons within the target are held there by electric forces. To just escape from the target, an electron must pick up a certain minimum energy $\phi$, where $\phi$ is a property of the target material called its **work function.** If the energy $hf$ transferred to an electron by a photon exceeds the work function of the material (if $hf > \phi$), the electron can escape the target.

## The Photoelectric Equation

Einstein summed up the results of such photoelectric experiments in the equation

$$hf = K_{max} + \Phi \quad \text{(photoelectric equation).} \tag{38-5}$$

Let us rewrite Eq. 38-5 by substituting for $K_{max}$ from Eq. 38-4 ($K_{max} = eV_{stop}$). After a little rearranging we get

$$V_{stop} = \left(\frac{h}{e}\right)f - \frac{\Phi}{e}. \tag{38-6}$$

The ratios $h/e$ and $\Phi/e$ are constants, and so we would expect a plot of the measured stopping potential $V_{stop}$ versus the frequency $f$ of the light to be a straight line, as it is in Fig. 38-2. Further, the slope of that straight line should be $h/e$. As a check, we measure $ab$ and $bc$ in Fig. 38-2 and write

$$\begin{aligned}\frac{h}{e} = \frac{ab}{bc} &= \frac{2.35\text{ V} - 0.72\text{ V}}{(11.2 \times 10^{14} - 7.2 \times 10^{14})\text{ Hz}} \\ &= 4.1 \times 10^{-15}\text{ V}\cdot\text{s}.\end{aligned}$$

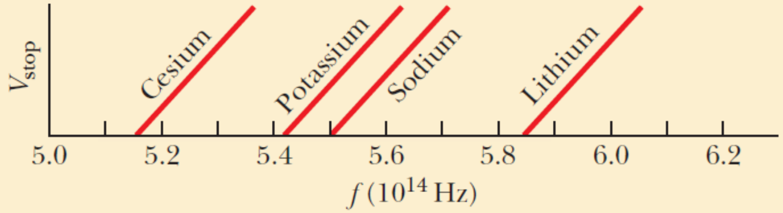
Multiplying this result by the elementary charge $e$, we find

$$h = (4.1 \times 10^{-15}\text{ V}\cdot\text{s})(1.6 \times 10^{-19}\text{ C}) = 6.6 \times 10^{-34}\text{ J}\cdot\text{s},$$

which agrees with values measured by many other methods.

## Checkpoint 2

The figure shows data like those of Fig. 38-2 for targets of cesium, potassium, sodium, and lithium. The plots are parallel. (a) Rank the targets according to their work functions, greatest first. (b) Rank the plots according to the value of $h$ they yield, greatest first.




(a) lithium, sodium, potassium, cesium; (b) all tie

$$V_{stop} = \left(\frac{h}{e}\right)f - \frac{\Phi}{e}.$$

### Sample Problem 38.02 Photoelectric effect and work function

Find the work function $\Phi$ of sodium from Fig. 38-2.

**KEY IDEAS**

We can find the work function $\Phi$ from the cutoff frequency $f_0$ (which we can measure on the plot). The reasoning is this: At the cutoff frequency, the kinetic energy $K_{max}$ in Eq. 38-5 is zero. Thus, all the energy $hf$ that is transferred from a photon to an electron goes into the electron's escape, which requires an energy of $\Phi$.

***Calculations:*** From that last idea, Eq. 38-5 then gives us, with $f = f_0$,

$$hf_0 = 0 + \Phi = \Phi.$$

In Fig. 38-2, the cutoff frequency $f_0$ is the frequency at which the plotted line intercepts the horizontal frequency axis, about $5.5 \times 10^{14}$ Hz. We then have

$$\Phi = hf_0 = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(5.5 \times 10^{14}\ \text{Hz})$$

$$= 3.6 \times 10^{-19}\ \text{J} = 2.3\ \text{eV}. \qquad \text{(Answer)}$$

# 38.3: Photons Have Momentum: Compton Effect:

## Photons Have Momentum

In 1916, Einstein extended his concept of light quanta (photons) by proposing that a quantum of light has linear momentum. For a photon with energy $hf$, the magnitude of that momentum is

$$p = \frac{hf}{c} = \frac{h}{\lambda} \qquad \text{(photon momentum)}$$

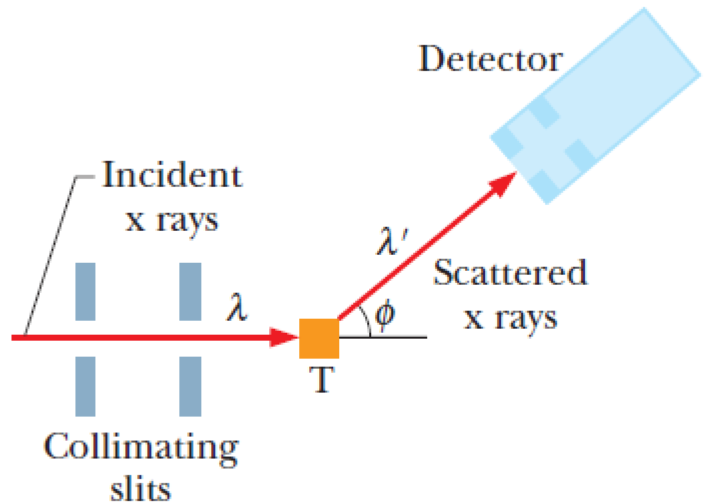



**Fig. 38-3** Compton's apparatus. A beam of x rays of wavelength $\lambda = 71.1$ pm is directed onto a carbon target T. The x rays scattered from the target are observed at various angles $\phi$ to the direction of the incident beam. The detector measures both the intensity of the scattered x rays and their wavelength.

# 38.3: Photons Have Momentum, Compton Effect:

Figure 38-4 shows his results. Although there is only a single wavelength ($\lambda$ = 71.1 pm) in the incident x-ray beam, we see that the scattered x rays contain a range of wavelengths with two prominent intensity peaks. One peak is centered about the incident wavelength $\lambda$, the other about a wavelength $\lambda'$ that is longer than $\lambda$ by an amount $\Delta\lambda$, which is called the **Compton shift.** The value of the Compton shift varies with the angle at which the scattered x rays are detected and is greater for a greater angle.

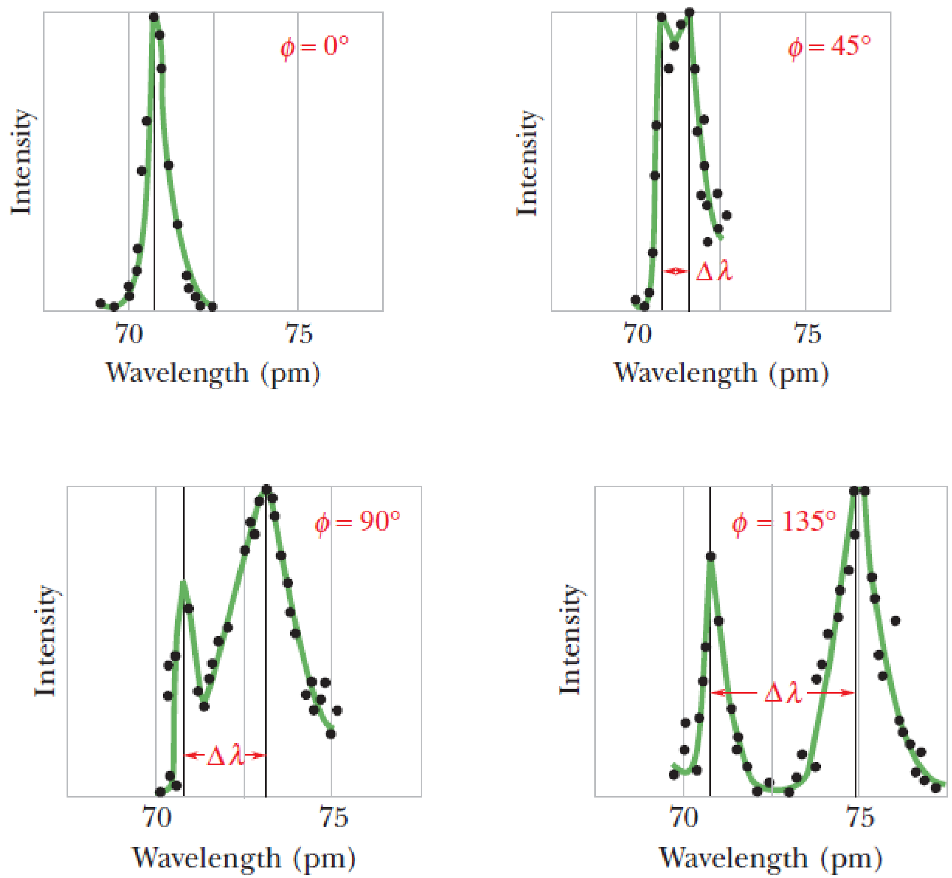



**Fig. 38-4** Compton's results for four values of the scattering angle $\phi$. Note that the Compton shift $\Delta\lambda$ increases as the scattering angle increases.

# 38.3: Photons Have Momentum, Compton Effect:

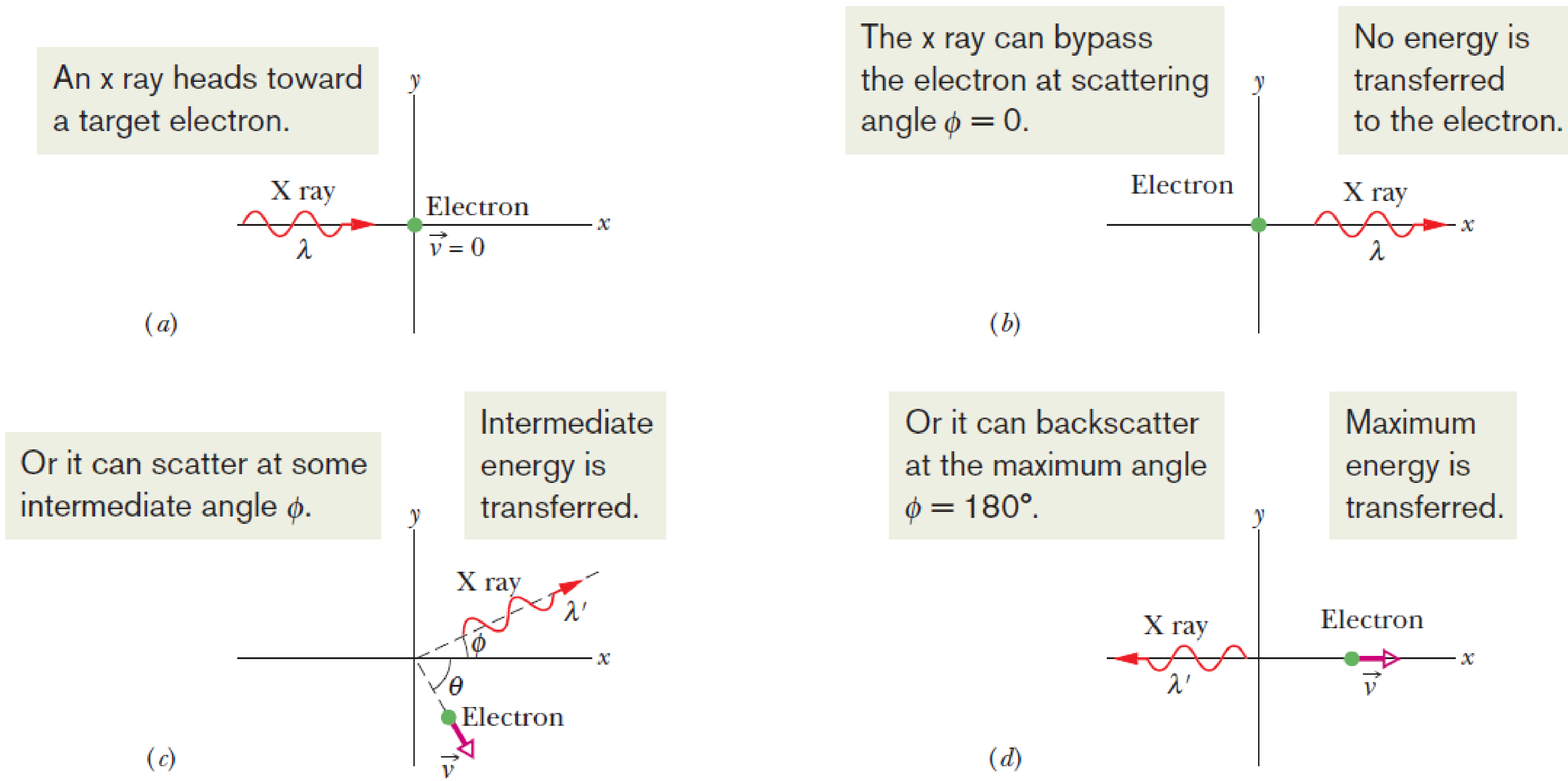



**Fig. 38-5** (*a*) An x ray approaches a stationary electron. The x ray can (*b*) bypass the electron (forward scatter) with no energy or momentum transfer, (*c*) scatter at some intermediate angle with an intermediate energy and momentum transfer, or (*d*) backscatter with the maximum energy and momentum transfer.

As a result of the collision, an x ray of wavelength $\lambda'$ moves off at an angle $\phi$ and the electron moves off at an angle $\theta$, as shown. Conservation of energy then gives us

$$hf = hf' + K$$

$$hf = hf' + K$$

Here *hf* is the energy of the incident x-ray photon, *hf'* is the energy of the scattered x-ray photon, and *K* is the kinetic energy of the recoiling electron. Since the electron may recoil with a speed comparable to that of light,

$$K = mc^2(\gamma - 1), \qquad \gamma = \frac{1}{\sqrt{1 - (v/c)^2}}.$$

$$hf = hf' + mc^2(\gamma - 1).$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} + mc(\gamma - 1).$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'} \cos\phi + \gamma m v \cos\theta \qquad (x \text{ axis})$$

$$0 = \frac{h}{\lambda'} \sin\phi - \gamma m v \sin\theta \qquad (y \text{ axis}).$$

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi) \qquad \text{(Compton shift)}.$$

The quantity *h/mc* in Eq. 38-11 is a constant called the ***Compton wavelength.***

## Checkpoint 3

Compare Compton scattering for x rays ($\lambda \approx 20$ pm) and visible light ($\lambda \approx 500$ nm) at a particular angle of scattering. Which has the greater (a) Compton shift, (b) fractional wavelength shift, (c) fractional energy loss, and (d) energy imparted to the electron?

(a) same; (b)–(d) x rays

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi) \quad \text{(Compton shift).}$$

# Example, Compton Scattering of Light by Electrons:

X rays of wavelength $\lambda = 22$ pm (photon energy = 56 keV) are scattered from a carbon target, and the scattered rays are detected at 85° to the incident beam.

(a) What is the Compton shift of the scattered rays?

**KEY IDEA**

The Compton shift is the wavelength change of the x rays due to scattering from loosely bound electrons in a target. Further, that shift depends on the angle at which the scattered x rays are detected, according to Eq. 38-11. The shift is zero for forward scattering at angle $\phi = 0°$, and it is maximum for back scattering at angle $\phi = 180°$. Here we have an intermediate situation at angle $\phi = 85°$.

***Calculation:*** Substituting 85° for that angle and $9.11 \times 10^{-31}$ kg for the electron mass (because the scattering is from electrons) in Eq. 38-11 gives us

$$\begin{aligned}\Delta\lambda &= \frac{h}{mc}(1 - \cos\phi) \\ &= \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})(1 - \cos 85°)}{(9.11 \times 10^{-31}\ \mathrm{kg})(3.00 \times 10^{8}\ \mathrm{m/s})} \\ &= 2.21 \times 10^{-12}\ \mathrm{m} \approx 2.2\ \mathrm{pm}. \qquad \text{(Answer)}\end{aligned}$$

(b) What percentage of the initial x-ray photon energy is transferred to an electron in such scattering?

**KEY IDEA**

We need to find the *fractional energy loss* (let us call it *frac*) for photons that scatter from the electrons:

$$frac = \frac{\text{energy loss}}{\text{initial energy}} = \frac{E - E'}{E}.$$

***Calculations:*** From Eq. 38-2 ($E = hf$), we can substitute for the initial energy $E$ and the detected energy $E'$ of the x rays in terms of frequencies. Then, from Eq. 38-1 ($f = c/\lambda$), we can substitute for those frequencies in terms of the wavelengths. We find

$$\begin{aligned}frac &= \frac{hf - hf'}{hf} = \frac{c/\lambda - c/\lambda'}{c/\lambda} = \frac{\lambda' - \lambda}{\lambda'} \\ &= \frac{\Delta\lambda}{\lambda + \Delta\lambda}. \qquad (38\text{-}12)\end{aligned}$$

Substitution of data yields

$$frac = \frac{2.21\ \mathrm{pm}}{22\ \mathrm{pm} + 2.21\ \mathrm{pm}} = 0.091, \text{ or } 9.1\%. \qquad \text{(Answer)}$$

Although the Compton shift $\Delta\lambda$ is independent of the wavelength $\lambda$ of the incident x rays (see Eq. 38-11), the *fractional* photon energy loss of the x rays does depend on $\lambda$, increasing as the wavelength of the incident radiation decreases, as indicated by Eq. 38-12.