Chapter 38

Photons and Matter Waves

Light as a Probability Wave

A fundamental mystery in physics is how light can be a wave (which spreads out over a region) in classical physics but be emitted and absorbed as photons (which originate and vanish at points) in quantum physics. The double-slit experiment of Module 35-2 lies at the heart of this mystery. Let us discuss three versions of it.

The Standard Version

The probabilistic description of a light wave is another way to view light. It is not only an electromagnetic wave but also a **probability wave.** That is, to every point in a light wave we can attach a numerical probability (per unit time interval) that a photon can be detected in any small volume centered on that point. This probability is directly related to the square of the amplitude electric field vector at that point.

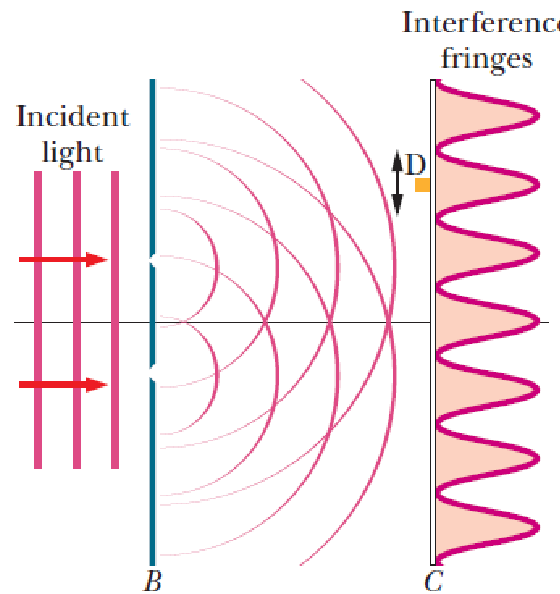


Fig. 38-6 Light is directed onto screen B, which contains two parallel slits. Light emerging from these slits spreads out by diffraction. The two diffracted waves overlap at screen C and form a pattern of interference fringes. A small photon detector D in the plane of screen C generates a sharp click for each photon that it absorbs.

The Single-Photon Version

Consider the double-slit experiment again. Since an interference pattern eventually builds up on the screen, we can speculate that each photon travels from source to screen as a wave that fills up the space between source and screen and then vanishes in a photon absorption at some point on the screen, with a transfer of energy and momentum to the screen at that point.

We cannot predict where this transfer will occur (where a photon will be detected) for any given photon originating at the source.

However, we can predict the probability that a transfer will occur at any given point on the screen.

Transfers will tend to occur (and thus photons will tend to be absorbed) in the regions of the bright fringes in the interference pattern that builds up on the screen. Transfers will tend not to occur (and thus photons will tend not to be absorbed) in the regions of the dark fringes in the built-up pattern.

Thus, we can say that the wave traveling from the source to the screen is a *probability wave,* which produces a pattern of “probability fringes” on the screen.

The Single-Photon, Wide-Angle Version

A *single* photon can take widely different paths and still interfere with itself.

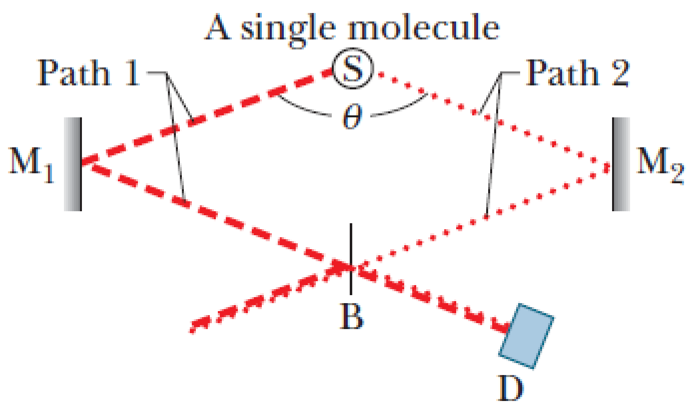


Fig. 38-7 The light from a single photon emission in source S travels over two widely separated paths and interferes with itself at detector D after being recombined by beam splitter B. (After Ming Lai and Jean-Claude Diels, *Journal of the Optical Society of America B*, **9**, 2290–2294, December 1992.)

When a molecule in the source emits a single photon, does that photon travel along path 1 or path 2 in the figure (or along any other path)? Or can it move in both directions at once?

To answer, we assume that when a molecule emits a photon, a probability wave radiates in all directions from it. The experiment samples this wave in two of those directions, chosen to be nearly opposite each other.

We see that we can interpret all three versions of the double-slit experiment if we assume that (1) light is generated in the source as photons, (2) light is absorbed in the detector as photons, and (3) light travels between source and detector as a probability wave.

38.4: The Birth of Quantum Physics

A debate among scientists started in the 1900 on discrepancy between theory and experiment of black body radiation that led to the birth Quantum Physics.

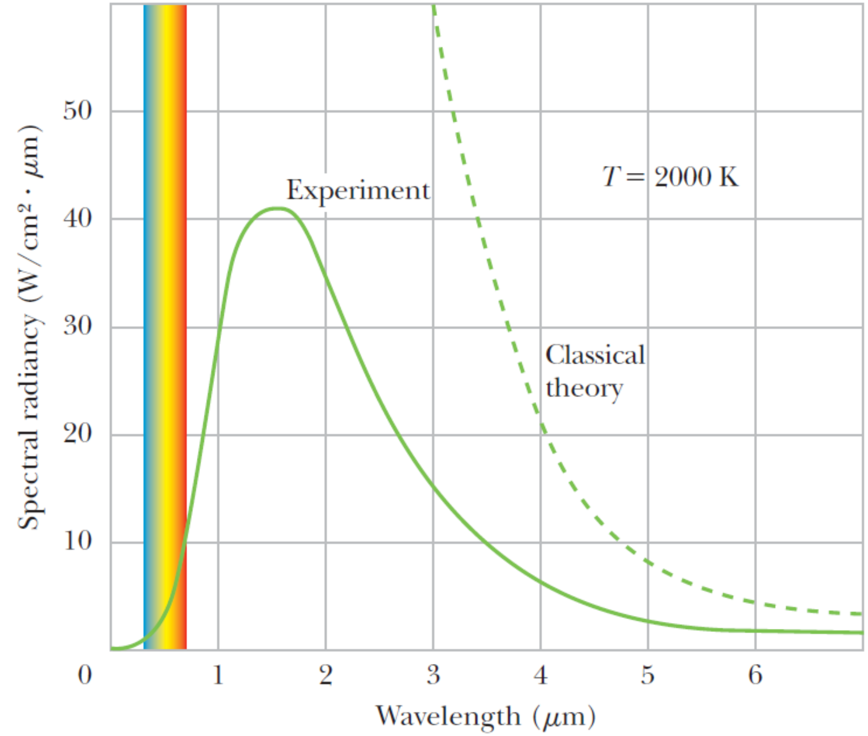


Figure 38-8 The solid curve shows the experimental spectral radiancy for a cavity at 2000 K. Note the failure of the classical theory, which is shown as a dashed curve. The range of visible wavelengths is indicated.

That intensity distribution is handled by defining a **spectral radiancy** $S(\lambda)$ of the radiation emitted at given wavelength λ:

$$S(\lambda) = \frac{\text{intensity}}{\left(\begin{array}{c}\text{unit}\\ \text{wavelength}\end{array}\right)} = \frac{\text{power}}{\left(\begin{array}{c}\text{unit area}\\ \text{of emitter}\end{array}\right)\left(\begin{array}{c}\text{unit}\\ \text{wavelength}\end{array}\right)}. \tag{38-12}$$

If we multiply $S(\lambda)$ by a narrow wavelength range $d\lambda$, we have the intensity (that is, the power per unit area of the hole in the wall) that is being emitted in the wavelength range λ to $\lambda + d\lambda$.

Theory. The prediction of classical physics for the spectral radiancy, for a given temperature T in kelvins, is

$$S(\lambda) = \frac{2\pi ckT}{\lambda^4} \quad \text{(classical radiation law)}, \tag{38-13}$$

where k is the Boltzmann constant (Eq. 19-7) with the value

$$k = 1.38 \times 10^{-23}\,\text{J/K} = 8.62 \times 10^{-5}\,\text{eV/K}.$$

This classical result is plotted in Fig. 38-8 for $T = 2000$ K. Although the theoretical and experimental results agree well at long wavelengths (off the graph to the right), they are not even close in the short wavelength region. Indeed, the theoretical prediction does not even include a maximum as seen in the measured results and instead "blows up" up to infinity (which was quite disturbing, even embarrassing, to the physicists).

Planck's Solution. In 1900, Planck devised a formula for $S(\lambda)$ that neatly fitted the experimental results for all wavelengths and for all temperatures:

$$S(\lambda) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \quad \text{(Planck's radiation law)}. \tag{38-14}$$

Einstein's Solution. No one understood Eq. 38-14 for 17 years, but then Einstein explained it with a very simple model with two key ideas: (1) The energies of the cavity-wall atoms that are emitting the radiation are indeed quantized. (2) The energies of the radiation in the cavity are also quantized in the form of quanta (what we now call photons), each with energy $E = hf$. In his model he explained the processes by which atoms can emit and absorb photons and how the atoms can be in equilibrium with the emitted and absorbed light.

Maximum Value. The wavelength λ_{max} at which the $S(\lambda)$ is maximum (for a given temperature T) can be found by taking the first derivative of Eq. 38-14 with respect to λ, setting the derivative to zero, and then solving for the wavelength. The result is known as Wien's law:

$$\lambda_{max} T = 2898\ \mu\text{m} \cdot \text{K} \qquad \text{(at maximum radiancy).} \tag{38-15}$$

For example, in Fig. 38-8 for which $T = 2000$ K, $\lambda_{max} = 1.5\ \mu$m, which is greater than the long wavelength end of the visible spectrum and is in the infrared region, as shown. If we increase the temperature, λ_{max} decreases and the peak in Fig. 38-8 changes shape and shifts more into the visible range.

Radiated Power. If we integrate Eq. 38-14 over all wavelengths (for a given temperature), we find the power per unit area of a thermal radiator. If we then multiply by the total surface area A, we find the total radiated power P. We have already seen the result in Eq. 18-38 (with some changes in notation):

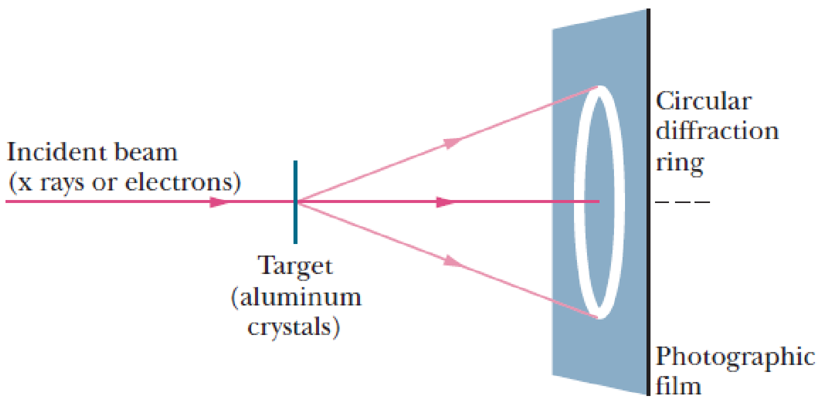
$$P = \sigma \varepsilon A T^4, \tag{38-16}$$

where σ ($= 5.6704 \times 10^{-8}$ W/m$^2 \cdot$ K^4) is the Stefan–Boltzmann constant and ε is the emissivity of the radiating surface ($\varepsilon = 1$ for an ideal blackbody radiator).

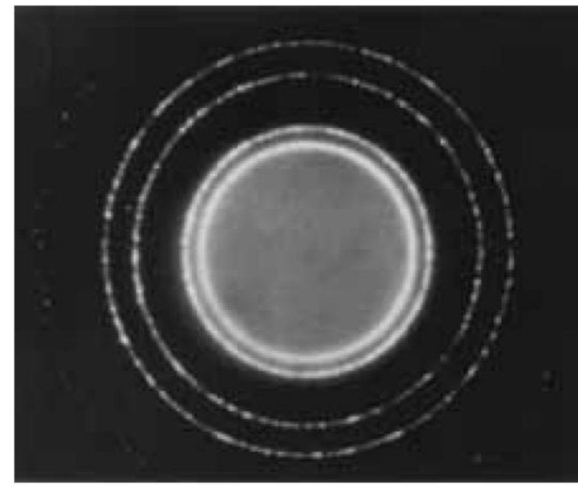
38.5: Electrons and Matter Waves:

$$\lambda = \frac{h}{p} \qquad \text{(de Broglie wavelength)}.$$

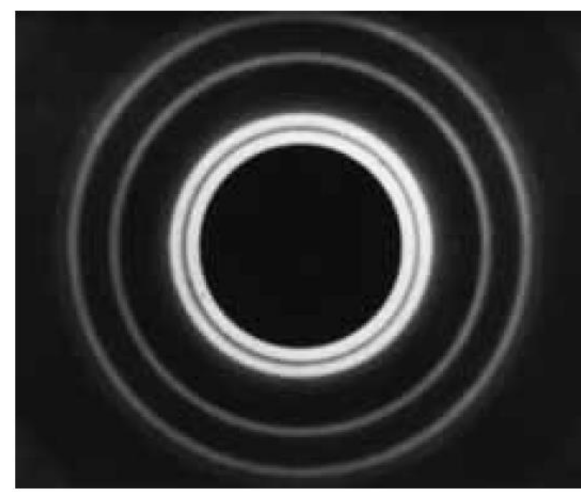
de Broglie suggested that $p = h/\lambda$ might apply not only to photons but also to electrons



(a)



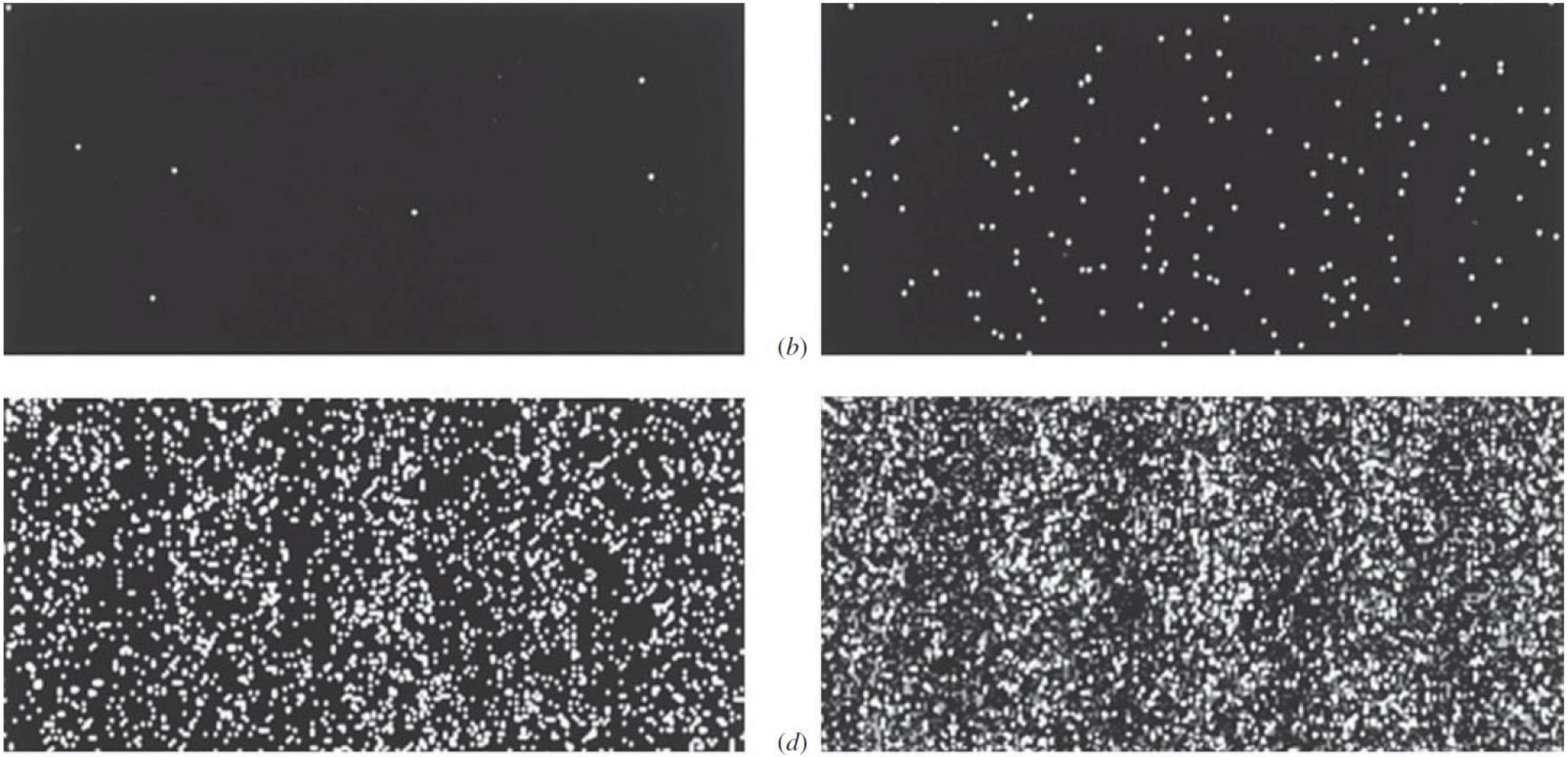
(b)



(c)

Fig. 38-9 (*a*) An experimental arrangement used to demonstrate, by diffraction techniques, the wave-like character of the incident beam. Photographs of the diffraction patterns when the incident beam is (*b*) an x-ray beam (light wave) and (*c*) an electron beam (matter wave). Note that the two patterns are geometrically identical to each other. *(Photo (b) Cameca, Inc. Photo (c) from PSSC film "Matter Waves," courtesy Education Development Center, Newton, Massachusetts)*

38.5: Electrons and Matter Waves:



Central Research Laboratory, Hitachi, Ltd., Kokubinju, Tokyo;
H. Ezawa, Department of Physics, Gakushuin University,
Mejiro, Tokyo

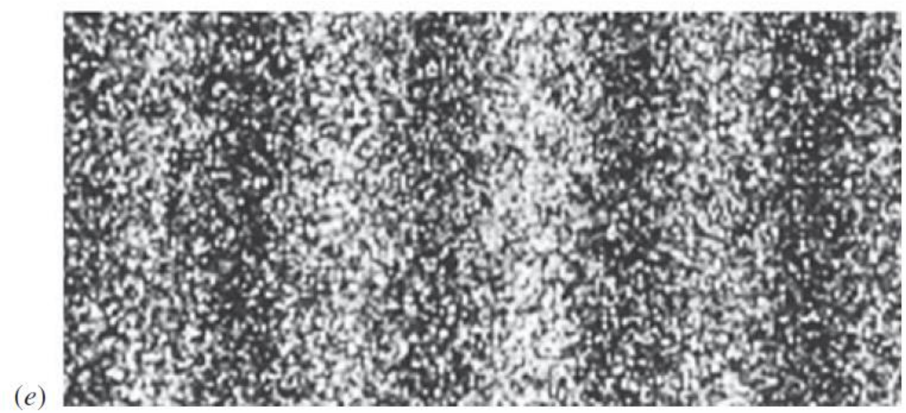


Figure 38-9 Photographs showing the buildup of an interference pattern by a beam of electrons in a two-slit interference experiment like that of Fig. 38-6. Matter waves, like light waves, are *probability waves.* The approximate numbers of electrons involved are (*a*) 7, (*b*) 100, (*c*) 3000, (*d*) 20 000, and (*e*) 70 000.

Checkpoint 4

For an electron and a proton that have the same (a) kinetic energy, (b) momentum, or (c) speed, which particle has the shorter de Broglie wavelength?

(a) proton; (b) same; (c) proton

Sample Problem 38.04 de Broglie wavelength of an electron

What is the de Broglie wavelength of an electron with a kinetic energy of 120 eV?

KEY IDEAS

(1) We can find the electron's de Broglie wavelength λ from Eq. 38-17 ($\lambda = h/p$) if we first find the magnitude of its momentum p. (2) We find p from the given kinetic energy K of the electron. That kinetic energy is much less than the rest energy of an electron (0.511 MeV, from Table 37-3). Thus, we can get by with the classical approximations for momentum p ($= mv$) and kinetic energy K ($= \frac{1}{2}mv^2$).

Calculations: We are given the value of the kinetic energy. So, in order to use the de Broglie relation, we first solve the kinetic energy equation for v and then substitute into the momentum equation, finding

$$\begin{aligned} p &= \sqrt{2mK} \\ &= \sqrt{(2)(9.11 \times 10^{-31}\ \text{kg})(120\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} \\ &= 5.91 \times 10^{-24}\ \text{kg}\cdot\text{m/s}. \end{aligned}$$

From Eq. 38-17 then

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{5.91 \times 10^{-24}\ \text{kg}\cdot\text{m/s}} \\ &= 1.12 \times 10^{-10}\ \text{m} = 112\ \text{pm}. \qquad \text{(Answer)} \end{aligned}$$

This wavelength associated with the electron is about the size of a typical atom. If we increase the electron's kinetic energy, the wavelength becomes even smaller.

38.6: Schrödinger's Equation:

If a **wave function,** $\psi(x, y, z, t)$, can be used to describe matter waves, then its space and time variables can be grouped separately and can be written in the form

$$\Psi(x, y, z, t) = \psi(x, y, z)\, e^{-i\omega t}$$

where $\omega = (2\pi f)$ is the angular frequency of the matter wave.

Suppose that a matter wave reaches a particle detector; then the probability that a particle will be detected in a specified time interval is proportional to $|\psi|^2$, where $|\psi|$ is the absolute value of the wave function at the location of the detector.

$|\psi|^2$ is always both real and positive, and it is called the **probability density,**

The probability (per unit time) of detecting a particle in a small volume centered on a given point in a matter wave is proportional to the value of $|\psi|^2$ at that point.

38.6: Schrödinger's Equation:

Matter waves are described by Schrödinger's Equation.

Suppose a particle traveling in the *x* direction through a region in which forces acting on the particle cause it to have a potential energy *U(x).* In this special case, Schrödinger's equation can be written as:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U(x)]\psi = 0 \qquad \text{(Schrödinger's equation, one-dimensional motion)}$$

For a free particle, *U(x)* is zero, that equation describes a free particle where a moving particle on which no net force acting on it. The particle's total energy in this case is all kinetic, and the equation becomes:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}\left(\frac{mv^2}{2}\right)\psi = 0, \rightarrow \frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0.$$

Using the concept of de Broglie wavelength and the definition of wave number,

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad \text{(Schrödinger's equation, free particle).}$$

The solution to this is: $\psi(x) = Ae^{ikx} + Be^{-ikx},$

Here A and B are constants.

38.6: Schrödinger’s Equation, Finding the Probability Density:

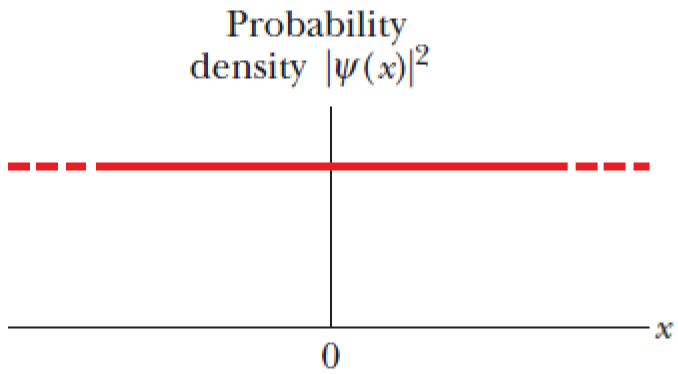


Fig. 38-12 A plot of the probability density $|\psi|^2$ for a free particle moving in the positive x direction. Since $|\psi|^2$ has the same constant value for all values of x, the particle has the same probability of detection at all points along its path.

$$\psi(x) = Ae^{ikx} + Be^{-ikx},$$

Consider a free particle that travels only in the positive direction of x. Let the arbitrary constant B be zero. At the same time, let us relabel the constant A as ψ_0.

$$\psi(x) = \psi_0 \, e^{ikx}.$$

$$\downarrow$$

$$|\psi|^2 = |\psi_0 \, e^{ikx}|^2 = (\psi_0^2)|e^{ikx}|^2.$$

$$\downarrow$$

$$|e^{ikx}|^2 = (e^{ikx})(e^{ikx})^* = e^{ikx} \, e^{-ikx} = e^{ikx-ikx} = e^0 = 1,$$

$$|\psi|^2 = (\psi_0^2)(1)^2 = \psi_0^2 \qquad \text{(a constant).}$$