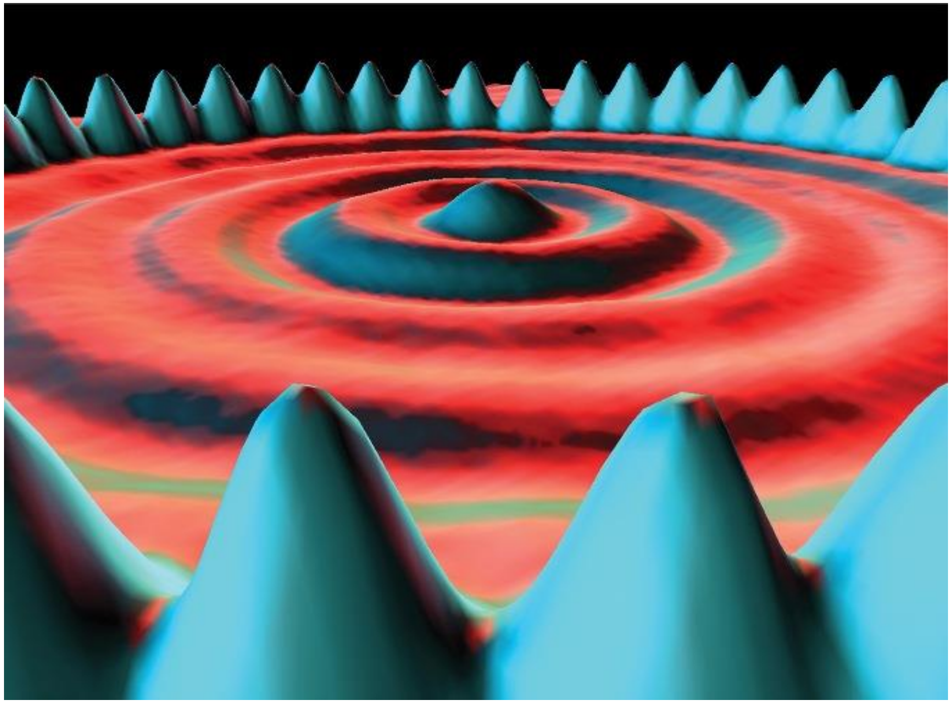
Chapter 39

More About Matter Waves



39.2: String Waves and Matter Waves:

Confinement of a wave leads to quantization—that is, to the existence of discrete states with discrete energies. The wave can have only those energies.

This observation applies to waves of all kinds, including matter waves.

For matter waves, however, it is more convenient to deal with the energy E of the associated particle than with the frequency f of the wave.

39.3: Energies of a Trapped Electron: One-dimensional trap:

An electron can be trapped in the $V = 0$ region.

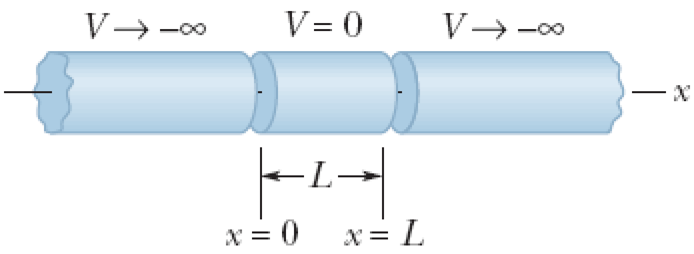


Fig. 39-1 The elements of an idealized "trap" designed to confine an electron to the central cylinder. We take the semi-infinitely long end cylinders to be at an infinitely great negative potential and the central cylinder to be at zero potential.

$$L = \frac{n\lambda}{2}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

Each value of n identifies a state of the oscillating string; the integer n is a **quantum number.**

For each state of the string, the transverse displacement of the string at any position x along the string is given by

$$y_n(x) = A \sin\left(\frac{n\pi}{L} x\right), \qquad \text{for } n = 1, 2, 3, \ldots ,$$

39.3: Energies of a Trapped Electron: Finding the quantized energies:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}.$$

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2, \qquad \text{for } n = 1, 2, 3, \ldots .$$

An electron can be trapped in the $U = 0$ region.

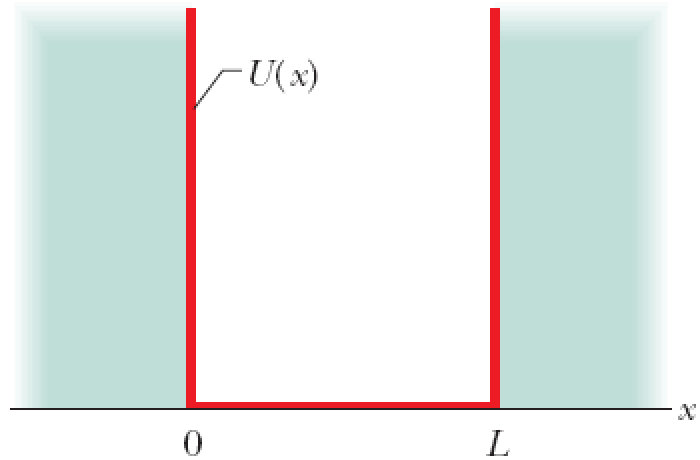


Fig. 39-2 The electric potential energy $U(x)$ of an electron confined to the central cylinder of the idealized trap of Fig. 39-1. We see that $U = 0$ for $0 < x < L$, and $U \rightarrow \infty$ for $x < 0$ and $x > L$.

39.3: Energies of a Trapped Electron: Finding the quantized energies:

These are the lowest five energy levels allowed the electron. (No intermediate levels are allowed.)

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2, \qquad \text{for } n = 1, 2, 3, \ldots .$$

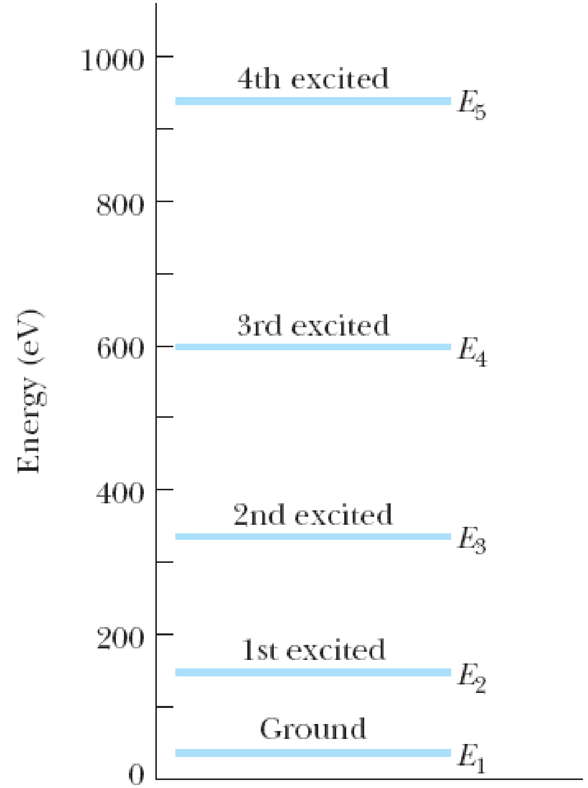
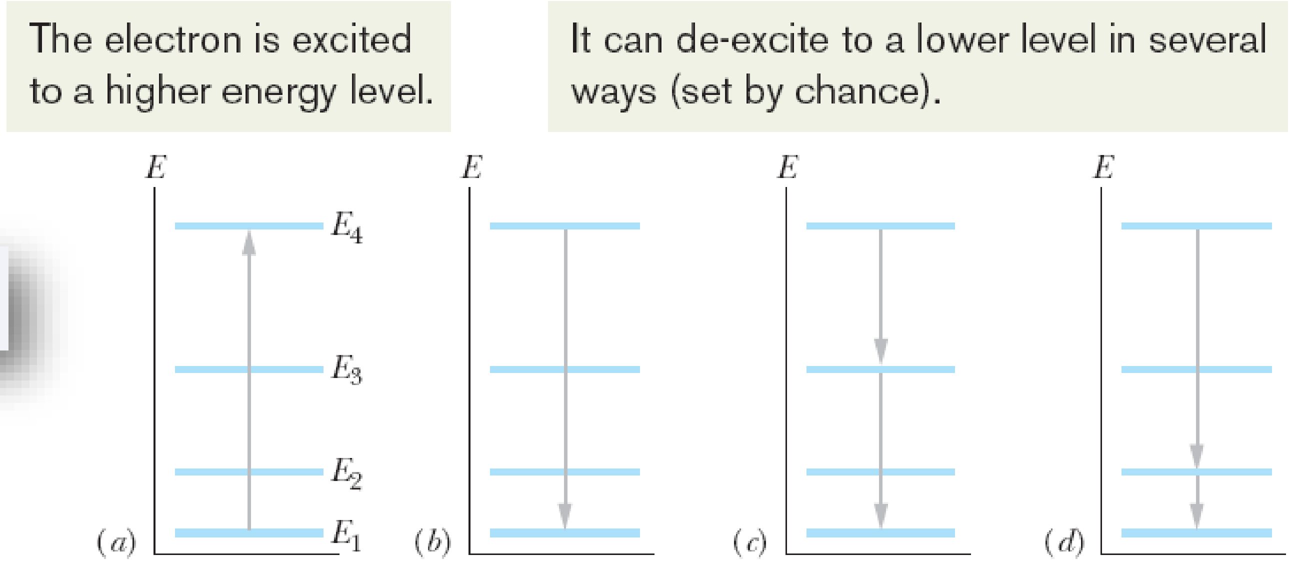


Fig. 39-3 Several of the allowed energies given by Eq. 39-4 for an electron confined to the infinite well of Fig. 39-2. Here width $L = 100$ pm. Such a plot is called an *energy-level diagram*.

39.3: Energies of a Trapped Electron: Finding the quantized energies:

Fig. 39-4 (*a*) Excitation of a trapped electron from the energy level of its ground state to the level of its third excited state. (*b*)–(*d*) Three of four possible ways the electron can de-excite to return to the energy level of its ground state. (Which way is not shown?)

$$\Delta E = E_{\text{high}} - E_{\text{low}}.$$



If a confined electron is to absorb a photon, the energy hf of the photon must equal the energy difference ΔE between the initial energy level of the electron and a higher level.

$$hf = \Delta E = E_{\text{high}} - E_{\text{low}}.$$

Checkpoint 1

Rank the following pairs of quantum states for an electron confined to an infinite well according to the energy differences between the states, greatest first: (a) $n = 3$ and $n = 1$, (b) $n = 5$ and $n = 4$, (c) $n = 4$ and $n = 3$.

b, a, c

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2, \qquad \text{for } n = 1, 2, 3, \ldots .$$

Example, Energy levels in a 1-D infinite potential well:

An electron is confined to a one-dimensional, infinitely deep potential energy well of width $L = 100$ pm.

(a) What is the smallest amount of energy the electron can have?

Lowest energy level: Here, the collection of constants in front of n^2 in Eq. 39-4 is evaluated as

$$\frac{h^2}{8mL^2} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})^2}{(8)(9.11 \times 10^{-31}\ \mathrm{kg})(100 \times 10^{-12}\ \mathrm{m})^2}$$
$$= 6.031 \times 10^{-18}\ \mathrm{J}. \qquad (39\text{-}7)$$

The smallest amount of energy the electron can have corresponds to the lowest quantum number, which is $n = 1$ for the ground state of the electron. Thus, Eqs. 39-4 and 39-7 give us

$$E_1 = \left(\frac{h^2}{8mL^2}\right)n^2 = (6.031 \times 10^{-18}\ \mathrm{J})(1^2)$$
$$\approx 6.03 \times 10^{-18}\ \mathrm{J} = 37.7\ \mathrm{eV}. \qquad \text{(Answer)}$$

(b) How much energy must be transferred to the electron if it is to make a quantum jump from its ground state to its second excited state?

$$\Delta E_{31} = \left(\frac{h^2}{8mL^2}\right)(3)^2 - \left(\frac{h^2}{8mL^2}\right)(1)^2$$
$$= \frac{h^2}{8mL^2}(3^2 - 1^2)$$
$$= (6.031 \times 10^{-18}\ \mathrm{J})(8)$$
$$= 4.83 \times 10^{-17}\ \mathrm{J} = 301\ \mathrm{eV}. \qquad \text{(Answer)}$$

Upward jump: The energies E_3 and E_1 depend on the quantum number n, according to Eq. 39-4. Therefore, substituting that equation into Eq. 39-8 for energies E_3 and E_1 and

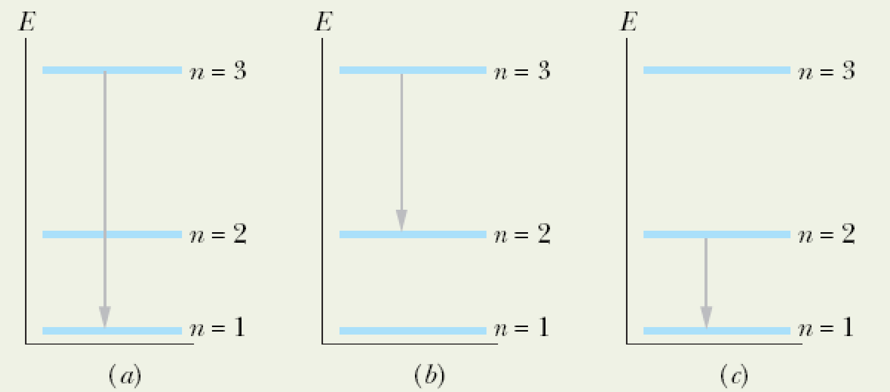


Fig. 39-5 De-excitation from the second excited state to the ground state either directly (a) or via the first excited state (b, c).