CH30

30-1 Faraday's Law and Lenz's Law

Faraday's Law. With the notion of magnetic flux, we can state Faraday's law in a more quantitative and useful way:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

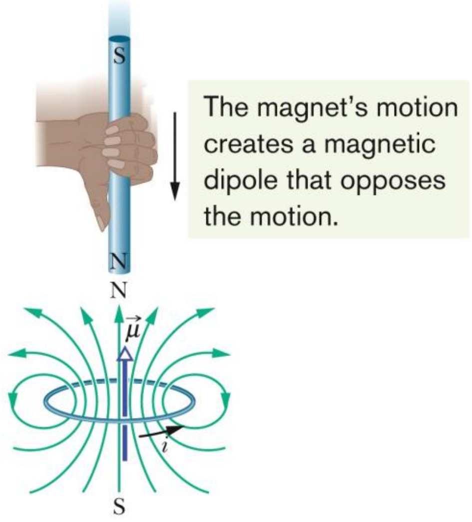
the induced emf tends to oppose the flux change and the minus sign indicates this opposition. This minus sign is referred to as Lenz's Law.

30-1 Faraday's Law and Lenz's Law

Lenz's Law

An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current. The induced emf has the same direction as the induced current.

Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.



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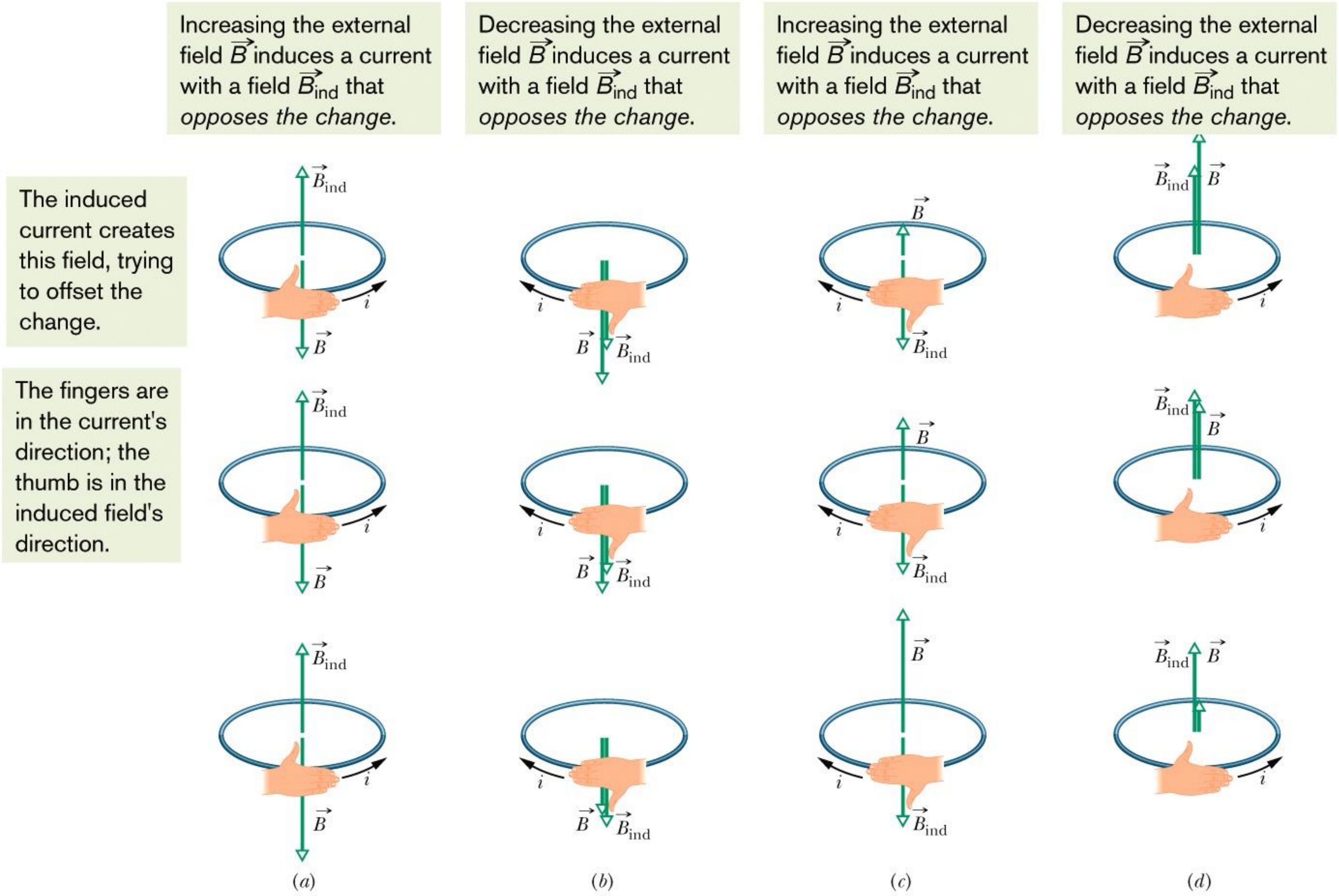
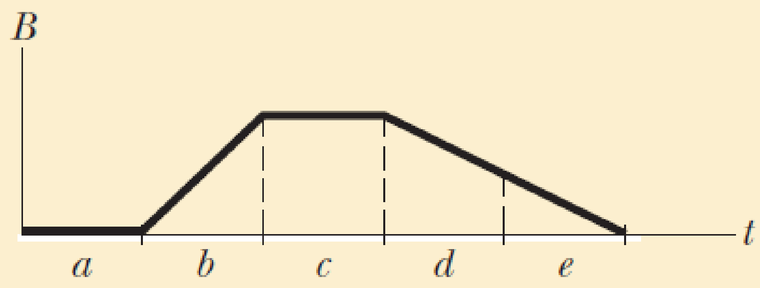
Lenz's Law



Checkpoint 1

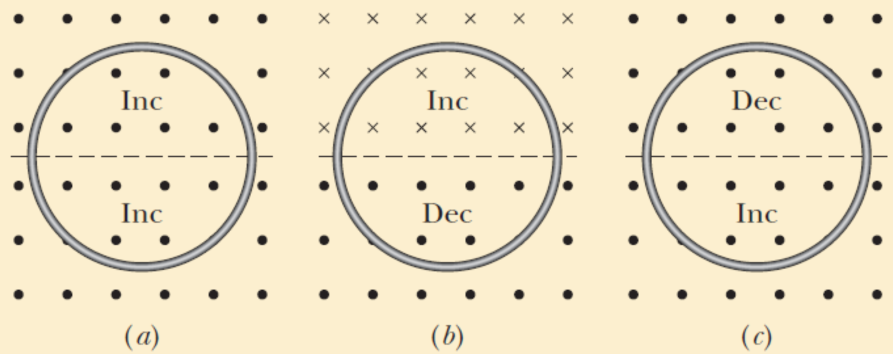
The graph gives the magnitude $B(t)$ of a uniform magnetic field that exists throughout a conducting loop, with the direction of the field perpendicular to the plane of the loop. Rank the five regions of the graph according to the magnitude of the emf induced in the loop, greatest first.



1. *b*, then *d* and *e* tie, and then *a* and *c* tie (zero)

Checkpoint 2

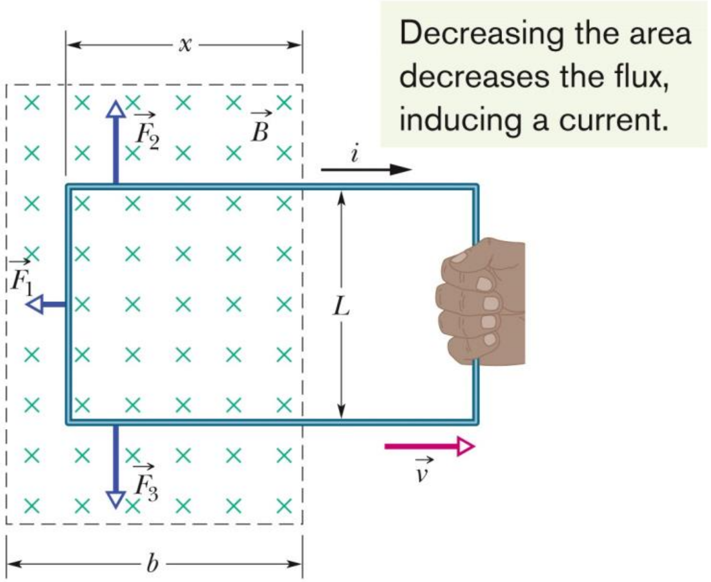
The figure shows three situations in which identical circular conducting loops are in uniform magnetic fields that are either increasing (Inc) or decreasing (Dec) in magnitude at identical rates. In each, the dashed line coincides with a diameter. Rank the situations according to the magnitude of the current induced in the loops, greatest first.



a and *b* tie, then *c* (zero)

30-2 Induction and Energy Transfer

In the figure, a rectangular loop of wire of width L has one end in a uniform external magnetic field that is directed perpendicularly into the plane of the loop. This field may be produced, for example, by a large electromagnet. The dashed lines in the figure show the assumed limits of the magnetic field; the fringing of the field at its edges is neglected. You are to pull this loop to the right at a constant velocity $\vec{v}$.



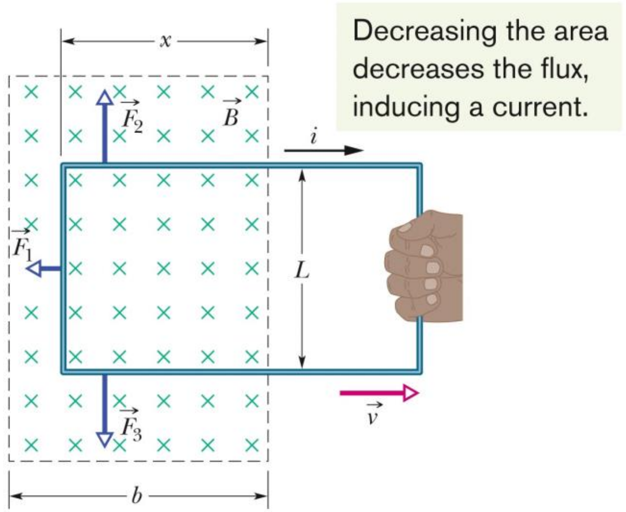
$$\Phi_B = BA = BLx.$$

$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BL\frac{dx}{dt} = BLv,$$

30-2 Induction and Energy Transfer

Induced Current: Figure (bottom) shows the loop as a circuit: induced emf is represented on the left, and the collective resistance *R* of the loop is represented on the right. To find the magnitude of the induced current, we can apply the equation $i = \frac{\mathcal{E}}{R}$ which gives

$$i = \frac{BLv}{R}.$$



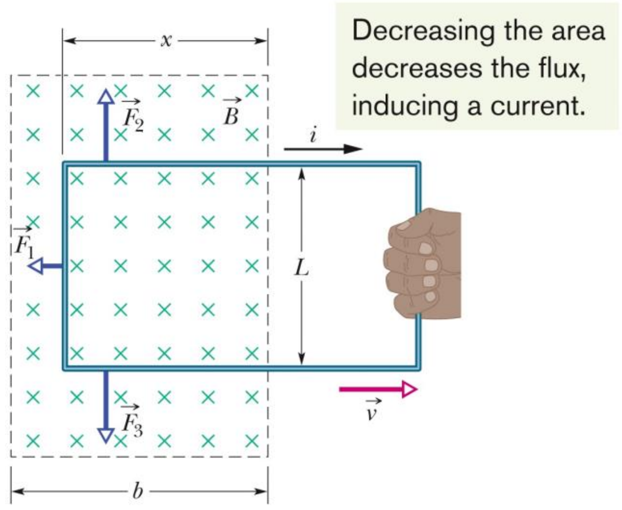
30-2 Induction and Energy Transfer

So, $\vec{F} = -\vec{F}_1$, the magnitude of $\vec{F}_1$ thus

$$F = F_1 = iLB\sin 90° = iLB. \quad \left(\text{from } \vec{F}_d = i\vec{L}\times\vec{B}.\right)$$

where the angle between B and the length vector L for the left segment is 90°. This gives us

$$F = \frac{B^2L^2v}{R}.$$



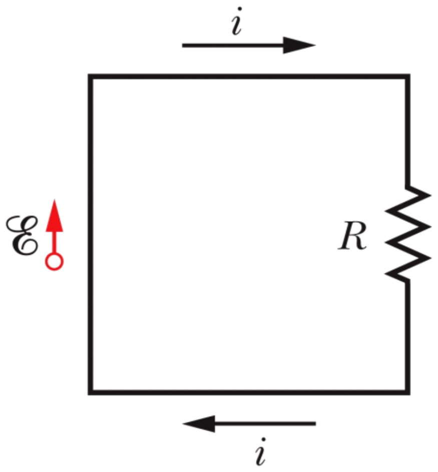
30-2 Induction and Energy Transfer

Because B, L, and R are constants, the speed v at which you move the loop is constant if the magnitude F of the force you apply to the loop is also constant.

Rate of Work: We find the rate at which you do work on the loop as you pull it from the magnetic field:

$$P = Fv = \frac{B^2L^2v^2}{R}$$

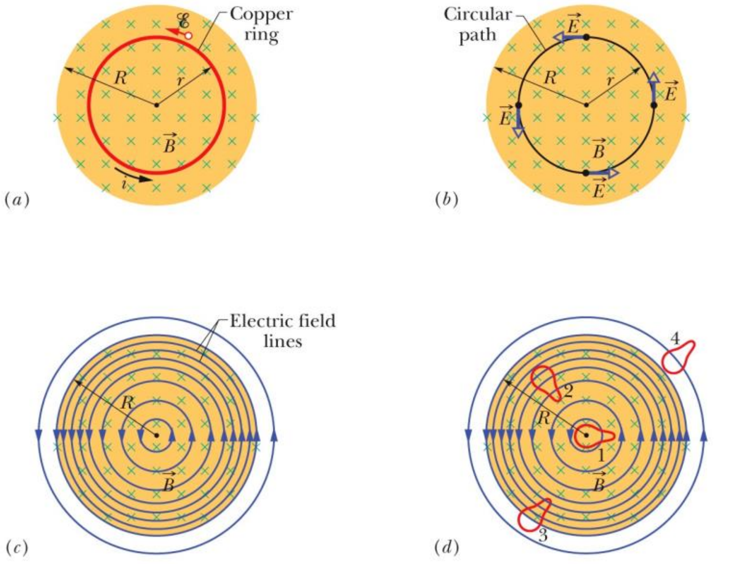
NOTE: The work that you do in pulling the loop through the magnetic field appears as **thermal energy** in the loop.



A circuit diagram for the loop of above figure while the loop is moving.

30-3 Induced Electric Field

(a) If the magnetic field increases at a steady rate, a constant induced current appears, as shown, in the copper ring of radius *r*. (b) An induced electric field exists even when the ring is removed; the electric field is shown at four points. (c) The complete picture of the induced electric field, displayed as field lines. (d) Four similar closed paths that enclose identical areas. Equal emfs are induced around paths 1 and 2, which lie entirely within the region of changing magnetic field. A smaller emf is induced around path 3, which only partially lies in that region. No net emf is induced around path 4, which lies entirely outside the magnetic field.



30-3 Induced Electric Field

Therefore, an emf is induced by a changing magnetic flux even if the loop through which the flux is changing is not a physical conductor but an imaginary line. The changing magnetic field induces an electric field $\vec{E}$ at every point of such a loop; the induced emf is related to $\vec{E}$ by

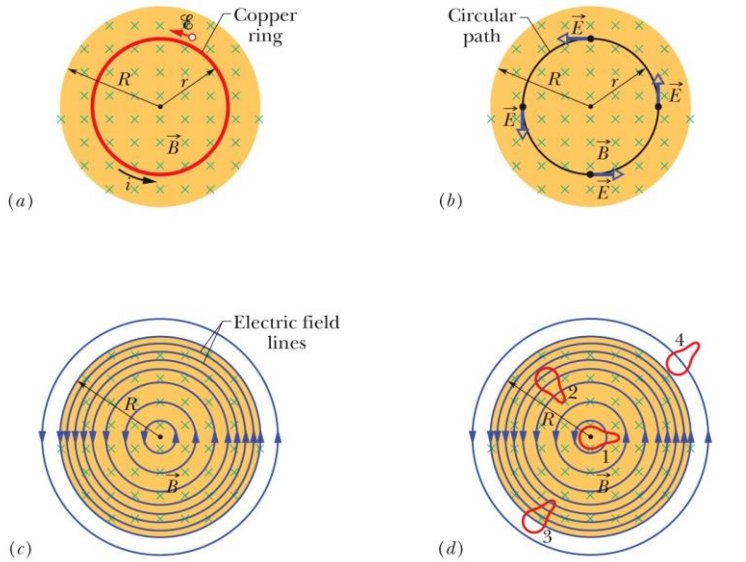
$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}.$$

30-3 Induced Electric Field

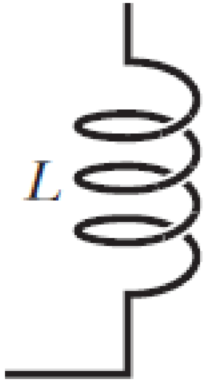
Using the induced electric field, we can write Faraday's law in its most general form as

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

A changing magnetic field produces an electric field.



30-4 Inductors and Inductance



An inductor is a device that can be used to produce a known magnetic field in a specified region. If a current i is established through each of the N windings of an inductor, a magnetic flux Φ_B links those windings. The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$

The SI unit of inductance is the henry (H), where $1\text{ henry} = 1\,\text{H} = 1\,\text{T}\cdot\text{m}^2/\text{A}$.

The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$