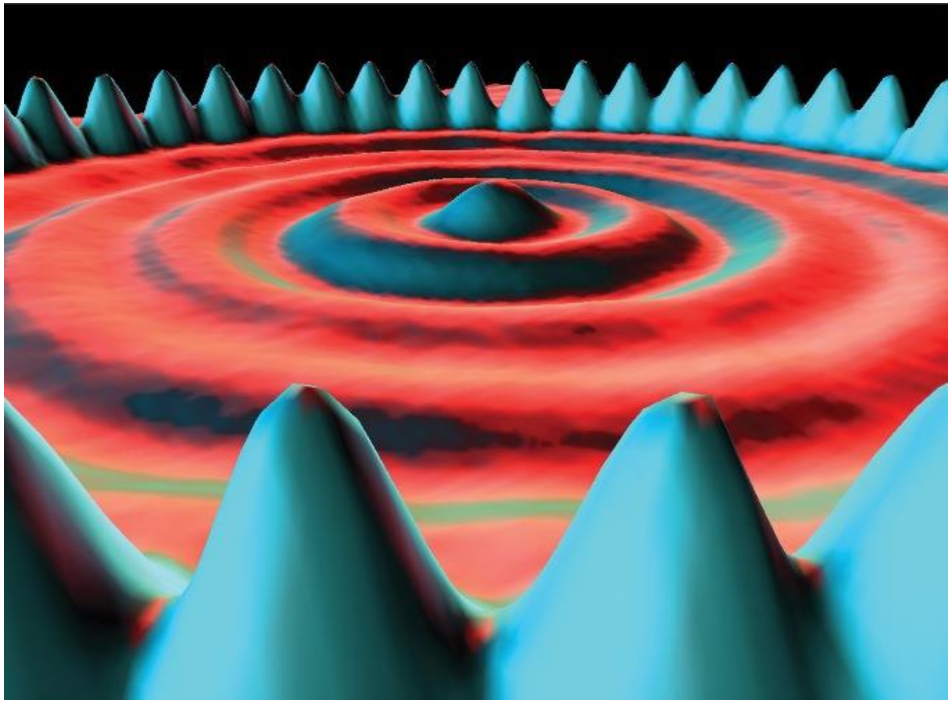
Chapter 39

More About Matter Waves



39.4: Two- and Three- Dimensional Electron Traps:

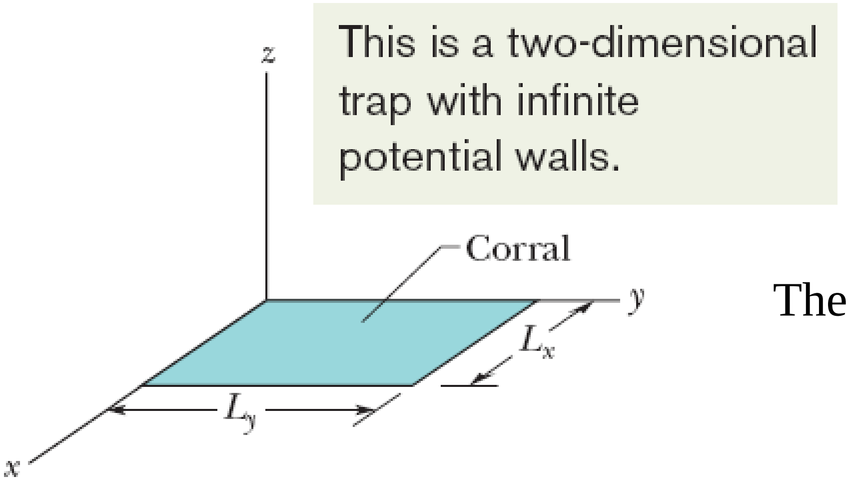


Fig. 39-13 A rectangular corral—a two-dimensional version of the infinite potential well of Fig. 39-2—with widths L_x and L_y.

The normalized wave function:

$$\psi_{nx,\,ny} = \sqrt{\frac{2}{L_x}}\sin\left(\frac{n_x\pi}{L}x\right)\sqrt{\frac{2}{L_y}}\sin\left(\frac{n_y\pi}{L}y\right),$$

The energy of the electron:

$$E_{nx,ny} = \left(\frac{h^2}{8mL_x^2}\right)n_x^2 + \left(\frac{h^2}{8mL_y^2}\right)n_y^2 = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right).$$

39.4: Two- and Three- Dimensional Electron Traps, Rectangular Box

The energy of an electron trapped in a 3-D infinite potential box:

$$E_{nx,ny,nz} = \frac{h^2}{8m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}\right).$$

This is a three-dimensional trap with infinite potential walls.

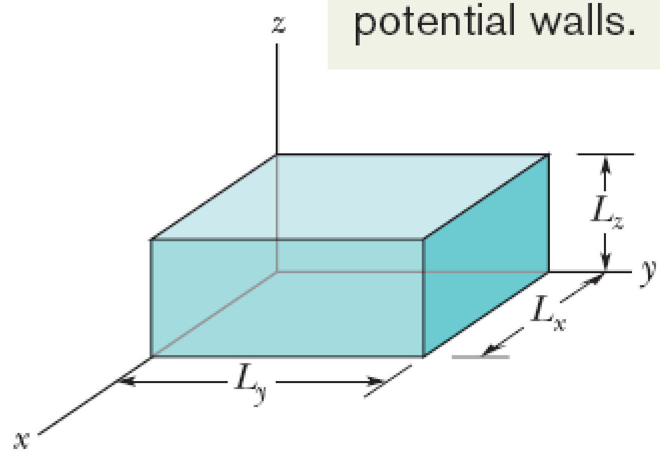


Fig. 39-14 A rectangular box—a three-dimensional version of the infinite potential well of Fig. 39-2—with widths L_x, L_y, and L_z.

Checkpoint 4

In the notation of Eq. 39-20, is $E_{0,0}$, $E_{1,0}$, $E_{0,1}$, or $E_{1,1}$ the ground-state energy of an electron in a (two-dimensional) rectangular corral?

$E_{1,1}$ (neither n_x nor n_y can be zero)

Example, Energy levels in a 2D infinite potential well:

An electron is trapped in a square corral that is a two-dimensional infinite potential well (Fig. 39-13) with widths $L_x = L_y$.

(a) Find the energies of the lowest five possible energy levels for this trapped electron, and construct the corresponding energy-level diagram.

Energy levels: Because the well here is square, we can let the widths be $L_x = L_y = L$. Then Eq. 39-20 simplifies to

$$E_{nx,ny} = \frac{h^2}{8mL^2}(n_x^2 + n_y^2). \qquad (39\text{-}22)$$

The lowest energy states correspond to low values of the quantum numbers n_x and n_y, which are the positive integers $1, 2, \ldots, \infty$. Substituting those integers for n_x and n_y in Eq. 39-22, starting with the lowest value 1, we can obtain the energy values as listed in Table 39-1. There we can see that several of the pairs of quantum numbers (n_x, n_y) give the same energy. For example, the (1, 2) and (2, 1) states both have an energy of $5(h^2/8mL^2)$. Each such pair is associated with degenerate energy levels. Note also that, perhaps surprisingly, the (4, 1) and (1, 4) states have less energy than the (3, 3) state.

From Table 39-1 (carefully keeping track of degenerate levels), we can construct the energy-level diagram of Fig. 39-15.

Table 39-1

Energy Levels

n_x	n_y	Energy[a]	n_x	n_y	Energy[a]
1	3	10	2	4	20
3	1	10	4	2	20
2	2	8	3	3	18
1	2	5	1	4	17
2	1	5	4	1	17
1	1	2	2	3	13
			3	2	13

[a]In multiples of $h^2/8mL^2$.

These are the lowest five energy levels allowed the electron. Different quantum states may have the same energy.

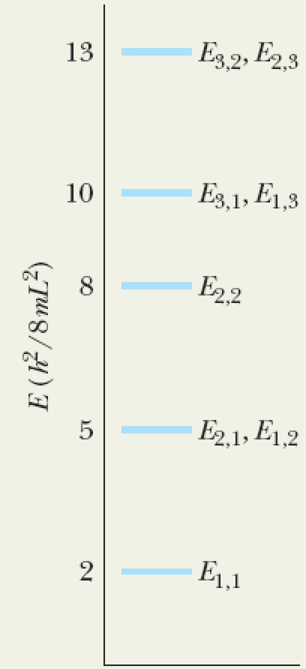


Fig. 39-15 Energy-level diagram for an electron trapped in a square corral.

Example, Energy levels in a 2D infinite potential well, cont.:

(b) As a multiple of $h^2/8mL^2$, what is the energy difference between the ground state and the third excited state?

Energy difference: From Fig. 39-15, we see that the ground state is the (1, 1) state, with an energy of $2(h^2/8mL^2)$. We also see that the third excited state (the third state up from the ground state in the energy-level diagram) is the degenerate (1, 3) and (3, 1) states, with an energy of $10(h^2/8mL^2)$. Thus, the difference ΔE between these two states is

$$\Delta E = 10\left(\frac{h^2}{8mL^2}\right) - 2\left(\frac{h^2}{8mL^2}\right) = 8\left(\frac{h^2}{8mL^2}\right).$$

(Answer)

These are the lowest five energy levels allowed the electron. Different quantum states may have the same energy.

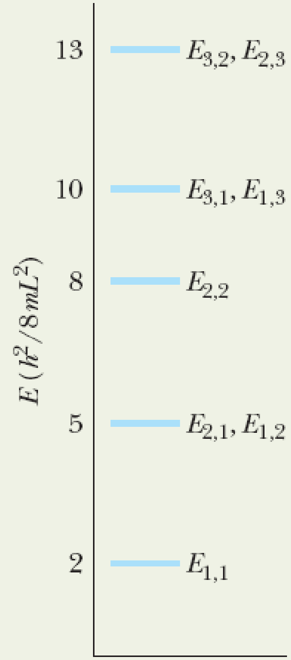
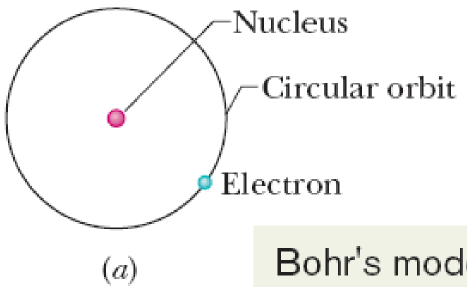
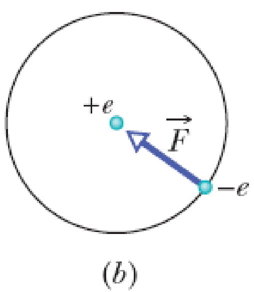


Fig. 39-15 Energy-level diagram for an electron trapped in a square corral.

39.5: The Bohr Model of the Hydrogen Atom:





Bohr's model for hydrogen resembles the orbital model of a planet around a star.

Fig. 39-16 (*a*) Circular orbit of an electron in the Bohr model of the hydrogen atom. (*b*) The Coulomb force $\vec{F}$ on the electron is directed radially inward toward the nucleus.

$$F = k\frac{|q_1||q_2|}{r^2},$$

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} = m\left(-\frac{v^2}{r}\right),$$

The angular momentum: $\ell = rmv \sin\phi$,

$= rmv \sin 90°$

For quantization of *l*, $rmv = n\hbar$,

$$v = \frac{n\hbar}{rm}.$$

$$r = \frac{h^2\varepsilon_0}{\pi m e^2}n^2, \quad \text{for } n = 1, 2, 3, \ldots.$$

$$r = an^2, \quad \text{for } n = 1, 2, 3, \ldots,$$

$$a = \frac{h^2\varepsilon_0}{\pi m e^2} = 5.291\,772 \times 10^{-11}\text{ m} \approx 52.92\text{ pm}.$$

39.5: The Bohr Model of the Hydrogen Atom, Orbital energy is quantized:

$$E = K + U$$
$$= \tfrac{1}{2}mv^2 + \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right).$$

$$\Rightarrow \quad E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}.$$

$$\Rightarrow \quad E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2}, \qquad \text{for } n = 1, 2, 3, \ldots,$$

$$\Rightarrow \quad \boxed{E_n = -\frac{2.180 \times 10^{-18}\ \mathrm{J}}{n^2} = -\frac{13.61\ \mathrm{eV}}{n^2}, \quad \text{for } n = 1, 2, 3, \ldots.}$$

39.5: The Bohr Model of the Hydrogen Atom, Energy Changes:

$$hf = \Delta E = E_{\text{high}} - E_{\text{low}}.$$

$$\Downarrow$$

$$\frac{1}{\lambda} = -\frac{me^4}{8\varepsilon_0^2 h^3 c}\left(\frac{1}{n_{\text{high}}^2} - \frac{1}{n_{\text{low}}^2}\right).$$

$$\Downarrow$$

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2}\right),}$$

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c} = 1.097\,373 \times 10^7\ \text{m}^{-1} \quad \text{(Rydberg Constant)}$$

39.9: Schrodinger's Equation and The Hydrogen Atom:

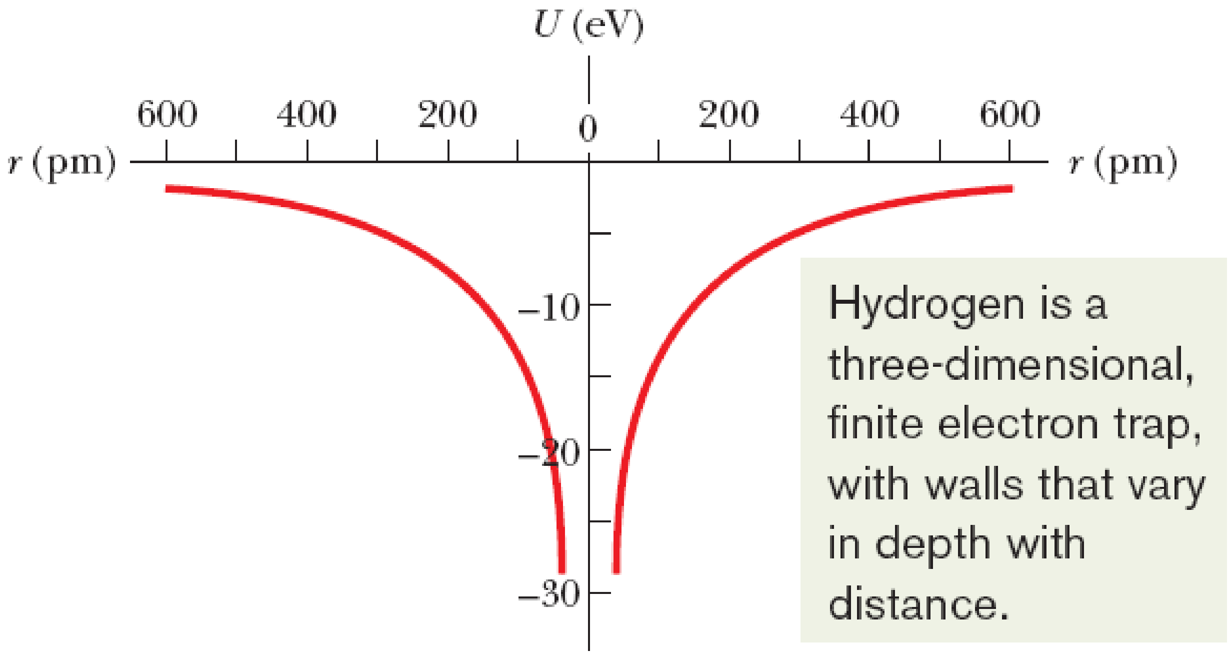


Fig. 39-17 The potential energy U of a hydrogen atom as a function of the separation r between the electron and the central proton. The plot is shown twice (on the left and on the right) to suggest the three-dimensional spherically symmetric trap in which the electron is confined.

Schrodinger's Equation and The Hydrogen Atom:

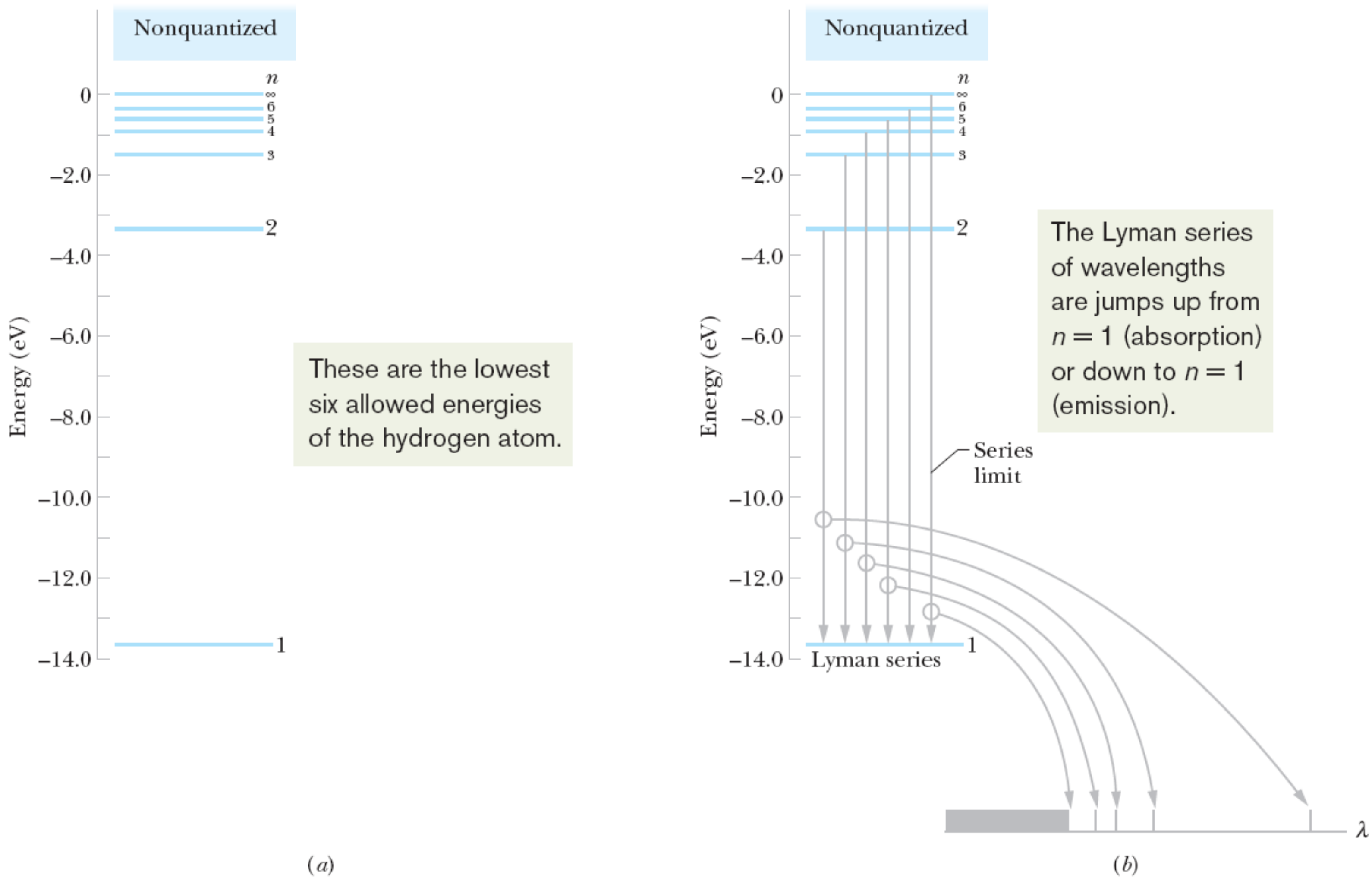
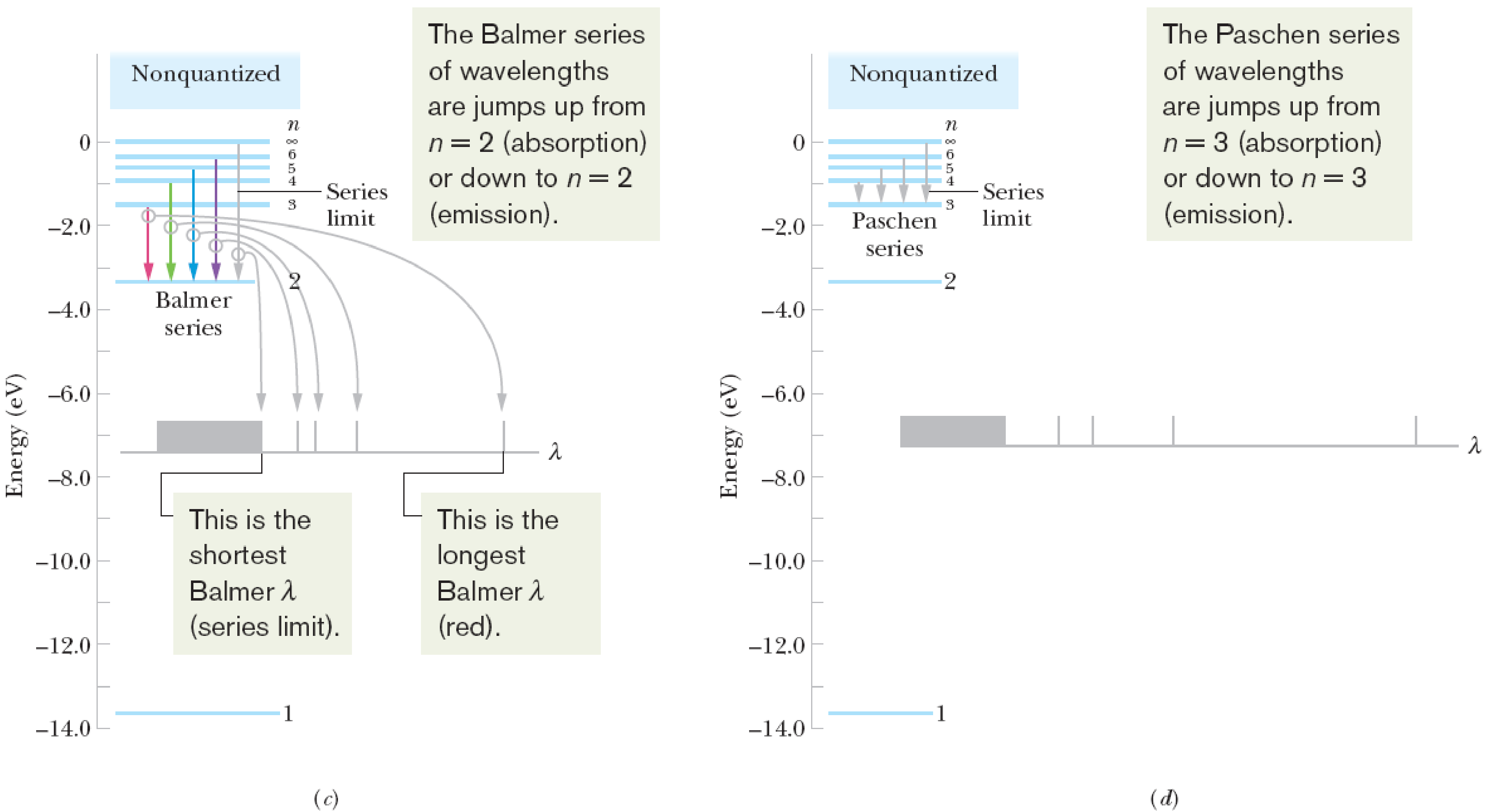


Fig. 39-18 (a) An energy-level diagram for the hydrogen atom. Some of the transitions for (b) the Lyman series. For each, the longest four wavelengths and the series-limit wavelength are plotted on a wavelength axis. Any wavelength shorter than the series-limit wavelength is allowed.

Schrodinger's Equation and The Hydrogen Atom:

Fig. 39-18 Some of the transitions for (c) the Balmer series, and (d) the Paschen series. For each, the longest four wavelengths and the series-limit wavelength are plotted on a wavelength axis. Any wavelength shorter than the series-limit wavelength is allowed.



Quantum Numbers and The Hydrogen Atom:

Each set of quantum numbers (n, l, m_l) identifies the wave function of a particular quantum state. The quantum number n, is called the **principal quantum number.** The **orbital quantum number** l is a measure of the magnitude of the angular momentum associated with the quantum state. The **orbital magnetic quantum number** m_l is related to the orientation in space of this angular momentum vector.

The restrictions on the values of the quantum numbers for the hydrogen atom, as listed in Table 39-2, are not arbitrary but come out of the solution to Schrödinger's equation.

Table 39-2

Quantum Numbers for the Hydrogen Atom

Symbol	Name	Allowed Values
n	Principal quantum number	$1, 2, 3, \ldots$
ℓ	Orbital quantum number	$0, 1, 2, \ldots, n-1$
m_ℓ	Orbital magnetic quantum number	$-\ell, -(\ell-1), \ldots, +(\ell-1), +\ell$