Chapter 40

All About Atoms

40.2: The Stern-Gerlach Experiment: *Quantization of magnetic moment*

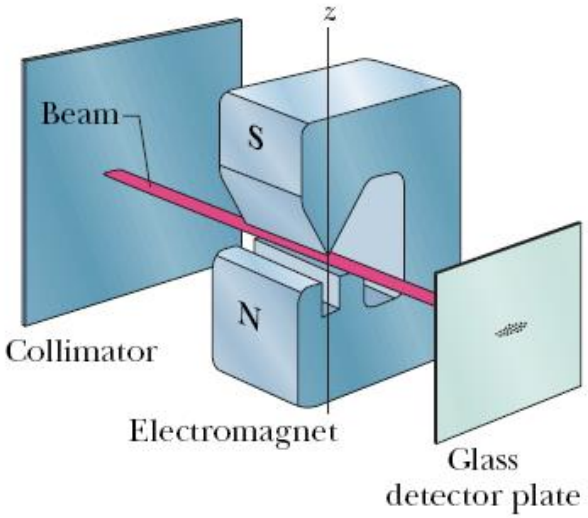
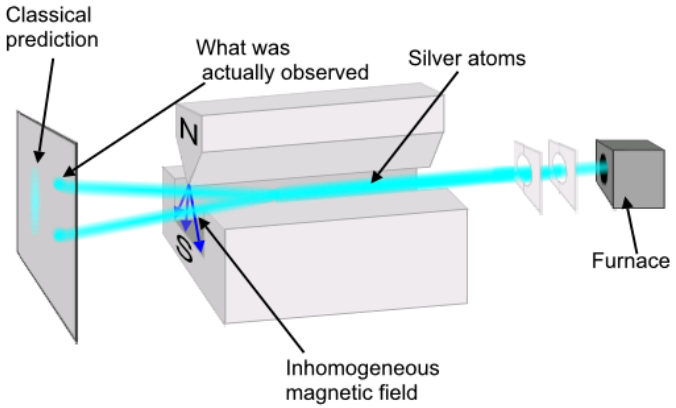


Fig. 40-8 Apparatus used by Stern and Gerlach.



In 1922, Otto Stern and Walther Gerlach showed experimentally that the magnetic moment of silver atoms is quantized. In the experiment silver is vaporized in an oven, and some of the atoms in that vapor escape through a narrow slit in the oven wall and pass into an evacuated tube. The beam passes between the poles of an electromagnet and then lands on a glass detector plate where it forms a silver deposit. By analyzing the silver deposit on the plate, one can determine what deflections the atoms underwent in the magnetic field.

Why silver (Ag)?

$$\text{Ag: } \overbrace{1s^2 2s^2 2sp^6 3s^2 3p^6 3d^{10} 4s^2 4p^6}^{[\text{Kr}]} 4d^{10} 5s^1$$

40.2: The Stern-Gerlach Experiment:

The type of magnetic force the silver atom experiences in the Stern-Gerlach experiment is due to an interaction between the magnetic field of the electromagnet and the magnetic dipole of the individual silver atom.

The energy U of the dipole in the magnetic field is $U = -\vec{\mu} \cdot \vec{B},$ where $\boldsymbol{\mu}$ is the magnetic dipole moment of the silver atom.

Therefore $U = -\mu_z B.$

And $$F_z = -\frac{dU}{dz} = \mu_z \frac{dB}{dz}.$$

$$\mu_{s,z} = -2m_s\mu_B,$$

$\mu_{s,z} = -2(+\frac{1}{2})\mu_B = -\mu_B$ and $\mu_{s,z} = -2(-\frac{1}{2})\mu_B = +\mu_B.$

$$F_z = -\mu_B\left(\frac{dB}{dz}\right) \quad \text{and} \quad F_z = +\mu_B\left(\frac{dB}{dz}\right),$$

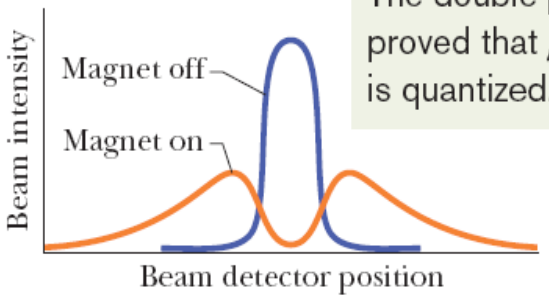


Fig. 40-9 Results of a modern repetition of the Stern–Gerlach experiment. With the electromagnet turned off, there is only a single beam; with the electromagnet turned on, the original beam splits into two sub-beams. The two subbeams correspond to parallel and antiparallel alignment of the magnetic moments of cesium atoms with the external magnetic field.

Example, Beam Separation in a Stern-Gerlach experiment:

In the Stern–Gerlach experiment of Fig. 40-8, a beam of silver atoms passes through a magnetic field gradient dB/dz of magnitude 1.4 T/mm that is set up along the z axis. This region has a length w of 3.5 cm in the direction of the original beam. The speed of the atoms is 750 m/s. By what distance d have the atoms been deflected when they leave the region of the field gradient? The mass M of a silver atom is 1.8×10^{-25} kg.

KEY IDEAS

(1) The deflection of a silver atom in the beam is due to an interaction between the magnetic dipole of the atom and the magnetic field, because of the gradient dB/dz. The deflecting force is directed along the field gradient (along the z axis) and is given by Eqs. 40-19. Let us consider only deflection in the positive direction of z; thus, we shall use $F_z = \mu_B(dB/dz)$ from Eqs. 40-19.

(2) We assume the field gradient dB/dz has the same value throughout the region through which the silver atoms travel. Thus, force component F_z is constant in that region, and from Newton's second law, the acceleration a_z of an atom along the z axis due to F_z is also constant.

Calculations: Putting these ideas together, we write the acceleration as

$$a_z = \frac{F_z}{M} = \frac{\mu_B(dB/dz)}{M}.$$

Because this acceleration is constant, we can use Eq. 2-15 (from Table 2-1) to write the deflection d parallel to the z axis as

$$d = v_{0z}t + \tfrac{1}{2}a_z t^2 = 0t + \tfrac{1}{2}\left(\frac{\mu_B(dB/dz)}{M}\right)t^2. \qquad (40\text{-}20)$$

Because the deflecting force on the atom acts perpendicular to the atom's original direction of travel, the component v of the atom's velocity along the original direction of travel is not changed by the force. Thus, the atom requires time $t = w/v$ to travel through length w in that direction. Substituting w/v for t into Eq. 40-20, we find

$$\begin{aligned} d &= \tfrac{1}{2}\left(\frac{\mu_B(dB/dz)}{M}\right)\left(\frac{w}{v}\right)^2 = \frac{\mu_B(dB/dz)w^2}{2Mv^2} \\ &= (9.27 \times 10^{-24}\ \text{J/T})(1.4 \times 10^{3}\ \text{T/m}) \\ &\quad \times \frac{(3.5 \times 10^{-2}\ \text{m})^2}{(2)(1.8 \times 10^{-25}\ \text{kg})(750\ \text{m/s})^2} \\ &= 7.85 \times 10^{-5}\ \text{m} \approx 0.08\ \text{mm}. \qquad \text{(Answer)} \end{aligned}$$

The separation between the two subbeams is twice this, or 0.16 mm. This separation is not large but is easily measured.

40.3: Magnetic Resonance:

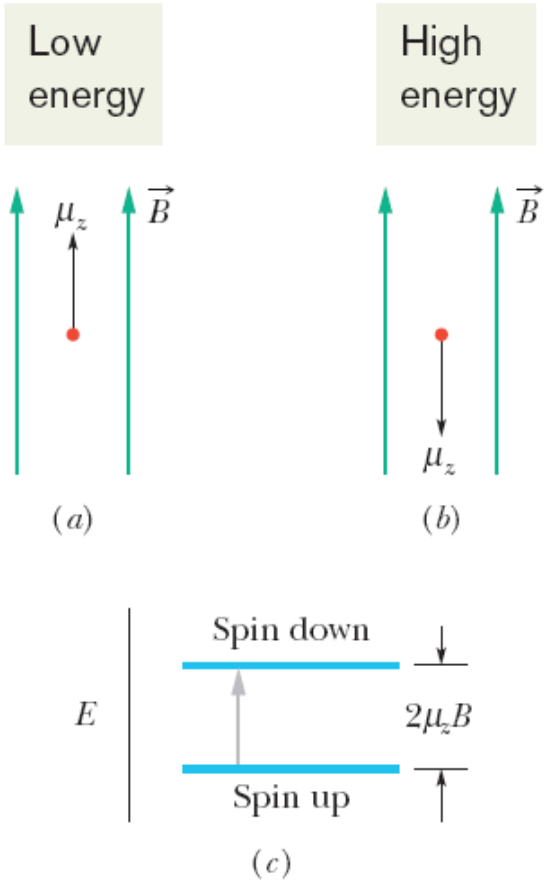


Fig. 40-10 The z component of $\vec{\mu}$ for a proton in the (a) lower-energy (spin-up) and (b) higher-energy (spin-down) state. (c) An energy-level diagram for the states, showing the upward quantum jump the proton makes when its spin flips from up to down.

$$\Delta E = \mu_z B - (-\mu_z B) = 2\mu_z B.$$

$$hf = 2\mu_z B. \quad \text{(Energy of absorbing photon)}$$

Such absorption is called ***magnetic resonance*** or, ***nuclear magnetic resonance (NMR),*** and the consequent reversal of S_z is called ***spin-flipping***.

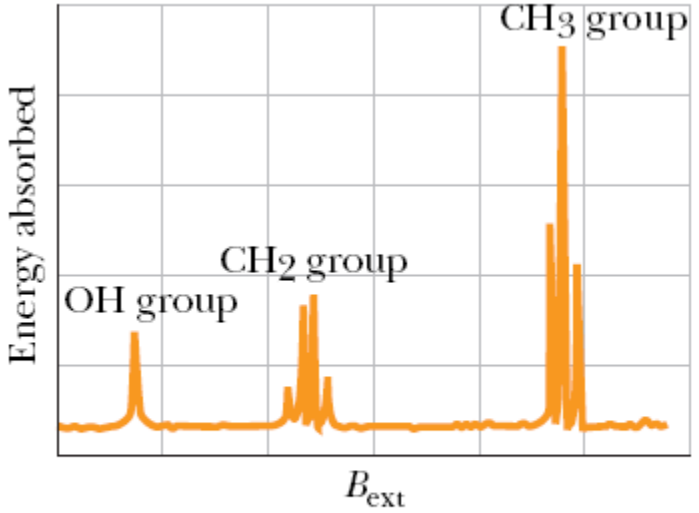


Fig. 40-11 A nuclear magnetic resonance spectrum for ethanol, CH_3CH_2OH.

40.4: The Pauli Exclusion Principle and Multiple Electrons in a Trap:

The Pauli Exclusion Principle

No two electrons confined to the same trap can have the same set of values for their quantum numbers.

As we shall discuss in Module 40-5, this principle means that no two electrons in an atom can have the same four values for the quantum numbers n, ℓ, m_ℓ, and m_s. All electrons have the same quantum number $s = \frac{1}{2}$. Thus, any two electrons in an atom must differ in at least one of these other quantum numbers. Were this not true, atoms would collapse, and thus you and the world could not exist.

This principle applies not only to electrons but also to protons and neutrons, all of which have s=½. The principle is known as the **Pauli exclusion principle** after Wolfgang Pauli, who formulated it in 1925

Multiple Electrons in Rectangular Traps:

- The Pauli exclusion principle disallows any more electrons from occupying that lowest energy level, and the next electron must occupy the next higher level.

- When an energy level cannot be occupied by more electrons because of the Pauli exclusion principle, we say that level is ***full or fully occupied***.

- In contrast, a level that is not occupied by any electrons is ***empty or unoccupied***.

- For intermediate situations, the level is ***partially occupied***.

- The ***electron configuration of a system of trapped*** electrons is a listing or drawing either of the energy levels the electrons occupy or of the set of the quantum numbers of the electrons.

Sample Problem 40.02 Energy levels of multiple electrons in a 2D infinite potential well

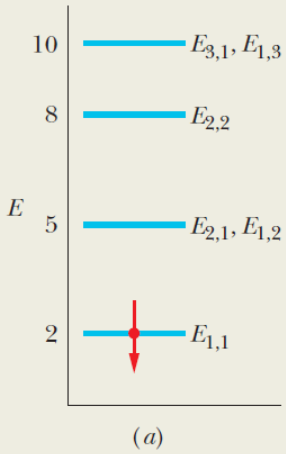
Seven electrons are confined to a square corral (two-dimensional infinite potential well) with widths $L_x = L_y = L$ (Fig. 39-13). Assume that the electrons do not electrically interact with one another.

(a) What is the electron configuration for the ground state of the system of seven electrons?

One-electron diagram: We can determine the electron configuration of the system by placing the seven electrons in the corral one by one, to build up the system. Because we assume the electrons do not electrically interact with one another, we can use the energy-level diagram for a single trapped electron in order to keep track of how we place the seven electrons in the corral. That *one-electron energy-level diagram* is given in Fig. 39-15 and partially reproduced here as Fig. 40-12*a*. Recall that the levels are labeled as $E_{nx,ny}$ for their associated energy. For example, the lowest level is for energy $E_{1,1}$, where quantum number n_x is 1 and quantum number n_y is 1.

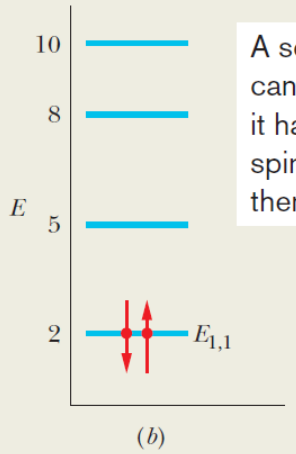
Pauli principle: The trapped electrons must obey the Pauli exclusion principle; that is, no two electrons can have the same set of values for their quantum numbers n_x, n_y, and m_s. The first electron goes into energy level $E_{1,1}$ and can have $m_s = \frac{1}{2}$ or $m_s = -\frac{1}{2}$. We arbitrarily choose the latter

Figure 40-12 (*a*) Energy-level diagram for one electron in a square corral. (Energy E is in multiples of $h^2/8mL^2$.) A spin-down electron occupies the lowest level. (*b*) Two electrons (one spin down, the other spin up) occupy the lowest level of the one-electron energy-level diagram. (*c*) A third electron occupies the next energy level. (*d*) Four electrons can be put into the second level. (*e*) The system's ground-state configuration. (*f*) Three transitions to consider for the first excited state. (*g*) The system's lowest three total energies.



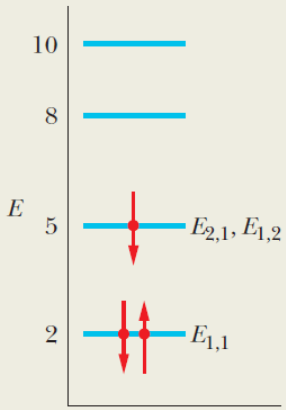
(*a*)

These are the four lowest energy levels of the corral. The first electron is in the lowest level.



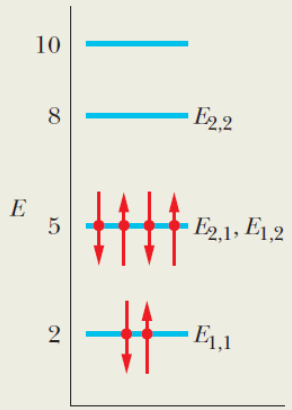
(*b*)

A second electron can be there only if it has the opposite spin. The level is then full.



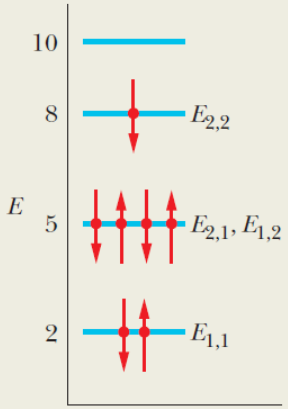
(*c*)

The lowest energy for a third electron is on the next level up.



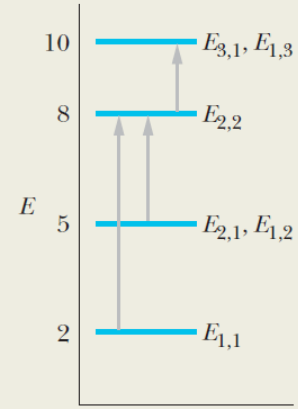
(*d*)

Two quantum states have that energy. Two electrons (with opposite spins) can be in each state. Then that level is also full.



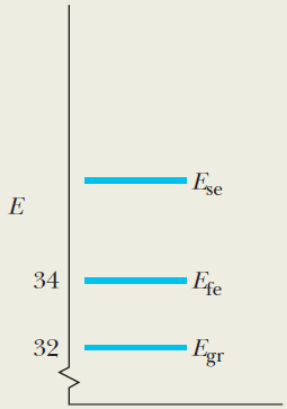
(*e*)

The lowest energy for the seventh electron is on the next level up. The system of 7 electrons is in its lowest energy (system ground state).



(*f*)

Electrons can jump up only to levels that are not full. Here are three allowed jumps. Which uses the least energy? If that jump is made, the system is then in its first excited state.



(*g*)

Here are the three lowest energy levels of the system.

(b) What is the total energy of the seven-electron system in its ground state, as a multiple of $h^2/8mL^2$?

KEY IDEA

The total energy E_{gr} is the sum of the energies of the individual electrons in the system's ground-state configuration.

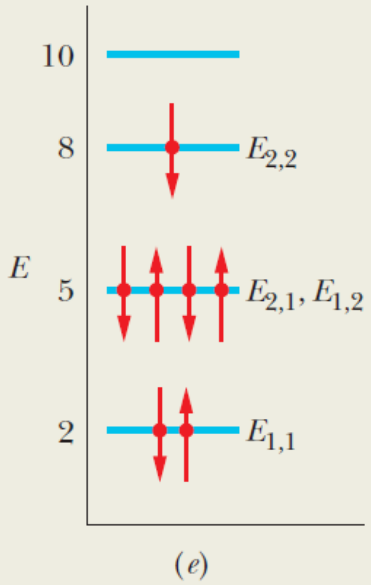
Ground-state energy: The energy of each electron can be read from Table 39-1, which is partially reproduced in Table 40-2, or from Fig. 40-12*e*. Because there are two electrons in the first (lowest) level, four in the second level, and one in the third level, we have

$$E_{gr} = 2\left(2\frac{h^2}{8mL^2}\right) + 4\left(5\frac{h^2}{8mL^2}\right) + 1\left(8\frac{h^2}{8mL^2}\right)$$
$$= 32\frac{h^2}{8mL^2}. \qquad \text{(Answer)}$$

Table 40-2 Ground-State Configuration and Energies

n_x	n_y	m_s	Energy[a]
2	2	$-\frac{1}{2}$	8
2	1	$+\frac{1}{2}$	5
2	1	$-\frac{1}{2}$	5
1	2	$+\frac{1}{2}$	5
1	2	$-\frac{1}{2}$	5
1	1	$+\frac{1}{2}$	2
1	1	$-\frac{1}{2}$	2
			Total 32

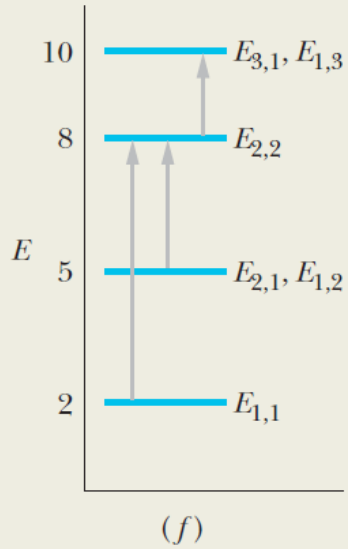
[a]In multiples of $h^2/8mL^2$.



(*e*)

The lowest energy for the seventh electron is on the next level up. The system of 7 electrons is in its lowest energy (system ground state).

(c) How much energy must be transferred to the system for it to jump to its first excited state, and what is the energy of that state?



Electrons can jump up only to levels that are not full. Here are three allowed jumps. Which uses the least energy? If that jump is made, the system is then in its first excited state.

First-excited-state energy: Let us consider the three jumps shown in Fig. 40-12*f*; all are allowed by the Pauli exclusion principle because they are jumps to either empty or partially occupied states. In one of those possible jumps, an electron jumps from the $E_{1,1}$ level to the partially occupied $E_{2,2}$ level. The change in the energy is

$$\Delta E = E_{2,2} - E_{1,1} = 8\frac{h^2}{8mL^2} - 2\frac{h^2}{8mL^2} = 6\frac{h^2}{8mL^2}.$$

(We shall assume that the spin orientation of the electron making the jump can change as needed.)

In another of the possible jumps in Fig. 40-12*f*, an electron jumps from the degenerate level of $E_{2,1}$ and $E_{1,2}$ to the partially occupied $E_{2,2}$ level. The change in the energy is

$$\Delta E = E_{2,2} - E_{2,1} = 8\frac{h^2}{8mL^2} - 5\frac{h^2}{8mL^2} = 3\frac{h^2}{8mL^2}.$$

In the third possible jump in Fig. 40-12*f*, the electron in the $E_{2,2}$ level jumps to the unoccupied, degenerate level of $E_{1,3}$ and $E_{3,1}$. The change in energy is

$$\Delta E = E_{1,3} - E_{2,2} = 10\frac{h^2}{8mL^2} - 8\frac{h^2}{8mL^2} = 2\frac{h^2}{8mL^2}.$$

Of these three possible jumps, the one requiring the least energy change ΔE is the last one. We could consider even more possible jumps, but none would require less energy. Thus, for the system to jump from its ground state to its first excited state, the electron in the $E_{2,2}$ level must jump to the unoccupied, degenerate level of $E_{1,3}$ and $E_{3,1}$, and the required energy is

$$\Delta E = 2\frac{h^2}{8mL^2}. \qquad \text{(Answer)}$$

The energy E_{fe} of the first excited state of the system is then

$$E_{fe} = E_{gr} + \Delta E = 32\frac{h^2}{8mL^2} + 2\frac{h^2}{8mL^2} = 34\frac{h^2}{8mL^2}. \qquad \text{(Answer)}$$

We can represent this energy and the energy E_{gr} for the ground state of the system on an energy-level diagram *for the system*, as shown in Fig. 40-12*g*.

40.5: Building The Periodic Table:

For the purpose of labeling subshells, the values of ℓ are represented by letters:

$\ell =$	0	1	2	3	4	5	...
	s	p	d	f	g	h	....

For example, the $n = 3, \ell = 2$ subshell would be labeled the $3d$ subshell.

Neon: The atom has 10 electrons. It has three closed subshells (*1s, 2s,* and *2p)* and, like the other **noble gases** that form the right-hand column of the periodic table, is almost chemically inert.

Sodium: The atom has 11 electrons. Ten of them form a closed neon-like core, and has zero angular momentum. The remaining electron is largely outside this inert core, in the $3s$ subshell. This is the **valence electron** of the atom, and the atom's angular momentum and magnetic dipole moment must be due entirely to the spin of this single electron.

Chlorine: This, with 17 electrons, has the outermost 7 electrons in *3p* subshell, leaving a "hole" in this state. It is receptive to interacting with other atoms that have a valence electron that might fill this hole. Chlorine, like the other **halogens** that form column VIIA of the periodic table, is chemically active.

Iron: $1s^2 \quad 2s^2\,2p^6 \quad 3s^2 \quad 3p^6 \quad 3d^6 \quad 4s^2$. This atom of 26 electrons, has the first 18 electrons form the five filled subshells that are marked off by the bracket. 6 of the remaining 8 electrons go into the *3d* subshell, and the remaining two go into the *4s* subshell. The configuration $3d^6 4s^2$ is of lower energy than $3d^8$.

40.6: X-Rays and the Ordering of the Elements:

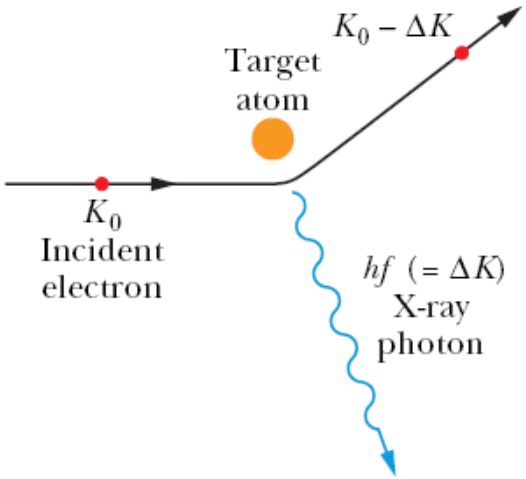


Fig. 40-14 An electron of kinetic energy K_0 passing near an atom in the target may generate an x-ray photon, the electron losing part of its energy in the process. The continuous x-ray spectrum arises in this way.

The minimum possible x-ray wavelength:

$$K_0 = hf = \frac{hc}{\lambda_{min}},$$

$$\lambda_{min} = \frac{hc}{K_0} \qquad \text{(cutoff wavelength).}$$

Checkpoint 2

Does the cutoff wavelength λ_{min} of the continuous x-ray spectrum increase, decrease, or remain the same if you (a) increase the kinetic energy of the electrons that strike the x-ray target, (b) allow the electrons to strike a thin foil rather than a thick block of the target material, (c) change the target to an element of higher atomic number?

(a) decrease; (b)−(c) remain the same

40.6: X Rays and the Ordering of the Elements: The Characteristic X-ray Spectrum

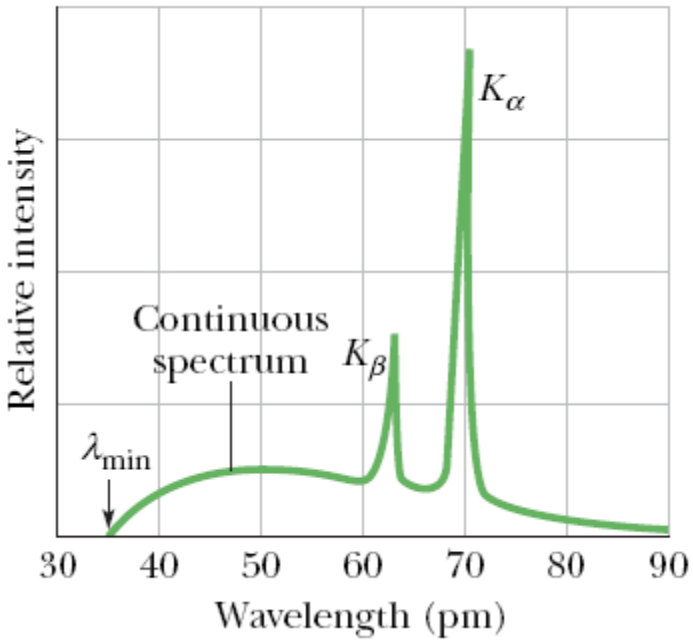


Fig. 40-13 The distribution by wavelength of the x rays produced when 35 keV electrons strike a molybdenum target

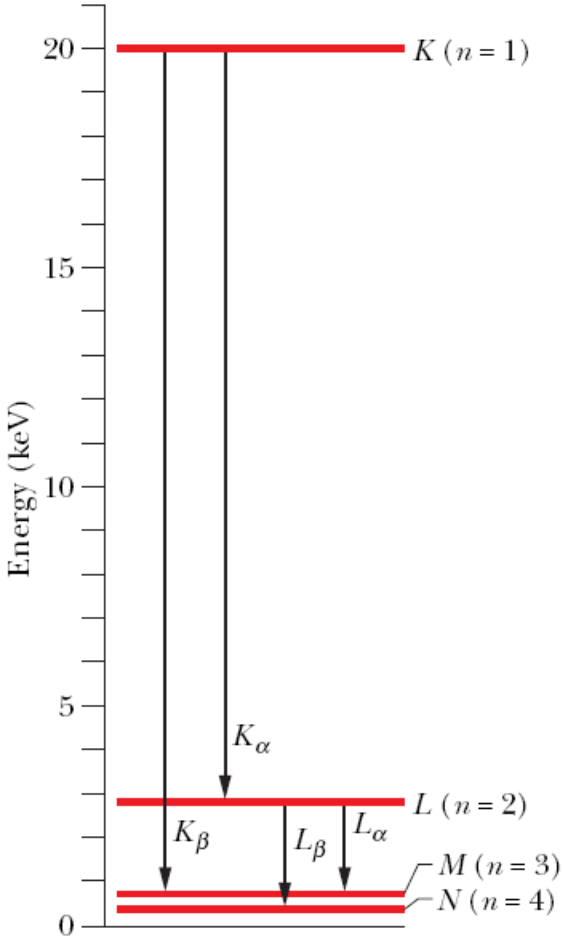


Fig. 40-15 A simplified energy-level diagram for a molybdenum atom, showing the transitions (of holes rather than electrons) that give rise to some of the characteristic x rays of that element. Each horizontal line represents the energy of the atom with a hole (a missing electron) in the shell indicated.