Chapter 40

All About Atoms

Accounting for Moseley's Plot:

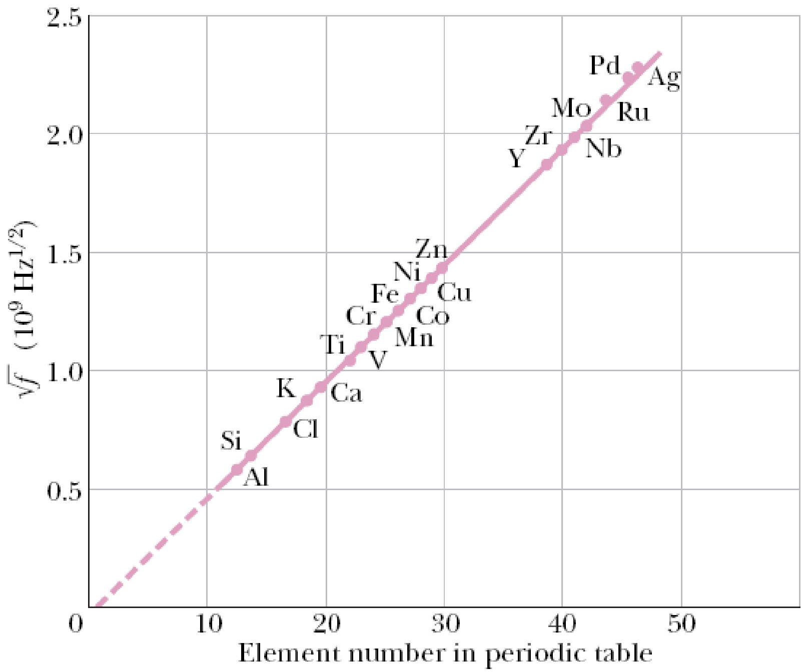


Fig. 40-16 A Moseley plot of the K_α line of the characteristic x-ray spectra of 21 elements. The frequency is calculated from the measured wavelength.

For hydrogen atom:

$$E_n = -\frac{me^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = -\frac{13.60 \text{ eV}}{n^2}, \qquad \text{for } n = 1, 2, 3, \ldots .$$

For one of the two innermost electrons in the *K* shell of a multi-electron atom, because of the presence of the other *K-shell* electron, it "sees" an effective nuclear charge of approximately (*Z -1)e,* where *e* is the electronic charge. Therefore the effective energy of the atom is:

$$E_n = -\frac{(13.60 \text{ eV})(Z-1)^2}{n^2}.$$

Therefore,

$$\Delta E = E_2 - E_1$$

$$= \frac{-(13.60 \text{ eV})(Z-1)^2}{2^2} - \frac{-(13.60 \text{ eV})(Z-1)^2}{1^2}$$

$$= (10.2 \text{ eV})(Z-1)^2.$$

And,

$$f = \frac{\Delta E}{h} = \frac{(10.2 \text{ eV})(Z-1)^2}{(4.14 \times 10^{-15} \text{ eV}\cdot\text{s})}$$

$$= (2.46 \times 10^{15} \text{ Hz})(Z-1)^2.$$

$$\boxed{\sqrt{f} = CZ - C,}$$

C is a constant ($= 4.96 \times 10^7$ Hz$^{1/2}$).

Example, Characteristic spectrum in x-ray production:

A cobalt target is bombarded with electrons, and the wavelengths of its characteristic x-ray spectrum are measured. There is also a second, fainter characteristic spectrum, which is due to an impurity in the cobalt. The wavelengths of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity), and the proton number for cobalt is $Z_{\text{Co}} = 27$. Determine the impurity using only these data.

KEY IDEA

The wavelengths of the K_α lines for both the cobalt (Co) and the impurity (X) fall on a K_α Moseley plot, and Eq. 40-27 is the equation for that plot.

Calculations: Substituting c/λ for f in Eq. 40-27, we obtain

$$\sqrt{\frac{c}{\lambda_{\text{Co}}}} = CZ_{\text{Co}} - C \quad \text{and} \quad \sqrt{\frac{c}{\lambda_{\text{X}}}} = CZ_{\text{X}} - C.$$

Dividing the second equation by the first neatly eliminates C, yielding

$$\sqrt{\frac{\lambda_{\text{Co}}}{\lambda_{\text{X}}}} = \frac{Z_{\text{X}} - 1}{Z_{\text{Co}} - 1}.$$

Substituting the given data yields

$$\sqrt{\frac{178.9 \text{ pm}}{143.5 \text{ pm}}} = \frac{Z_{\text{X}} - 1}{27 - 1}.$$

Solving for the unknown, we find that

$$Z_{\text{X}} = 30.0. \qquad \text{(Answer)}$$

Thus, the number of protons in the impurity nucleus is 30, and a glance at the periodic table identifies the impurity as zinc. Note that with a larger value of Z than cobalt, zinc has a smaller value of the K_α line. This means that the energy associated with that jump must be greater in zinc than cobalt.

40.7: Lasers and Laser Light:

1. *Laser light is highly monochromatic.*
2. *Laser light is highly coherent.*
3. *Laser light is highly directional.*
4. *Laser light can be sharply focused.*

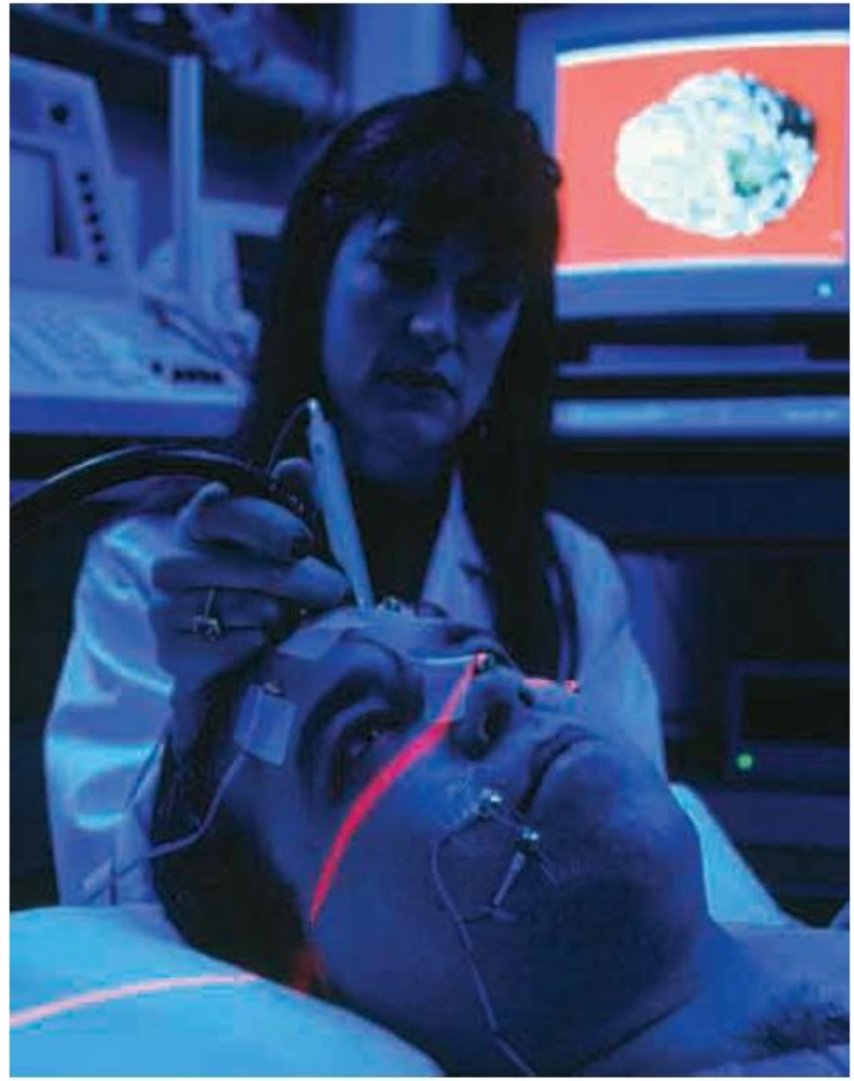


Fig. 40-17 A patient's head is scanned and mapped by (red) laser light in preparation for brain surgery. During the surgery, the laser-derived image of the head will be superimposed on the model of the brain shown on the monitor, to guide the surgical team into the region shown in green on the model. *(Sam Ogden/Photo Researchers)*

How Lasers Work:

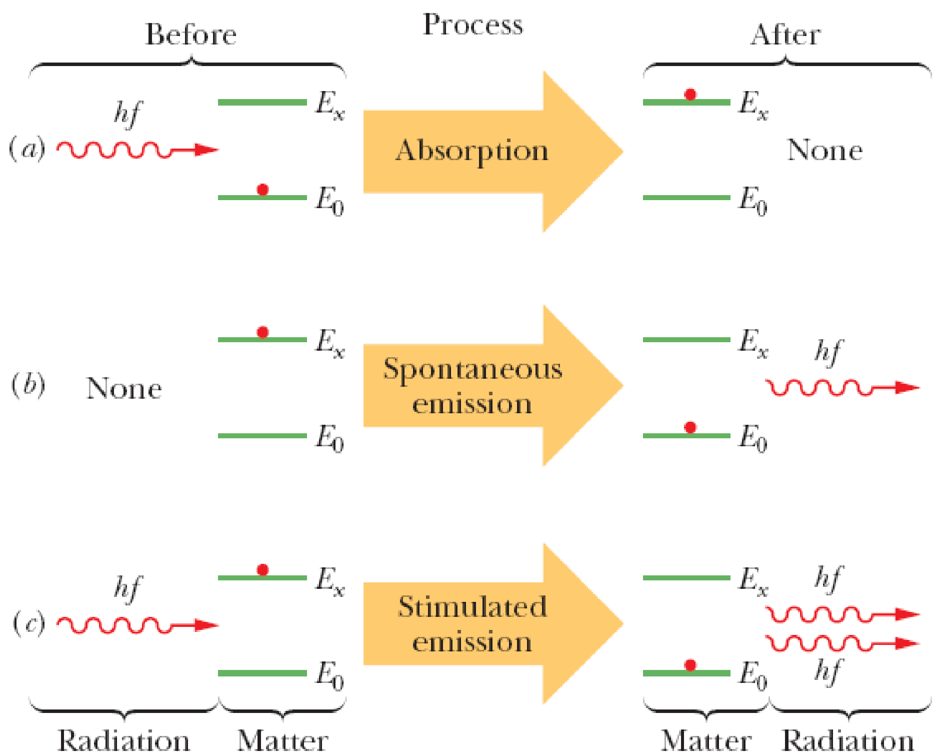


Fig. 40-18 The interaction of radiation and matter in the processes of (*a*) absorption, (*b*) spontaneous emission, and (*c*) stimulated emission. An atom (matter) is represented by the red dot; the atom is in either a lower quantum state with energy E_0 or a higher quantum state with energy E_x. In (*a*) the atom absorbs a photon of energy hf from a passing light wave. In (*b*) it emits a light wave by emitting a photon of energy hf. In (*c*) a passing light wave with photon energy hf causes the atom to emit a photon of the same energy, increasing the energy of the light wave.

How Lasers Work:

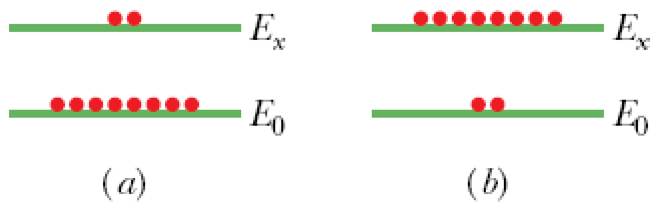


Fig. 40-19 (*a*) The equilibrium distribution of atoms between the ground state E_0 and excited state E_x accounted for by thermal agitation. (*b*) An inverted population, obtained by special methods. Such a population inversion is essential for laser action.

If the atoms of Fig. 40-19*a* are flooded with photons of energy $(E_x - E_0)$, photons will disappear via absorption by ground-state atoms and photons will be generated largely via stimulated emission of excited-state atoms. Thus, because there are more atoms in the ground state, the *net effect* will be the absorption of photons.

To produce laser light, one must have more photons emitted than absorbed; that is, one must have a situation in which stimulated emission dominates. Thus, one needs more atoms in the excited state than in the ground state, as in Fig. 40-19*b*.

40.12: The Helium-Neon Gas Laser:

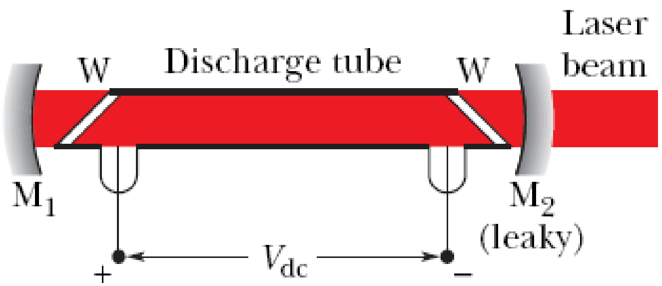
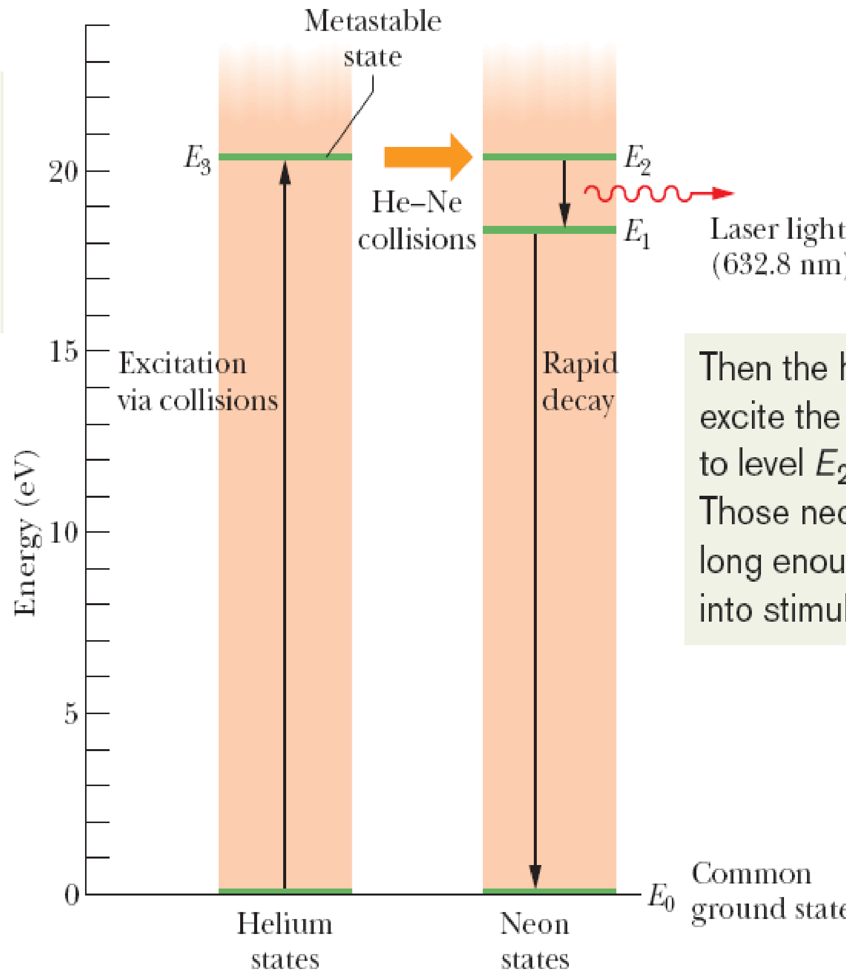


Fig. 40-20 The elements of a helium–neon gas laser. An applied potential V_{dc} sends electrons through a discharge tube containing a mixture of helium gas and neon gas. Electrons collide with helium atoms, which then collide with neon atoms, which emit light along the length of the tube. The light passes through transparent windows W and reflects back and forth through the tube from mirrors M_1 and M_2 to cause more neon atom emissions. Some of the light leaks through mirror M_2 to form the laser beam.



Then the helium atoms excite the neon atoms to level E_2 by collisions. Those neon atoms stay long enough to be forced into stimulated emission.

Fig. 40-21 Five essential energy levels for helium and neon atoms in a helium–neon gas laser. Laser action occurs between levels E_2 and E_1 of neon when more atoms are at the E_2 level than at the E_1 level.

Checkpoint 3

The wavelength of light from laser A (a helium–neon gas laser) is 632.8 nm; that from laser B (a carbon dioxide gas laser) is 10.6 μm; that from laser C (a gallium arsenide semiconductor laser) is 840 nm. Rank these lasers according to the energy interval between the two quantum states responsible for laser action, greatest first.

$$A, C, B$$

Example, Population inversion in a laser:

In the helium–neon laser of Fig. 40-20, laser action occurs between two excited states of the neon atom. However, in many lasers, laser action (*lasing*) occurs between the ground state and an excited state, as suggested in Fig. 40-19*b*.

(a) Consider such a laser that emits at wavelength $\lambda = 550$ nm. If a population inversion is not generated, what is the ratio of the population of atoms in state E_x to the population in the ground state E_0, with the atoms at room temperature?

KEY IDEAS

(1) The naturally occurring population ratio N_x/N_0 of the two states is due to thermal agitation of the gas atoms (Eq. 40-29):

$$N_x/N_0 = e^{-(E_x - E_0)/kT}. \qquad (40\text{-}30)$$

To find N_x/N_0 with Eq. 40-30, we need to find the energy separation $E_x - E_0$ between the two states. (2) We can obtain $E_x - E_0$ from the given wavelength of 550 nm for the lasing between those two states.

Calculation: The lasing wavelength gives us

$$\begin{aligned} E_x - E_0 = hf &= \frac{hc}{\lambda} \\ &= \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{(550 \times 10^{-9}\ \mathrm{m})(1.60 \times 10^{-19}\ \mathrm{J/eV})} \\ &= 2.26\ \mathrm{eV}. \end{aligned}$$

To solve Eq. 40-30, we also need the mean energy of thermal agitation kT for an atom at room temperature (assumed to be 300 K), which is

$$kT = (8.62 \times 10^{-5}\ \mathrm{eV/K})(300\ \mathrm{K}) = 0.0259\ \mathrm{eV},$$

in which k is Boltzmann's constant.

Substituting the last two results into Eq. 40-30 gives us the population ratio at room temperature:

$$\begin{aligned} N_x/N_0 &= e^{-(2.26\ \mathrm{eV})/(0.0259\ \mathrm{eV})} \\ &\approx 1.3 \times 10^{-38}. \qquad \text{(Answer)} \end{aligned}$$

This is an extremely small number. It is not unreasonable, however. Atoms with a mean thermal agitation energy of only 0.0259 eV will not often impart an energy of 2.26 eV to another atom in a collision.

(b) For the conditions of (a), at what temperature would the ratio N_x/N_0 be 1/2?

Calculation: Now we want the temperature T such that thermal agitation has bumped enough neon atoms up to the higher-energy state to give $N_x/N_0 = 1/2$. Substituting that ratio into Eq. 40-30, taking the natural logarithm of both sides, and solving for T yield

$$\begin{aligned} T = \frac{E_x - E_0}{k(\ln 2)} &= \frac{2.26\ \mathrm{eV}}{(8.62 \times 10^{-5}\ \mathrm{eV/K})(\ln 2)} \\ &= 38\,000\ \mathrm{K}. \qquad \text{(Answer)} \end{aligned}$$