Chapter 41

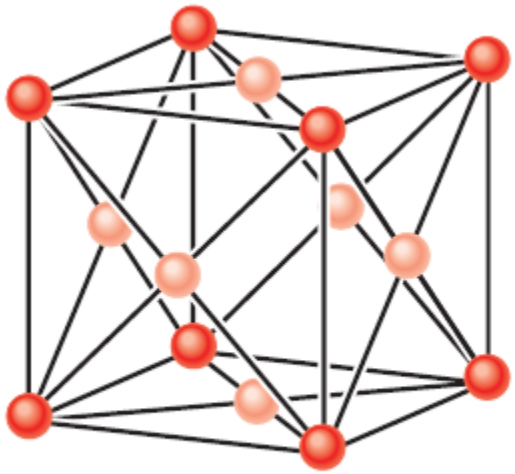
Conduction of Electricity in Solids

41.1: The Electrical Properties of Solids:

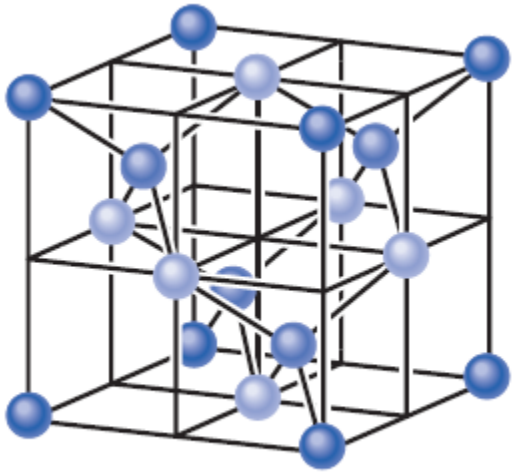
The electrical properties of solids can be categorized into following classes:

1. Their **resistivity** ρ at room temperature, with the SI unit ohm-meter ($\Omega \cdot m$); resistivity is defined in Section 26-4.
2. Their **temperature coefficient of resistivity** α, defined as $\alpha = (1/\rho)(d\rho/dT)$ in Eq. 26-17 and having the SI unit inverse kelvin (K^{-1}). We can evaluate α for any solid by measuring ρ over a range of temperatures.
3. Their **number density of charge carriers** n. This quantity, the number of charge carriers per unit volume, can be found from measurements of the Hall effect, as discussed in Section 28-4, and from other measurements. It has the SI unit inverse cubic meter (m^{-3}).

Fig. 41-1 (*a*) The unit cell for copper is a cube. There is one copper atom (darker) at each corner of the cube and one copper atom (lighter) at the center of each face of the cube. The arrangement is called *face-centered cubic*. (*b*) The unit cell for either silicon or the carbon atoms in diamond is also a cube, the atoms being arranged in what is called a *diamond lattice*. There is one atom (darkest) at each corner of the cube and one atom (lightest) at the center of each cube face; in addition, four atoms (medium color) lie within the cube. Every atom is bonded to its four nearest neighbors by a two-electron covalent bond (only the four atoms within the cube show all four nearest neighbors).



(*a*)



(*b*)

Energy Levels in a Crystalline Solid:

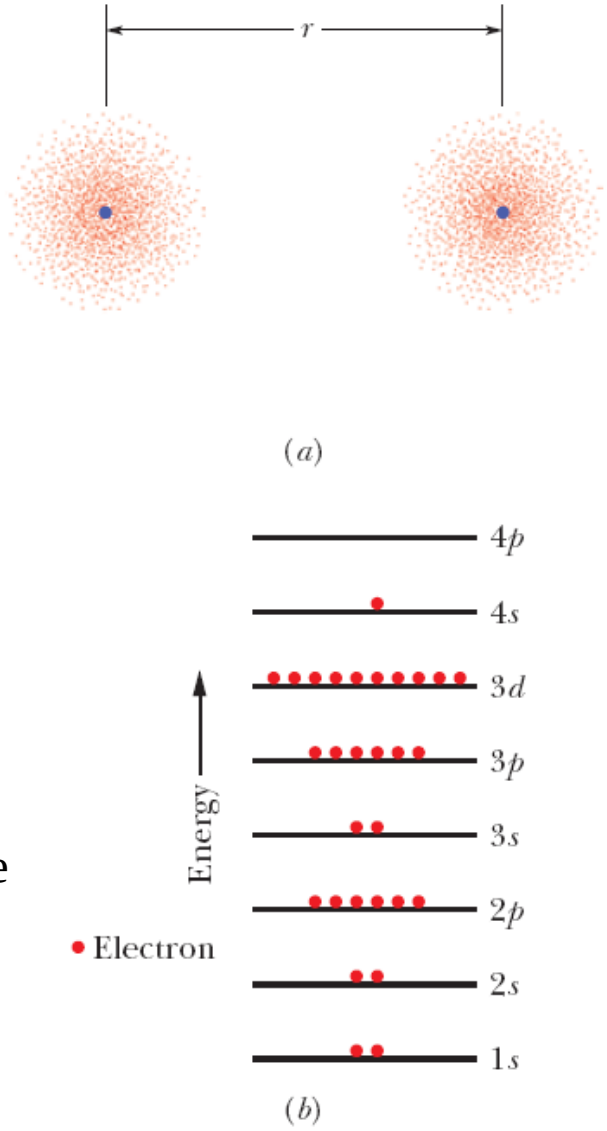


Fig. 41-2 (*a*) Two copper atoms separated by a large distance; their electron distributions are represented by dot plots. (*b*) Each copper atom has 29 electrons distributed among a set of subshells. In the neutral atom in its ground state, all subshells up through the 3*d* level are filled, the 4*s* subshell contains one electron (it can hold two), and higher subshells are empty. For simplicity, the subshells are shown as being evenly spaced in energy.

If we bring the atoms of Fig. 41-2*a* close together, their wave functions will overlap, beginning with those of the outermost electrons. Then we have a single two-atom system; here the system contains 2 x29 =58 electrons.

If we bring up more atoms, we gradually assemble a lattice of solid copper. If the lattice contains N atoms, then each level of an isolated copper atom must split into N levels in the solid.

Thus, the individual energy levels of the solid form energy ***bands,*** adjacent bands being separated by an energy ***gap,*** with the gap representing a range of energies that no electron can possess.

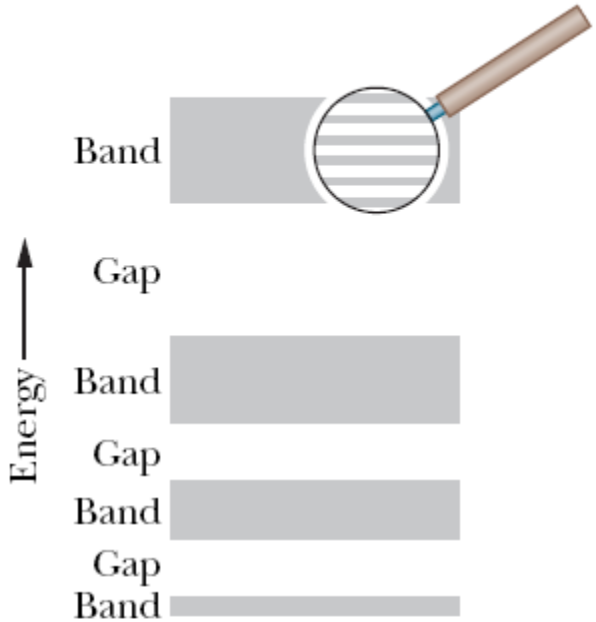


Fig. 41-3 The band–gap pattern of energy levels for an idealized crystalline solid. As the magnified view suggests, each band consists of a very large number of very closely spaced energy levels. (In many solids, adjacent bands may overlap; for clarity, we have not shown this condition.)

Table 41-1

Some Electrical Properties of Two Materials[a]

Property	Unit	Material	
		Copper	Silicon
Type of conductor		Metal	Semiconductor
Resistivity, ρ	$\Omega \cdot m$	2×10^{-8}	3×10^{3}
Temperature coefficient of resistivity, α	K^{-1}	$+4 \times 10^{-3}$	-70×10^{-3}
Number density of charge carriers, n	m^{-3}	9×10^{28}	1×10^{16}

[a] All values are for room temperature.

Insulators:

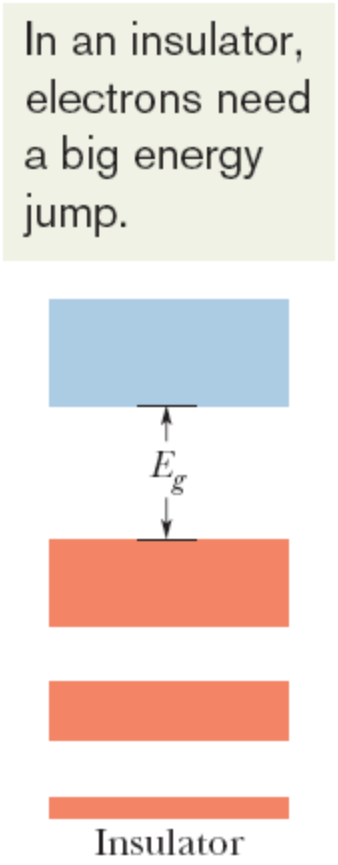


Fig. 41-4 The band–gap pattern for an insulator; filled levels are shown in red and empty levels in blue.

Sample Problem 41.01 Probability of electron excitation in an insulator

Approximately what is the probability that, at room temperature (300 K), an electron at the top of the highest filled band in diamond (an insulator) will jump the energy gap E_g in Fig. 41-4? For diamond, E_g is 5.5 eV.

KEY IDEA

In Chapter 40 we used Eq. 40-29,

$$\frac{N_x}{N_0} = e^{-(E_x - E_0)/kT}, \tag{41-1}$$

to relate the population N_x of atoms at energy level E_x to the population N_0 at energy level E_0, where the atoms are part of a system at temperature T (measured in kelvins); k is the Boltzmann constant (8.62×10^{-5} eV/K). In this chapter we can use Eq. 41-1 to *approximate* the probability P that an electron in an insulator will jump the energy gap E_g in Fig. 41-4.

Calculations: We first set the energy difference $E_x - E_0$ to E_g. Then the probability P of the jump is approximately equal to the ratio N_x/N_0 of the number of electrons just above the energy gap to the number of electrons just below the gap.

For diamond, the exponent in Eq. 41-1 is

$$-\frac{E_g}{kT} = -\frac{5.5 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(300 \text{ K})} = -213.$$

The required probability is then

$$P = \frac{N_x}{N_0} = e^{-(E_g/kT)} = e^{-213} \approx 3 \times 10^{-93}. \quad \text{(Answer)}$$

This result tells us that approximately 3 electrons out of 10^{93} electrons would jump across the energy gap. Because any diamond stone has fewer than 10^{23} electrons, we see that the probability of the jump is vanishingly small. No wonder diamond is such a good insulator.

Metals:

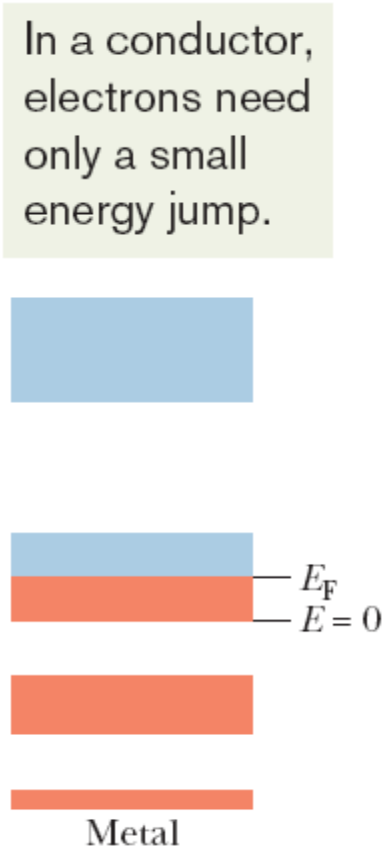


Fig. 41-5 The band–gap pattern for a metal. The highest filled level, called the Fermi level, lies near the middle of a band. Since vacant levels are available within that band, electrons in the band can easily change levels, and conduction can take place.

If the electric potential energy U of a conduction electron is uniform throughout the lattice, let's set $U = 0$, so that the mechanical energy E is entirely kinetic. Then the level at the bottom of the partially filled band of Fig. 41-5 corresponds to $E = 0$. The highest occupied level in this band at absolute zero ($T = 0$ K) is called the ***Fermi level,*** and the energy corresponding to it is called the ***Fermi energy*** E_F; for copper, $E_F = 7.0$ eV.

The electron speed corresponding to the Fermi energy is called the ***Fermi speed*** v_F. For copper the Fermi speed is $= 1.6 \times 10^6$ m/s. All motion does not cease at absolute zero; at that temperature the conduction electrons are stacked up in the partially filled band of Fig. 41-5 with energies that range from zero to the Fermi energy.

Metals: How Many Conduction Electrons Are There?

$$\begin{pmatrix}\text{number of conduction}\\ \text{electrons in sample}\end{pmatrix} = \begin{pmatrix}\text{number of atoms}\\ \text{in sample}\end{pmatrix}\begin{pmatrix}\text{number of valence}\\ \text{electrons per atom}\end{pmatrix}.$$

$$n = \frac{\text{number of conduction electrons in sample}}{\text{sample volume } V}.$$

$$\begin{pmatrix}\text{number of atoms}\\ \text{in sample}\end{pmatrix} = \frac{\text{sample mass } M_{\text{sam}}}{\text{atomic mass}} = \frac{\text{sample mass } M_{\text{sam}}}{(\text{molar mass } M)/N_{\text{A}}}$$

$$= \frac{(\text{material's density})(\text{sample volume } V)}{(\text{molar mass } M)/N_{\text{A}}},$$

where the molar mass M is the mass of one mole of the material in the sample and N_{A} is Avogadro's number ($6.02 \times 10^{23}\ \text{mol}^{-1}$).

Sample Problem 41.02 Number of conduction electrons in a metal

How many conduction electrons are in a cube of magnesium of volume $2.00 \times 10^{-6}\ m^3$? Magnesium atoms are bivalent.

KEY IDEAS

1. Because magnesium atoms are bivalent, each magnesium atom contributes two conduction electrons.
2. The cube's number of conduction electrons is related to its number of magnesium atoms by Eq. 41-2.
3. We can find the number of atoms with Eq. 41-4 and known data about the cube's volume and magnesium's properties.

Calculations: We can write Eq. 41-4 as

$$\begin{pmatrix}\text{number}\\ \text{of atoms}\\ \text{in sample}\end{pmatrix} = \frac{(\text{density})(\text{sample volume } V)N_A}{\text{molar mass } M}.$$

Magnesium has density $1.738\ g/cm^3$ $(= 1.738 \times 10^3\ kg/m^3)$ and molar mass $24.312\ g/mol$ $(= 24.312 \times 10^{-3}\ kg/mol)$ (see Appendix F). The numerator gives us

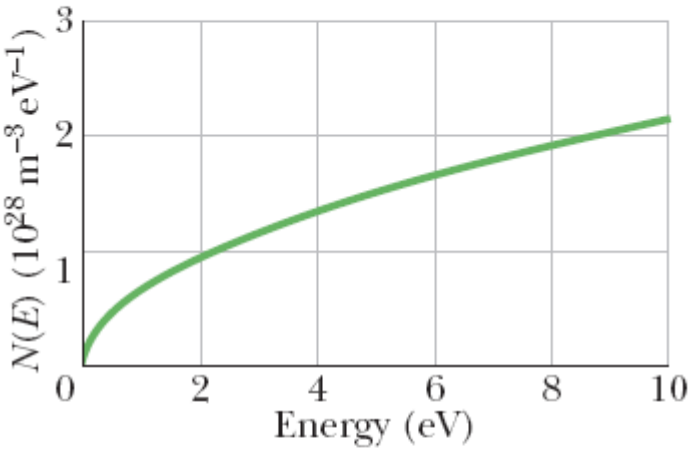
$$(1.738 \times 10^3\ kg/m^3)(2.00 \times 10^{-6}\ m^3) \times (6.02 \times 10^{23}\ \text{atoms/mol}) = 2.0926 \times 10^{21}\ kg/mol.$$

Thus,
$$\begin{pmatrix}\text{number of atoms}\\ \text{in sample}\end{pmatrix} = \frac{2.0926 \times 10^{21}\ kg/mol}{24.312 \times 10^{-3}\ kg/mol} = 8.61 \times 10^{22}.$$

Using this result and the fact that magnesium atoms are bivalent, we find that Eq. 41-2 yields

$$\begin{pmatrix}\text{number of}\\ \text{conduction electrons}\\ \text{in sample}\end{pmatrix} = (8.61 \times 10^{22}\ \text{atoms})\left(2\ \frac{\text{electrons}}{\text{atom}}\right)$$
$$= 1.72 \times 10^{23}\ \text{electrons.} \quad \text{(Answer)}$$

The density of energy levels increases upward in a band.



$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} \qquad \text{(density of states, m}^{-3}\text{ J}^{-1}\text{)},$$

Fig. 41-6 The density of states $N(E)$—that is, the number of electron energy levels per unit energy interval per unit volume—plotted as a function of electron energy. The density of states function simply counts the available states; it says nothing about whether these states are occupied by electrons.

Checkpoint 1

Is the spacing between adjacent energy levels at $E = 4$ eV in copper larger than, the same as, or smaller than the spacing at $E = 6$ eV?

larger

Sample Problem 41.03 Number of states per electron volt in a metal

(a) Using the data of Fig. 41-6, determine the number of states per electron-volt at 7 eV in a metal sample with a volume V of 2×10^{-9} m^3.

KEY IDEA

We can obtain the number of states per electron-volt at a given energy by using the density of states $N(E)$ at that energy and the sample's volume V.

Calculations: At an energy of 7 eV, we write

$$\begin{pmatrix}\text{number of states}\\ \text{per eV at 7 eV}\end{pmatrix} = \begin{pmatrix}\text{density of states}\\ N(E)\text{ at 7 eV}\end{pmatrix}\begin{pmatrix}\text{volume } V\\ \text{of sample}\end{pmatrix}.$$

From Fig. 41-6, we see that at an energy E of 7 eV, the density of states is about 1.8×10^{28} m^{-3} eV^{-1}. Thus,

$$\begin{aligned}\begin{pmatrix}\text{number of states}\\ \text{per eV at 7 eV}\end{pmatrix} &= (1.8 \times 10^{28}\ \text{m}^{-3}\ \text{eV}^{-1})(2 \times 10^{-9}\ \text{m}^3)\\ &= 3.6 \times 10^{19}\ \text{eV}^{-1}\\ &\approx 4 \times 10^{19}\ \text{eV}^{-1}. \qquad \text{(Answer)}\end{aligned}$$

(b) Next, determine the number of states N in the sample within a *small* energy range ΔE of 0.003 eV centered at 7 eV (the range is small relative to the energy level in the band).

Calculation: From Eq. 41-5 and Fig. 41-6, we know that the density of states is a function of energy E. However, for an energy range ΔE that is small relative to E, we can approximate the density of states (and thus the number of states per electron-volt) to be constant. Thus, at an energy of 7 eV, we find the number of states N in the energy range ΔE of 0.003 eV as

$$\begin{pmatrix}\text{number of states } N\\ \text{in range } \Delta E\text{ at 7 eV}\end{pmatrix} = \begin{pmatrix}\text{number of states}\\ \text{per eV at 7 eV}\end{pmatrix}\begin{pmatrix}\text{energy}\\ \text{range } \Delta E\end{pmatrix}$$

or

$$\begin{aligned}N &= (3.6 \times 10^{19}\ \text{eV}^{-1})(0.003\ \text{eV})\\ &= 1.1 \times 10^{17} \approx 1 \times 10^{17}. \qquad \text{(Answer)}\end{aligned}$$

(When you are asked for the number of states in a certain energy range, first see if that range is small enough to allow this type of approximation.)