Chapter 42

Nuclear Physics

42.1: Discovering The Nucleus:

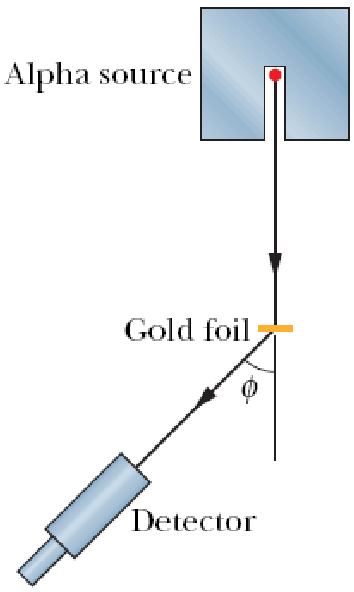
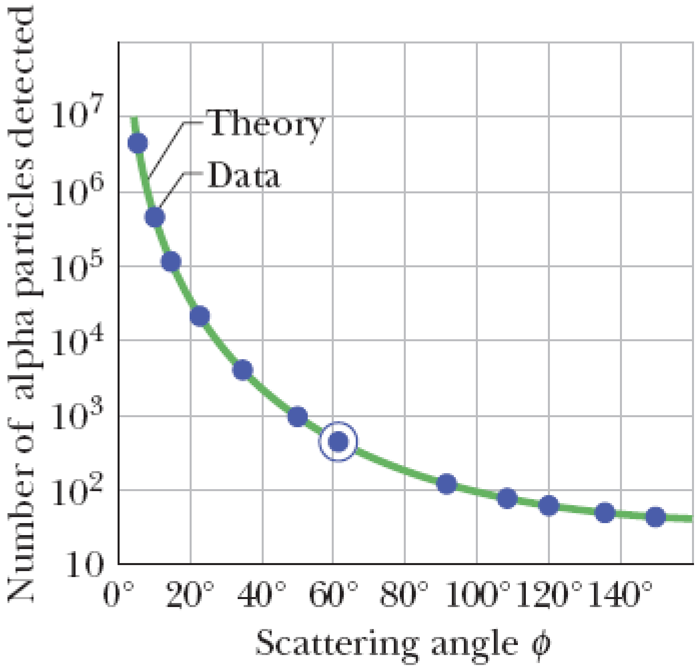


Fig. 42-1 An arrangement (top view) used in Rutherford's laboratory in 1911–1913 to study the scattering of α particles by thin metal foils. The detector can be rotated to various values of the scattering angle ϕ. The alpha source was radon gas, a decay product of radium. With this simple "tabletop" apparatus, the atomic nucleus was discovered.

In 1911 Ernest Rutherford proposed that the positive charge of the atom is densely concentrated at the center of the atom, forming its **nucleus,** and that, furthermore, the nucleus is responsible for most of the mass of the atom.

Fig. 42-2 The dots are alpha-particle scattering data for a gold foil, obtained by Geiger and Marsden using the apparatus of Fig. 42-1. The solid curve is the theoretical prediction, based on the assumption that the atom has a small, massive, positively charged nucleus. The data have been adjusted to fit the theoretical curve at the experimental point that is enclosed in a circle.



We see that most of the particles are scattered through rather small angles, but a very small fraction of them are scattered through very large angles, approaching 180°.

In Rutherford's words: "It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it [the shell] came back and hit you."

Rutherford saw that, to deflect the alpha particle backward, there must be a large force; this force could be provided if the positive charge, instead of being spread throughout the atom, were concentrated tightly at its center. Then the incoming alpha particle could get very close to the positive charge without penetrating it; such a close encounter would result in a large deflecting force.

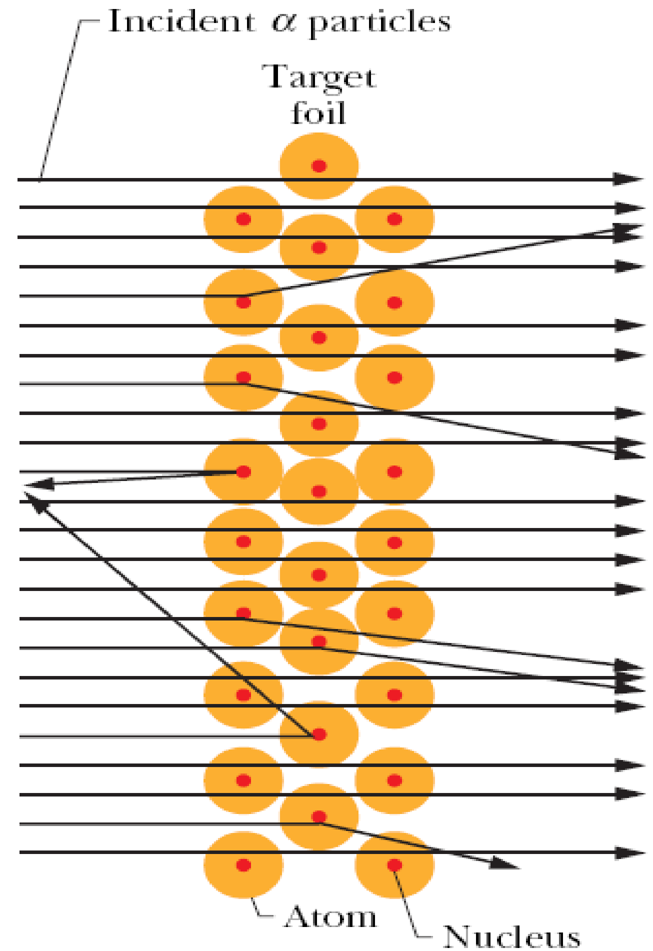
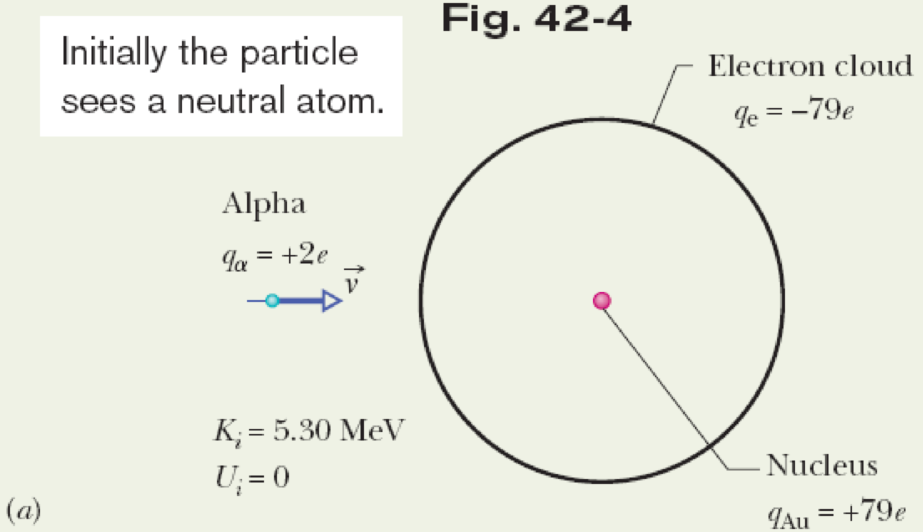


Fig. 42-3 The angle through which an incident alpha particle is scattered depends on how close the particle's path lies to an atomic nucleus. Large deflections result only from very close encounters.

Example, Rutherford scattering of an alpha particle by a gold nucleus:

An alpha particle with kinetic energy $K_i = 5.30$ MeV happens, by chance, to be headed directly toward the nucleus of a neutral gold atom (Fig. 42-4*a*). What is its *distance of closest approach d* (least center-to-center separation) to the nucleus? Assume that the atom remains stationary.

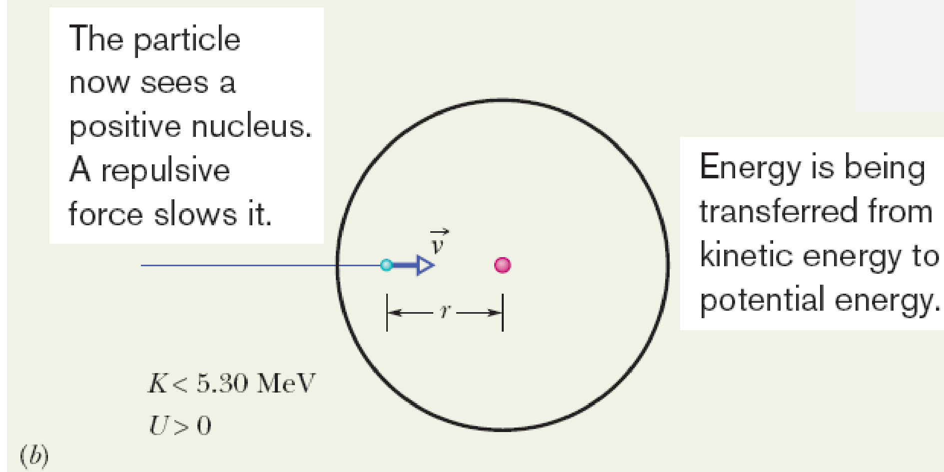
Fig. 42-4



Calculations: The alpha particle has a charge of $+2e$ because it contains two protons. The target nucleus has a charge of $q_{Au} = +79e$ because it contains 79 protons. However, that nuclear charge is surrounded by an electron "cloud" with a charge of $q_e = -79e$, and thus the alpha particle initially "sees" a neutral atom with a net charge of $q_{atom} = 0$. The electric force on the particle and the initial electric potential energy of the particle–atom system is $U_i = 0$.

Once the alpha particle enters the atom, we say that it passes through the electron cloud surrounding the nucleus. That cloud then acts as a closed conducting spherical shell and, by Gauss' law, has no effect on the (now internal) charged alpha particle. Then the alpha particle "sees" only the nuclear charge q_{Au}. Because q_α and q_{Au} are both positively charged, a repulsive electric force acts on the alpha particle, slowing it, and the particle–atom system has a potential energy

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{Au}}{r}$$

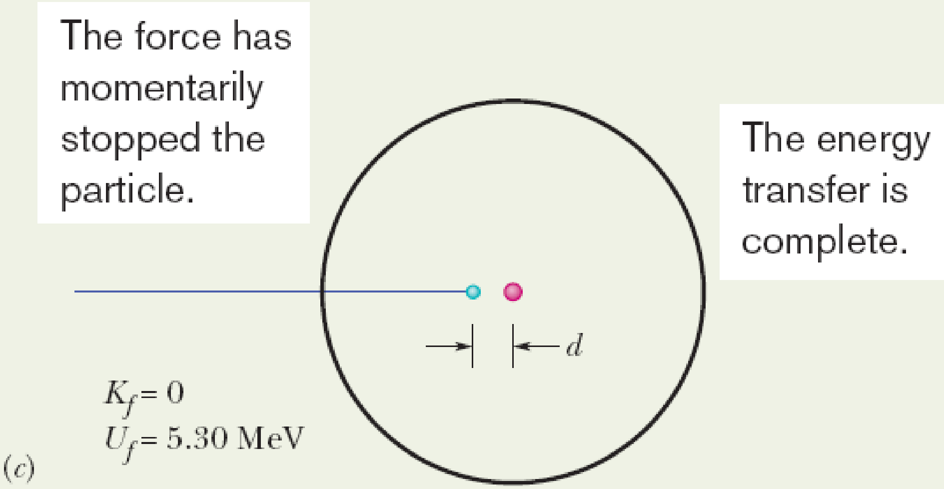


Example, Rutherford scattering of an alpha particle by a gold nucleus:

As the repulsive force slows the alpha particle, energy is transferred from kinetic energy to electric potential energy. The transfer is complete when the alpha particle momentarily stops at the distance of closest approach d to the target nucleus (Fig. 42-4c). Just then the kinetic energy is $K_f = 0$ and the particle–atom system has the electric potential energy

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\mathrm{Au}}}{d}.$$

Fig. 42-4



To find d, we conserve the total mechanical energy between the initial state i and this later state f, writing

$$K_i + U_i = K_f + U_f$$

and

$$K_i + 0 = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_\alpha q_{\mathrm{Au}}}{d}.$$

$$d = \frac{(2e)(79e)}{4\pi\varepsilon_0 K_\alpha}$$

$$= \frac{(2\times 79)(1.60\times 10^{-19}\ \mathrm{C})^2}{4\pi\varepsilon_0(5.30\ \mathrm{MeV})(1.60\times 10^{-13}\ \mathrm{J/MeV})}$$

$$= 4.29\times 10^{-14}\ \mathrm{m}. \qquad \text{(Answer)}$$

42.2: Some Nuclear Properties:

Nuclei are made up of protons and neutrons. The number of protons in a nucleus is called the **atomic number of the nucleus**, and is represented by the symbol ***Z;*** the number of neutrons is the **neutron number**, and is represented by the symbol ***N.***

The total number of neutrons and protons in a nucleus is called its **mass number *A.*** Neutrons and protons, when considered collectively, are called **nucleons.**

Table 42-1

Some Properties of Selected Nuclides

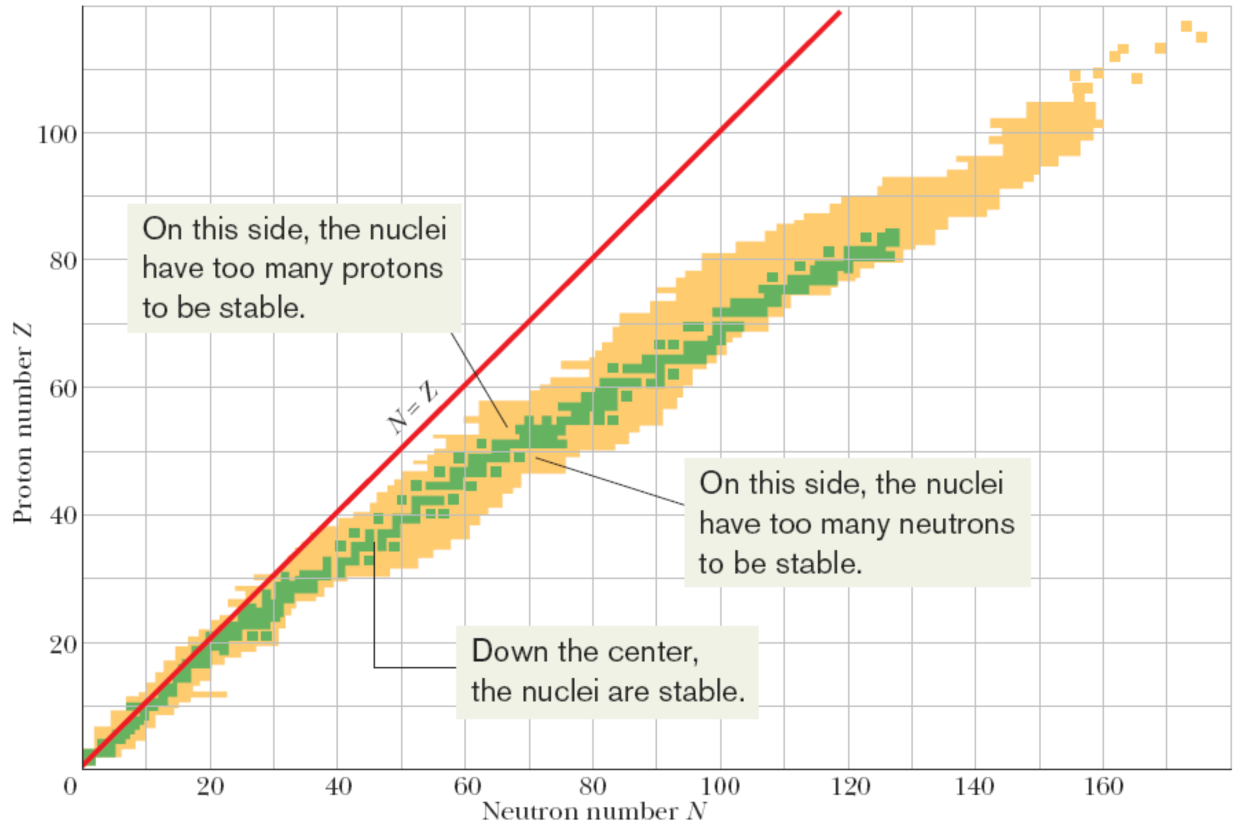
Nuclide	Z	N	A	Stability[a]	Mass[b] (u)	Spin[c]	Binding Energy (MeV/nucleon)
^{1}H	1	0	1	99.985%	1.007 825	$\frac{1}{2}$	—
^{7}Li	3	4	7	92.5%	7.016 004	$\frac{3}{2}$	5.60
^{31}P	15	16	31	100%	30.973 762	$\frac{1}{2}$	8.48
^{84}Kr	36	48	84	57.0%	83.911 507	0	8.72
^{120}Sn	50	70	120	32.4%	119.902 197	0	8.51
^{157}Gd	64	93	157	15.7%	156.923 957	$\frac{3}{2}$	8.21
^{197}Au	79	118	197	100%	196.966 552	$\frac{3}{2}$	7.91
^{227}Ac	89	138	227	21.8 y	227.027 747	$\frac{3}{2}$	7.65
^{239}Pu	94	145	239	24 100 y	239.052 157	$\frac{1}{2}$	7.56

[a]For stable nuclides, the **isotopic abundance** is given; this is the fraction of atoms of this type found in a typical sample of the element. For radioactive nuclides, the half-life is given.

[b]Following standard practice, the reported mass is that of the neutral atom, not that of the bare nucleus.

[c]Spin angular momentum in units of $\hbar$.

Fig. 42-5 A plot of the known nuclides. The green shading identifies the band of stable nuclides, the beige shading the radionuclides. Low-mass, stable nuclides have essentially equal numbers of neutrons and protons, but more massive nuclides have an increasing excess of neutrons. The figure shows that there are no stable nuclides with $Z > 83$ (bismuth).



Nuclides with the same atomic number Z but different neutron numbers N are called **isotopes** of one another. The element gold has 32 isotopes, ranging from ^{173}Au to ^{204}Au. Only one of them (^{197}Au) is stable; the remaining 31 are radioactive. Such **radionuclides** undergo decay (or disintegration) by emitting a particle and thereby transforming to a different nuclide.

Some Nuclear Properties: Organizing the Nuclides

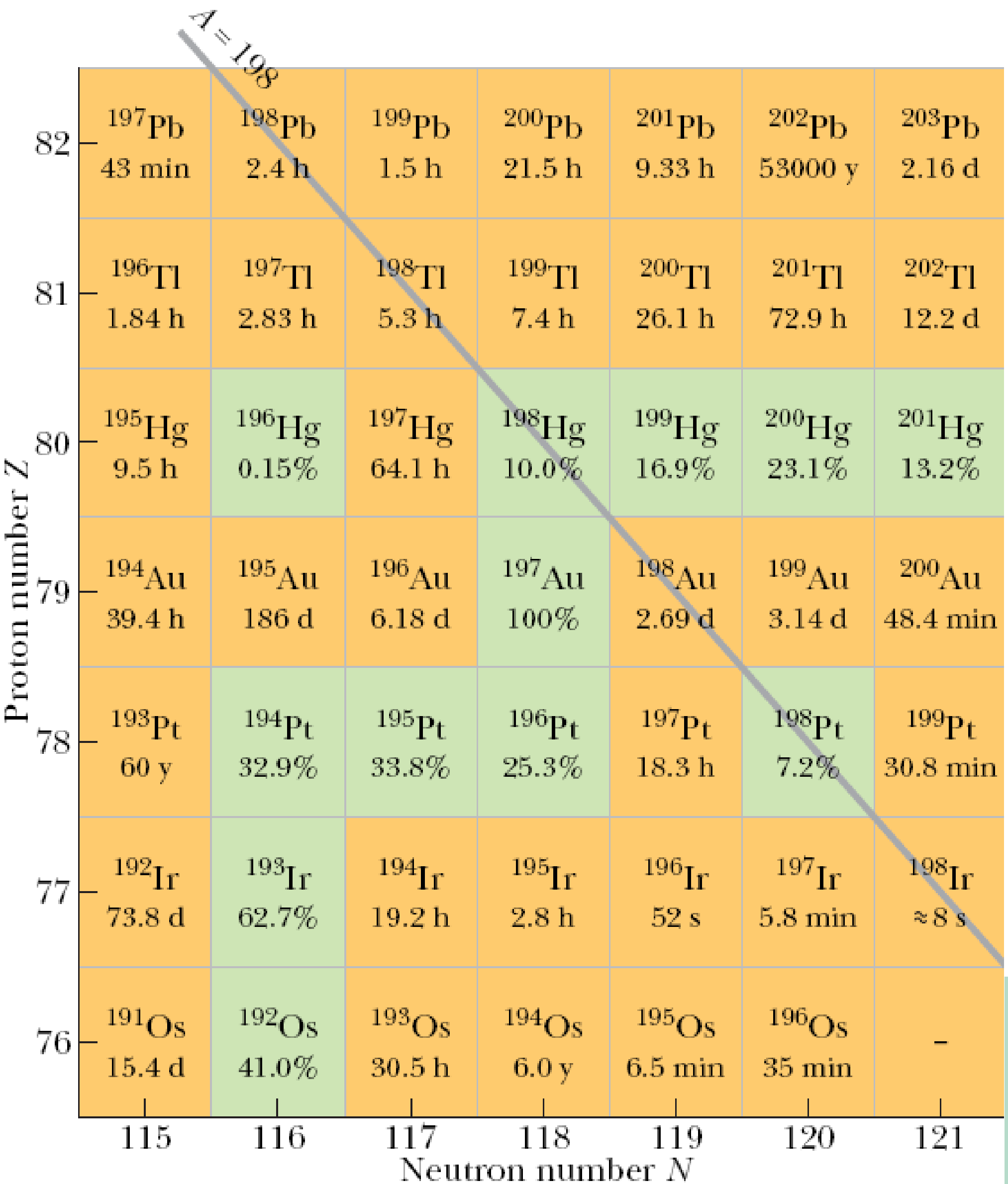


Fig. 42-6 An enlarged and detailed section of the nuclidic chart of Fig. 42-5, centered on ^{197}Au. Green squares represent stable nuclides, for which relative isotopic abundances are given. Beige squares represent radionuclides, for which half-lives are given. Isobaric lines of constant mass number A slope as shown by the example line for $A = 198$.

Some Nuclear Properties: Nuclear Radii

- The nucleus, like the atom, is not a solid object with a well-defined surface.
- Although most nuclides are spherical, some are notably ellipsoidal.
- Electron-scattering experiments (as well as experiments of other kinds) allow us to assign to each nuclide an effective radius given by

$$\boxed{r = r_0 A^{1/3},}$$

in which A is the mass number and $r_0 = 1.2\ fm.$

(1 femtometer =1 fermi =1 fm=10^{-15} m.

The above equation does not apply to *halo nuclides,* which are neutron-rich Nuclides, first produced in laboratories in the 1980s. These nuclides are larger than predicted by this equation, because some of the neutrons form a *halo* around a spherical core of the protons and the rest of the neutrons. Lithium isotopes are examples of this.

Some Nuclear Properties: Atomic Masses

•Atomic masses are often reported in *atomic mass units,* a system in which the atomic mass of neutral ^{12}C is defined to be exactly 12 u.

1 u =1.660 538 86 x10^{-27} kg.

•The mass number A of a nuclide gives such an approximate mass in atomic mass units. For example, the approximate mass of both the nucleus and the neutral atom for ^{197}Au is 197 u, which is close to the actual atomic mass of 196.966 552 u.

•If the total mass of the participants in a nuclear reaction changes by an amount Δm, there is an energy release or absorption given by $Q=mc^2$.

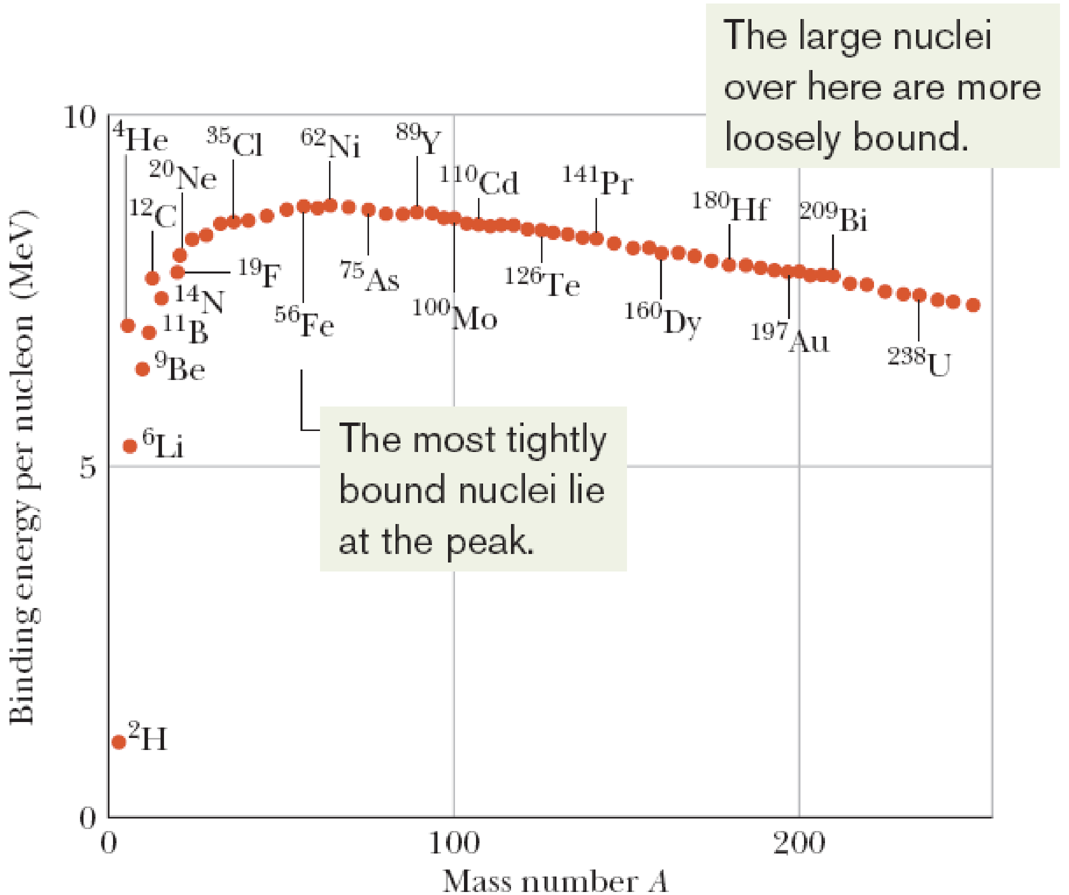
•The atom's *mass excess, Δ,* is defined as $\Delta = M - A$

Here, M is the actual mass of the atom in atomic units, and A is the mass number for that atom's nucleus.

Some Nuclear Properties: Nuclear Binding Energies

$$\Delta E_{be} = \Sigma(mc^2) - Mc^2 \qquad \text{(binding energy).}$$

Fig. 42-7 The binding energy per nucleon for some representative nuclides. The nickel nuclide ^{62}Ni has the highest binding energy per nucleon (about 8.794 60 MeV/nucleon) of any known stable nuclide. Note that the alpha particle (^{4}He) has a higher binding energy per nucleon than its neighbors in the periodic table and thus is also particularly stable.



If the nucleus splits into two nuclei, the process is called ***fission***, and occurs naturally with large (high mass number *A*).

If a pair of nuclei were to combine to form a single nucleus, the process is called ***fusion***, and occurs naturally in stars.

Some Nuclear Properties: Nuclear Energy Levels

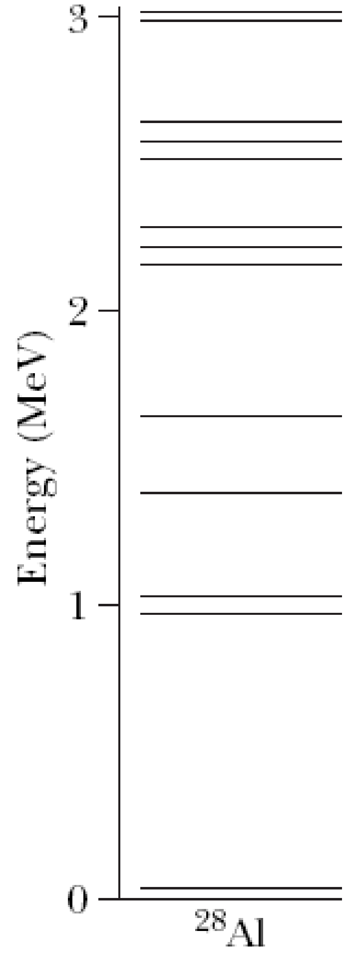


Fig. 42-8 Energy levels for the nuclide ^{28}Al, deduced from nuclear reaction experiments.

Example, Binding energy per nucleon:

What is the binding energy per nucleon for ^{120}Sn?

KEY IDEAS

1. We can find the binding energy per nucleon ΔE_{ben} if we first find the binding energy ΔE_{be} and then divide by the number of nucleons A in the nucleus, according to Eq. 42-8 ($\Delta E_{ben} = \Delta E_{be}/A$).

2. We can find ΔE_{be} by finding the difference between the mass energy Mc^2 of the nucleus and the total mass energy $\Sigma(mc^2)$ of the individual nucleons that make up the nucleus, according to Eq. 42-7 ($\Delta E_{be} = \Sigma(mc^2) - Mc^2$).

Calculations: From Table 42-1, we see that a ^{120}Sn nucleus consists of 50 protons ($Z = 50$) and 70 neutrons ($N = A - Z = 120 - 50 = 70$). Thus, we need to imagine a ^{120}Sn nucleus being separated into its 50 protons and 70 neutrons,

$$(^{120}\text{Sn nucleus}) \rightarrow 50\begin{pmatrix}\text{separate}\\ \text{protons}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\ \text{neutrons}\end{pmatrix}, \tag{42-9}$$

and then compute the resulting change in mass energy.

For that computation, we need the masses of a ^{120}Sn nucleus, a proton, and a neutron. However, because the mass of a neutral atom (nucleus *plus* electrons) is much easier to measure than the mass of a bare nucleus, calculations of binding energies are traditionally done with atomic masses. Thus, let's modify Eq. 42-9 so that it has a neutral ^{120}Sn atom on the left side. To do that, we include 50 electrons on the left side (to match the 50 protons in the ^{120}Sn nucleus). We must also add 50 electrons on the right side to balance Eq. 42-9. Those 50 electrons can be combined with the 50 protons, to form 50 neutral hydrogen atoms. We then have

$$(^{120}\text{Sn atom}) \rightarrow 50\begin{pmatrix}\text{separate}\\ \text{H atoms}\end{pmatrix} + 70\begin{pmatrix}\text{separate}\\ \text{neutrons}\end{pmatrix}. \tag{42-10}$$

From the mass column of Table 42-1, the mass M_{Sn} of a ^{120}Sn atom is 119.902 197 u and the mass m_H of a hydrogen atom is 1.007 825 u; the mass m_n of a neutron is 1.008 665 u. Thus, Eq. 42-7 yields

$$\begin{aligned}\Delta E_{be} &= \Sigma(mc^2) - Mc^2\\ &= 50(m_H c^2) + 70(m_n c^2) - M_{Sn}c^2\\ &= 50(1.007\,825\text{ u})c^2 + 70(1.008\,665\text{ u})c^2\\ &\quad - (119.902\,197\text{ u})c^2\\ &= (1.095\,603\text{ u})c^2\\ &= (1.095\,603\text{ u})(931.494\,013\text{ MeV/u})\\ &= 1020.5\text{ MeV},\end{aligned}$$

where Eq. 42-5 ($c^2 = 931.494\,013$ MeV/u) provides an easy unit conversion. Note that using atomic masses instead of nuclear masses does not affect the result because the mass of the 50 electrons in the ^{120}Sn atom subtracts out from the mass of the electrons in the 50 hydrogen atoms.

Now Eq. 42-8 gives us the binding energy per nucleon as

$$\begin{aligned}\Delta E_{ben} &= \frac{\Delta E_{be}}{A} = \frac{1020.5\text{ MeV}}{120}\\ &= 8.50\text{ MeV/nucleon.} \qquad \text{(Answer)}\end{aligned}$$

Example, Density of nuclear matter:

We can think of all nuclides as made up of a neutron–proton mixture that we can call *nuclear matter.* What is the density of nuclear matter?

KEY IDEA

We can find the (average) density ρ of a nucleus by dividing its total mass by its volume.

Calculations: Let m represent the mass of a nucleon (either a proton or a neutron, because those particles have about the same mass). Then the mass of a nucleus containing A nucleons is Am. Next, we assume the nucleus is spherical with radius r. Then its volume is $\frac{4}{3}\pi r^3$, and we can write the density of the nucleus as

$$\rho = \frac{Am}{\frac{4}{3}\pi r^3}.$$

The radius r is given by Eq. 42-3 ($r = r_0 A^{1/3}$), where r_0 is 1.2 fm ($= 1.2 \times 10^{-15}$ m). Substituting for r then leads to

$$\rho = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{m}{\frac{4}{3}\pi r_0^3}.$$

Note that A has canceled out; thus, this equation for density ρ applies to any nucleus that can be treated as spherical with a radius given by Eq. 42-3. Using 1.67×10^{-27} kg for the mass m of a nucleon, we then have

$$\rho = \frac{1.67 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi(1.2 \times 10^{-15}\ \text{m})^3} \approx 2 \times 10^{17}\ \text{kg/m}^3. \qquad \text{(Answer)}$$

This is about 2×10^{14} times the density of water and is the density of neutron stars, which contain only neutrons.