Chapter 42

Nuclear Physics

42.3: Radioactive Decay:

There is absolutely no way to predict whether any given nucleus in a radioactive sample will be among the small number of nuclei that decay during the next second. All have the same chance.

If a sample contains *N* radioactive nuclei, then the rate *(=dN/dt)* at which nuclei will decay is proportional to *N:*

$$-\frac{dN}{dt} = \lambda N,$$

Here λ is the disintegration or decay constant.

Therefore,

$$\frac{dN}{N} = -\lambda\, dt, \Rightarrow \int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt, \Rightarrow \ln \frac{N}{N_0} = -\lambda t.$$

$$\Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow N = N_0 e^{-\lambda t} \qquad \text{(radioactive decay),}$$

Here, N_o is the number of radioactive nuclei at time t = 0.

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay per second.}$$

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq.}$$

The decay rate:

$$R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t}$$

Therefore,

$$\boxed{R = R_0 e^{-\lambda t} \qquad \text{(radioactive decay),}}$$

The half life-time ($T_{1/2}$) is the time at which both N and R have been reduced to one-half their initial values

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}.$$

Therefore,

$$T_{1/2} = \frac{\ln 2}{\lambda}.$$

And,

$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.}$$

Here τ is the ***mean life time,*** which is the time at which both N and R have been reduced to e^{-1} of their initial values.

Example, Finding the disintegration constant and the half life-time:

The table that follows shows some measurements of the decay rate of a sample of ^{128}I, a radionuclide often used medically as a tracer to measure the rate at which iodine is absorbed by the thyroid gland.

Time (min)	R (counts/s)	Time (min)	R (counts/s)
4	392.2	132	10.9
36	161.4	164	4.56
68	65.5	196	1.86
100	26.8	218	1.00

Find the disintegration constant λ and the half-life $T_{1/2}$ for this radionuclide.

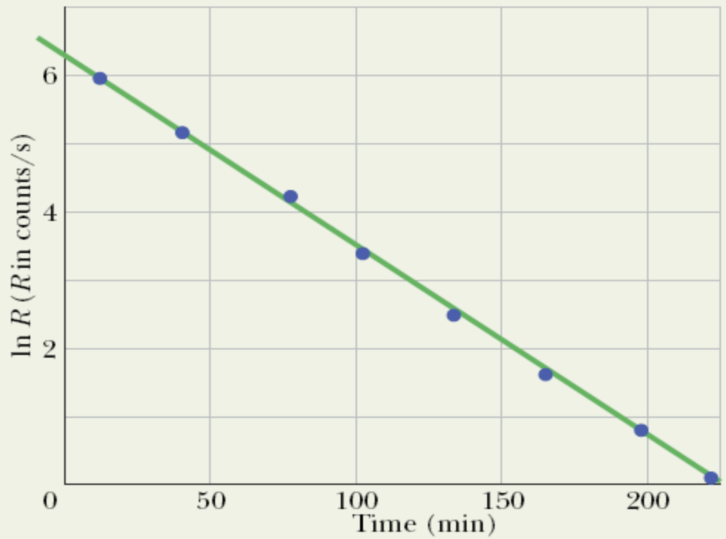


Fig. 42-9 A semilogarithmic plot of the decay of a sample of ^{128}I, based on the data in the table.

KEY IDEAS

The disintegration constant λ determines the exponential rate at which the decay rate R decreases with time t (as indicated by Eq. 42-16, $R = R_0 e^{-\lambda t}$). Therefore, we should be able to determine λ by plotting the measurements of R against the measurement times t. However, obtaining λ from a plot of R versus t is difficult because R decreases exponentially with t, according to Eq. 42-16. A neat solution is to transform Eq. 42-16 into a linear function of t, so that we can easily find λ. To do so, we take the natural logarithms of both sides of Eq. 42-16.

Calculations: We obtain

$$\ln R = \ln(R_0 e^{-\lambda t}) = \ln R_0 + \ln(e^{-\lambda t}) = \ln R_0 - \lambda t. \qquad (42\text{-}19)$$

Because Eq. 42-19 is of the form $y = b + mx$, with b and m constants, it is a linear equation giving the quantity $\ln R$ as a function of t. Thus, if we plot $\ln R$ (instead of R) versus t, we should get a straight line. Further, the slope of the line should be equal to $-\lambda$.

Figure 42-9 shows a plot of $\ln R$ versus time t for the given measurements. The slope of the straight line that fits through the plotted points is

$$\text{slope} = \frac{0 - 6.2}{225\ \text{min} - 0} = -0.0276\ \text{min}^{-1}.$$

Thus, $$-\lambda = -0.0276\ \text{min}^{-1}$$

or $$\lambda = 0.0276\ \text{min}^{-1} \approx 1.7\ \text{h}^{-1}. \qquad \text{(Answer)}$$

The time for the decay rate R to decrease by 1/2 is related to the disintegration constant λ via Eq. 42-18 ($T_{1/2} = (\ln 2)/\lambda$). From that equation, we find

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.0276\ \text{min}^{-1}} \approx 25\ \text{min}. \qquad \text{(Answer)}$$

Example, Finding the half life from the activity and the mass:

A 2.71 g sample of KCl from the chemistry stockroom is found to be radioactive, and it is decaying at a constant rate of 44.90 Bq. The decays are traced to the element potassium and in particular to the isotope ^{40}K, which constitutes 0.0117% of normal potassium. Calculate the half-life of this nuclide.

KEY IDEAS

1. Because the activity R of the sample is apparently constant, we cannot find the half-life $T_{1/2}$ by plotting ln R versus time t as in the preceding sample problem. (We would just get a horizontal plot.) However, we can use the following ideas.
2. We can relate the half-life $T_{1/2}$ to the disintegration constant λ via Eq. 42-18 ($T_{1/2} = (\ln 2)/\lambda$).
3. We can then relate λ to the given activity R of 44.90 Bq by means of Eq. 42-17 ($R = \lambda N$), where N is the number of ^{40}K nuclei (and thus atoms) in the sample.

Calculations: Combining Eqs. 42-18 and 42-17 yields

$$T_{1/2} = \frac{N \ln 2}{R}. \tag{42-20}$$

We know that N in this equation is 0.0117% of the total number N_K of potassium atoms in the sample. We also know that N_K must equal the number N_{KCl} of molecules in the sample. We can obtain N_{KCl} from the molar mass M_{KCl} of KCl (the mass of one mole of KCl) and the given mass M_{sam} of the sample by combining Eqs. 19-2 ($n = N/N_A$) and 19-3 ($n = M_{sam}/M$) to write

$$N_{KCl} = \begin{pmatrix} \text{number of moles} \\ \text{in sample} \end{pmatrix} N_A = \frac{M_{sam}}{M_{KCl}} N_A, \tag{42-21}$$

where N_A is Avogadro's number (6.02×10^{23} mol^{-1}). From Appendix F, we see that the molar mass of potassium is 39.102 g/mol and the molar mass of chlorine is 35.453 g/mol; thus, the molar mass of KCl is 74.555 g/mol. Equation 42-21 then gives us

$$N_{KCl} = \frac{(2.71\text{ g})(6.02 \times 10^{23}\text{ mol}^{-1})}{74.555\text{ g/mol}} = 2.188 \times 10^{22}$$

as the number of KCl molecules in the sample. Thus, the total number N_K of potassium atoms is also 2.188×10^{22}, and the number of ^{40}K in the sample must be

$$\begin{aligned} N = (0.000\,117)N_K &= (0.000\,117)(2.188 \times 10^{22}) \\ &= 2.560 \times 10^{18}. \end{aligned}$$

Substituting this value for N and the given activity of 44.90 Bq ($= 44.90$ s^{-1}) for R into Eq. 42-20 leads to

$$\begin{aligned} T_{1/2} &= \frac{(2.560 \times 10^{18}) \ln 2}{44.90\text{ s}^{-1}} \\ &= 3.95 \times 10^{16}\text{ s} = 1.25 \times 10^{9}\text{ y}. \qquad \text{(Answer)} \end{aligned}$$

This half-life of ^{40}K turns out to have the same order of magnitude as the age of the universe. Thus, the activity of ^{40}K in the stockroom sample decreases *very* slowly, too slowly for us to detect during a few days of observation or even an entire lifetime. A portion of the potassium in our bodies consists of this radioisotope, which means that we are all slightly radioactive.

42.4: Alpha Decay:

When a nucleus undergoes ***alpha decay,*** it transforms to a different nuclide by emitting an alpha particle (a helium nucleus, ^{4}He). For example, when uranium ^{238}U undergoes alpha decay, it transforms to thorium ^{234}Th: $^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}.$ The ***disintegration energy,*** *Q,* for the decay above is 4.25.

The potential energy shown in the figure below is a combination of the potential energy associated with the (attractive) strong nuclear force that acts in the nuclear interior and a Coulomb potential associated with the (repulsive) electric force that acts between the two particles (^{234}Th and ^{4}He) before and after the decay has occurred.

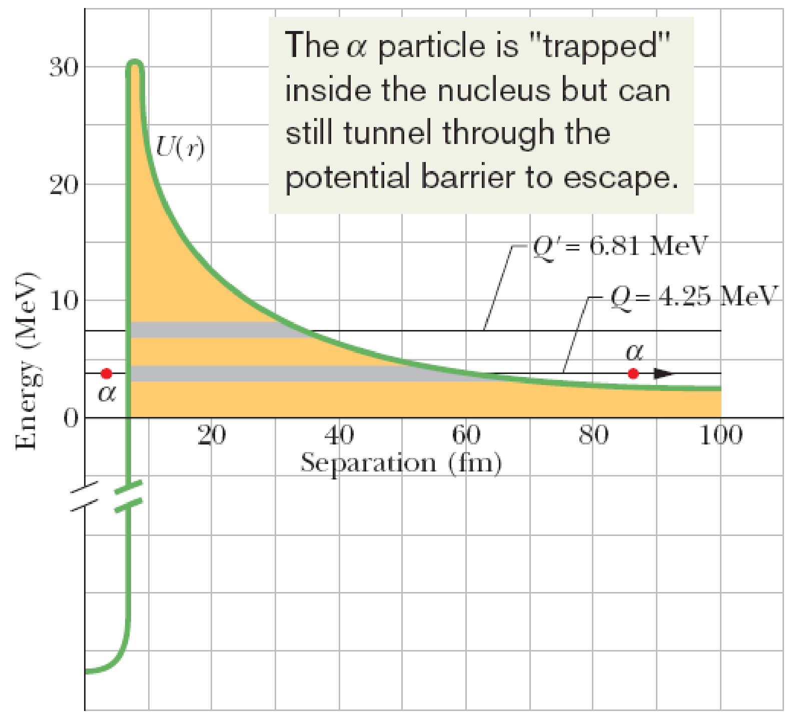


Fig. 42-10 A potential energy function for the emission of an alpha particle by ^{238}U. The horizontal black line marked $Q = 4.25\ MeV$ shows the disintegration energy for the process. The thick gray portion of this line represents separations r that are classically forbidden to the alpha particle. The alpha particle is represented by a dot, both inside this potential energy barrier (at the left) and outside it (at the right), after the particle has tunneled through. The horizontal black line marked $Q = 6.81\ MeV$ shows the disintegration energy for the alpha decay of ^{228}U. (Both isotopes have the same potential energy function because they have the same nuclear charge.)

Example, Q value of an alpha decay using masses:

We are given the following atomic masses:

^{238}U	238.050 79 u	^{4}He	4.002 60 u
^{234}Th	234.043 63 u	^{1}H	1.007 83 u
^{237}Pa	237.051 21 u		

Here Pa is the symbol for the element protactinium ($Z = 91$).

(a) Calculate the energy released during the alpha decay of ^{238}U. The decay process is

$$^{238}U \to {}^{234}Th + {}^{4}He.$$

Note, incidentally, how nuclear charge is conserved in this equation: The atomic numbers of thorium (90) and helium (2) add up to the atomic number of uranium (92). The number of nucleons is also conserved: $238 = 234 + 4$.

KEY IDEA

The energy released in the decay is the disintegration energy Q, which we can calculate from the change in mass ΔM due to the ^{238}U decay.

Calculation: To do this, we use Eq. 37-50,

$$Q = M_i c^2 - M_f c^2, \qquad (42\text{-}23)$$

where the initial mass M_i is that of ^{238}U and the final mass M_f is the sum of the ^{234}Th and ^{4}He masses. Using the atomic masses given in the problem statement, Eq. 42-23 becomes

$$\begin{aligned} Q &= (238.050\,79\text{ u})c^2 - (234.043\,63\text{ u} + 4.002\,60\text{ u})c^2 \\ &= (0.004\,56\text{ u})c^2 = (0.004\,56\text{ u})(931.494\,013\text{ MeV/u}) \\ &= 4.25\text{ MeV}. \qquad \text{(Answer)} \end{aligned}$$

Note that using atomic masses instead of nuclear masses does not affect the result because the total mass of the electrons in the products subtracts out from the mass of the nucleons + electrons in the original ^{238}U.

(b) Show that ^{238}U cannot spontaneously emit a proton; that is, protons do not leak out of the nucleus in spite of the proton–proton repulsion within the nucleus.

Solution: If this happened, the decay process would be

$$^{238}U \to {}^{237}Pa + {}^{1}H.$$

(You should verify that both nuclear charge and the number of nucleons are conserved in this process.) Using the same Key Idea as in part (a) and proceeding as we did there, we would find that the mass of the two decay products

$$237.051\,21\text{ u} + 1.007\,83\text{ u}$$

would *exceed* the mass of ^{238}U by $\Delta m = 0.008\,25$ u, with disintegration energy

$$Q = -7.68\text{ MeV}.$$

The minus sign indicates that we must *add* 7.68 MeV to a ^{238}U nucleus before it will emit a proton; it will certainly not do so spontaneously.

42.5: Beta Decay:

A nucleus that decays spontaneously by emitting an electron or a positron (a positively charged particle with the mass of an electron) is said to undergo ***beta decay.*** Like alpha decay, this is a spontaneous process, with a definite disintegration energy and half-life.

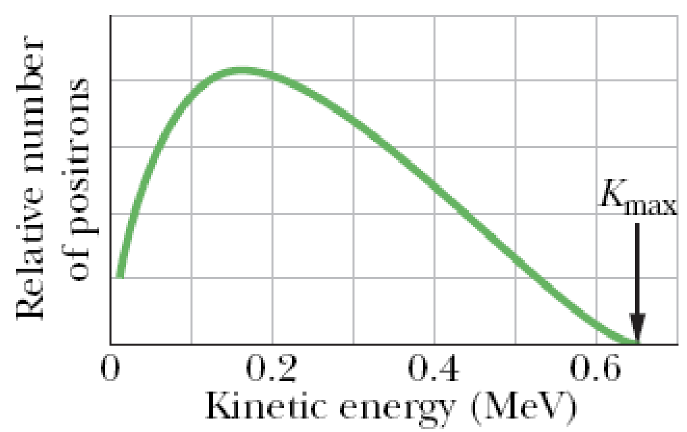
Examples: $^{32}\mathrm{P} \rightarrow {}^{32}\mathrm{S} + e^- + \nu \qquad (T_{1/2} = 14.3\ \mathrm{d}).$ (β^- decay)

$$^{64}\mathrm{Cu} \rightarrow {}^{64}\mathrm{Ni} + e^+ + \nu \qquad (T_{1/2} = 12.7\ \mathrm{h}).$$ (β^+ decay)

Here, ν is a neutrino, a neutral particle which has a very small mass, that is emitted from the nucleus along with the electron or positron during the decay process.

In a beta decay the energy of the emitted electrons or positrons may range from zero up to a certain maximum K_{max}, since, unlike the alpha decay, the Q energy is shared by two components.

Fig. 42-11 The distribution of the kinetic energies of positrons emitted in the beta decay of ^{64}Cu. The maximum kinetic energy of the distribution (K_{max}) is 0.653 MeV. In all ^{64}Cu decay events, this energy is shared between the positron and the neutrino, in varying proportions. The *most probable* energy for an emitted positron is about 0.15 MeV.



42.5: Beta Decay: The Neutrino

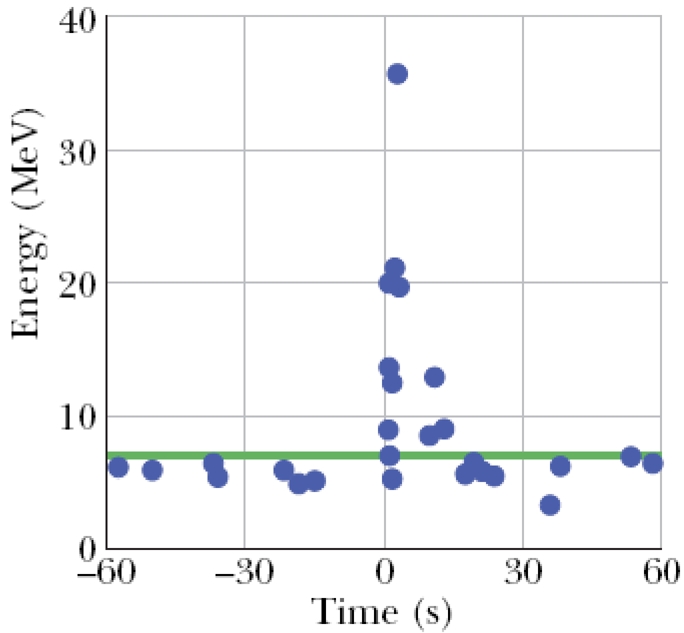


Fig. 42-12 A burst of neutrinos from the supernova SN 1987A, which occurred at (relative) time 0, stands out from the usual *background* of neutrinos. (For neutrinos, 10 is a "burst.") The particles were detected by an elaborate detector housed deep in a mine in Japan. The supernova was visible only in the Southern Hemisphere; so the neutrinos had to penetrate Earth (a trifling barrier for them) to reach the detector.

Wolfgang Pauli first suggested the existence of neutrinos in 1930.

Billions of them pass through our bodies every second, leaving no trace.

In spite of their elusive character, neutrinos have been detected in the laboratory. In spite of their elusive character, neutrinos have been detected in the laboratory.

Beta Decay: Radioactivity and the Nuclidic Chart

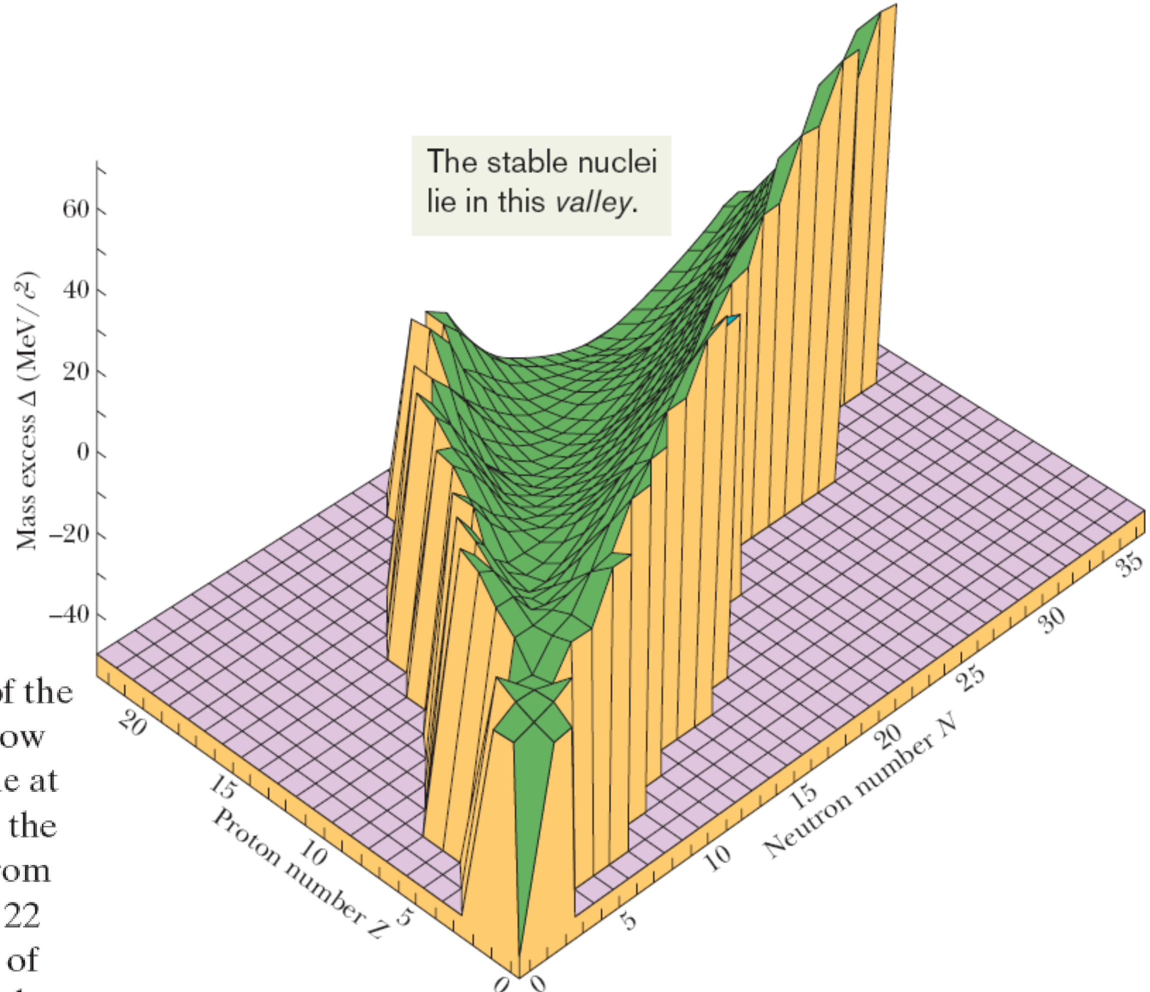


Fig. 42-13 A portion of the valley of the nuclides, showing only the nuclides of low mass. Deuterium, tritium, and helium lie at the near end of the plot, with helium at the high point. The valley stretches away from us, with the plot stopping at about $Z = 22$ and $N = 35$. Nuclides with large values of A, which would be plotted much beyond the valley, can decay into the valley by repeated alpha emissions and by fission (splitting of a nuclide).

Example, Q value of a beta decay using masses:

Calculate the disintegration energy Q for the beta decay of ^{32}P, as described by Eq. 42-24. The needed atomic masses are 31.973 91 u for ^{32}P and 31.972 07 u for ^{32}S.

KEY IDEA

The disintegration energy Q for the beta decay is the amount by which the mass energy is changed by the decay.

Calculations: Q is given by Eq. 37-50 ($Q = -\Delta M\ c^2$). However, we must be careful to distinguish between nuclear masses (which we do not know) and atomic masses (which we do know). Let the boldface symbols $\mathbf{m}_P$ and $\mathbf{m}_S$ represent the nuclear masses of ^{32}P and ^{32}S, and let the italic symbols m_P and m_S represent their atomic masses. Then we can write the change in mass for the decay of Eq. 42-24 as

$$\Delta m = (\mathbf{m}_S + m_e) - \mathbf{m}_P,$$

in which m_e is the mass of the electron. If we add and subtract $15m_e$ on the right side of this equation, we obtain

$$\Delta m = (\mathbf{m}_S + 16m_e) - (\mathbf{m}_P + 15m_e).$$

The quantities in parentheses are the atomic masses of ^{32}S and ^{32}P; so

$$\Delta m = m_S - m_P.$$

We thus see that if we subtract only the atomic masses, the mass of the emitted electron is automatically taken into account. (This procedure will not work for positron emission.)

The disintegration energy for the ^{32}P decay is then

$$\begin{aligned} Q &= -\Delta m\, c^2 \\ &= -(31.972\,07\text{ u} - 31.973\,91\text{ u})(931.494\,013\text{ MeV/u}) \\ &= 1.71\text{ MeV}. \qquad \text{(Answer)} \end{aligned}$$

Experimentally, this calculated quantity proves to be equal to K_{max}, the maximum energy the emitted electrons can have. Although 1.71 MeV is released every time a ^{32}P nucleus decays, in essentially every case the electron carries away less energy than this. The neutrino gets all the rest, carrying it stealthily out of the laboratory.