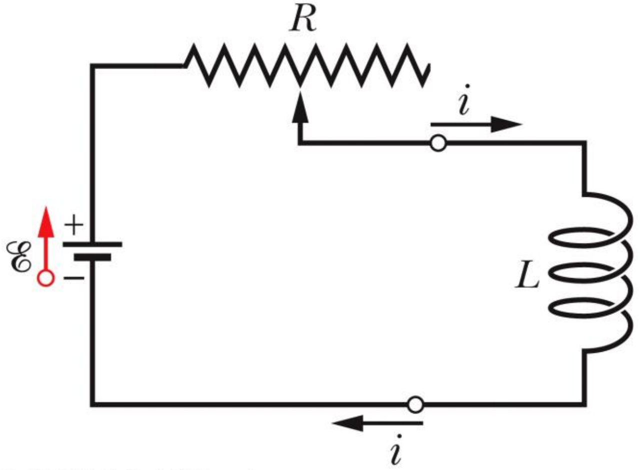
Induction and Inductance

Chapter 30

30-5 Self-Induction

When current flow in an inductor a process called self induction happen. This process (see Figure), and the emf that appears is called a self-induced emf. It obeys Faraday's law of induction just as other induced emfs do. For any inductor of inductance L and number of turns N and current i,



$$N\Phi_B = Li.$$

30-5 Self-Induction

Faraday's law tells us that

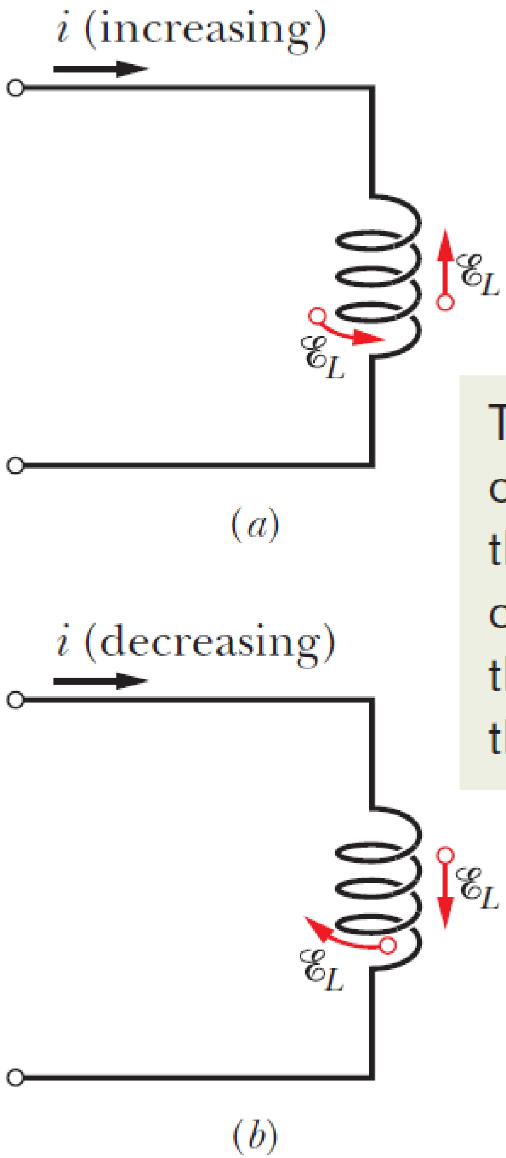
$$\mathcal{E}_L = -\frac{d\left(N\Phi_B\right)}{dt}.$$

By combining these equations, we can write

$$\mathcal{E}_L = -L\frac{di}{dt} \qquad \left(\text{self-induced emf}\right).$$

An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing.

Note: Thus, in any inductor (such as a coil, a solenoid, or a toroid) a self-induced emf appears whenever the current changes with time. The magnitude of the current has no influence on the magnitude of the induced emf; only the rate of change of the current counts.

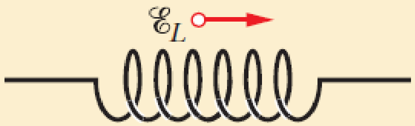


The changing current changes the flux, which creates an emf that opposes the change.

Figure 30-14 (*a*) The current i is increasing, and the self-induced emf $\mathscr{E}_L$ appears along the coil in a direction such that it opposes the increase. The arrow representing $\mathscr{E}_L$ can be drawn along a turn of the coil or alongside the coil. Both are shown. (*b*) The current i is decreasing, and the self-induced emf appears in a direction such that it opposes the decrease.

Checkpoint 5

The figure shows an emf $\mathscr{E}_L$ induced in a coil. Which of the following can describe the current through the coil: (a) constant and rightward, (b) constant and leftward, (c) increasing and rightward, (d) decreasing and rightward, (e) increasing and leftward, (f) decreasing and leftward?

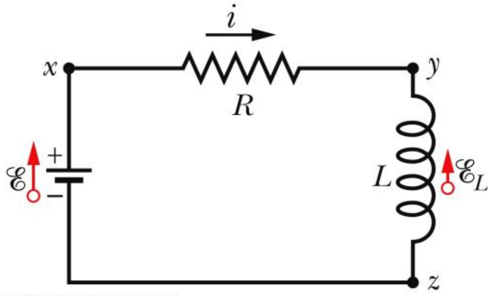


d and e

30-6 RL Circuits

If a constant emf $\mathcal{E}$ is introduced into a single-loop circuit containing a resistance R and an inductance L, the current rises to an equilibrium value of $\frac{\mathcal{E}}{R}$ according to

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{\frac{-t}{\tau_L}}\right)$$

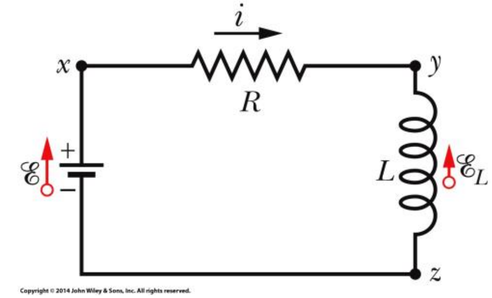


Here τ_L, the **inductive time constant**, is given by

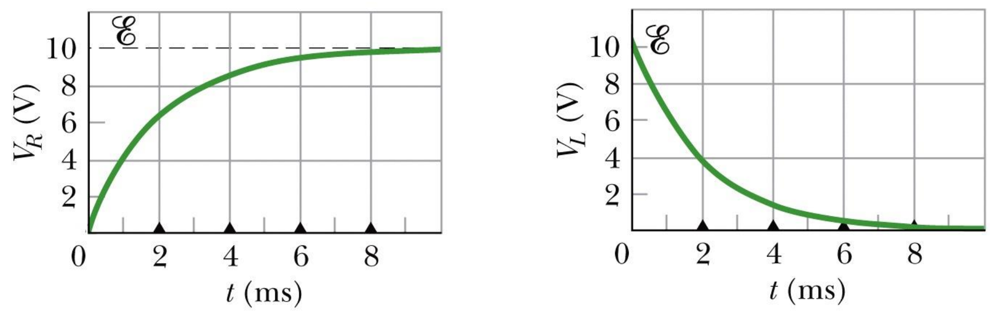
$$\tau_L = \frac{L}{R}$$

30-6 RL Circuits

Plot (a) and (b) shows how the potential differences V_R (= iR) across the resistor and $V_L\left(=L\frac{di}{dt}\right)$ across the inductor vary with time for particular values of $\mathcal{E}$, L, and R.



The resistor's potential difference turns on. The inductor's potential difference turns off.



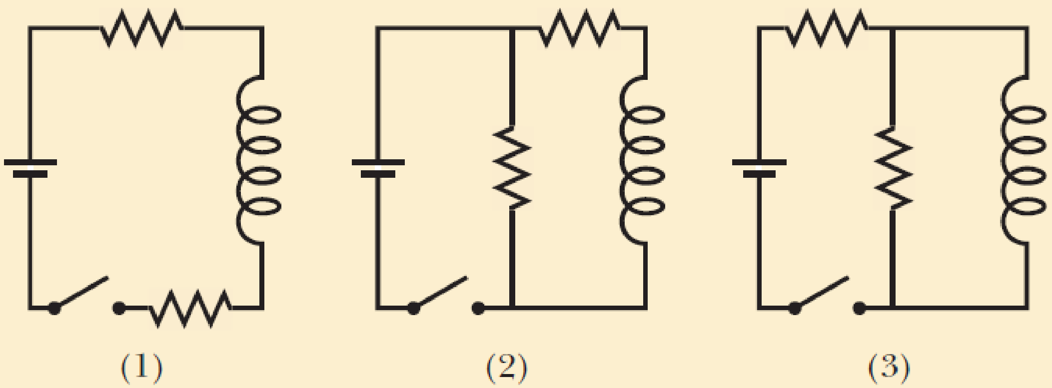
(*a*) (*b*)

When the source of constant e m f is removed and replaced by a conductor, the **current decays** from a value i_0 according to:

$$i = \frac{\mathcal{E}}{R} e^{\frac{-t}{\tau_L}} = i_0 e^{\frac{-t}{\tau_L}}$$

Checkpoint 6

The figure shows three circuits with identical batteries, inductors, and resistors. Rank the circuits according to the current through the battery (a) just after the switch is closed and (b) a long time later, greatest first. (If you have trouble here, work through the next sample problem and then try again.)

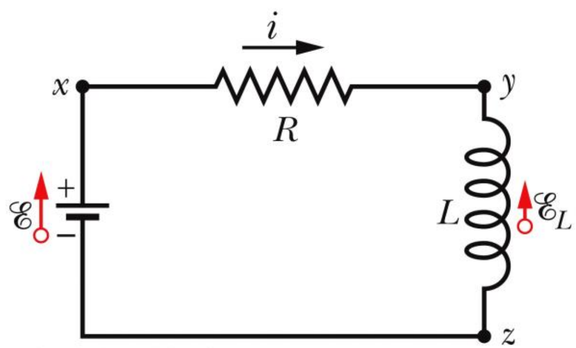


(a) 2, 3, 1 (zero); (b) 2, 3, 1

30-7 Energy Stored in a Magnetic Field

If an inductor *L* carries a current *i*, the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2} L i^2$$



An RL circuit.

Sample Problem 30.06 *RL* circuit, current during the transition

A solenoid has an inductance of 53 mH and a resistance of 0.37 Ω. If the solenoid is connected to a battery, how long will the current take to reach half its final equilibrium value? (This is a *real solenoid* because we are considering its small, but nonzero, internal resistance.)

KEY IDEA

We can mentally separate the solenoid into a resistance and an inductance that are wired in series with a battery, as in Fig. 30-16. Then application of the loop rule leads to Eq. 30-39, which has the solution of Eq. 30-41 for the current i in the circuit.

Calculations: According to that solution, current i increases exponentially from zero to its final equilibrium value of $\mathscr{E}/R$. Let t_0 be the time that current i takes to reach half its equilibrium value. Then Eq. 30-41 gives us

$$\frac{1}{2}\frac{\mathscr{E}}{R} = \frac{\mathscr{E}}{R}(1 - e^{-t_0/\tau_L}).$$

We solve for t_0 by canceling $\mathscr{E}/R$, isolating the exponential, and taking the natural logarithm of each side. We find

$$t_0 = \tau_L \ln 2 = \frac{L}{R}\ln 2 = \frac{53 \times 10^{-3}\ \text{H}}{0.37\ \Omega}\ln 2$$
$$= 0.10\ \text{s}. \qquad \text{(Answer)}$$

Sample Problem 30.07 Energy stored in a magnetic field

A coil has an inductance of 53 mH and a resistance of 0.35 Ω.

(a) If a 12 V emf is applied across the coil, how much energy is stored in the magnetic field after the current has built up to its equilibrium value?

KEY IDEA

The energy stored in the magnetic field of a coil at any time depends on the current through the coil at that time, according to Eq. 30-49 ($U_B = \frac{1}{2}Li^2$).

Calculations: Thus, to find the energy $U_{B\infty}$ stored at equilibrium, we must first find the equilibrium current. From Eq. 30-41, the equilibrium current is

$$i_\infty = \frac{\mathscr{E}}{R} = \frac{12\text{ V}}{0.35\ \Omega} = 34.3\text{ A}. \tag{30-51}$$

Then substitution yields

$$U_{B\infty} = \tfrac{1}{2}Li_\infty^2 = (\tfrac{1}{2})(53 \times 10^{-3}\text{ H})(34.3\text{ A})^2$$
$$= 31\text{ J}. \qquad \text{(Answer)}$$

(b) After how many time constants will half this equilibrium energy be stored in the magnetic field?

Calculations: Now we are being asked: At what time t will the relation

$$U_B = \tfrac{1}{2}U_{B\infty}$$

be satisfied? Using Eq. 30-49 twice allows us to rewrite this energy condition as

$$\tfrac{1}{2}Li^2 = (\tfrac{1}{2})\tfrac{1}{2}Li_\infty^2$$

or

$$i = \left(\frac{1}{\sqrt{2}}\right)i_\infty. \tag{30-52}$$

This equation tells us that, as the current increases from its initial value of 0 to its final value of i_∞, the magnetic field will have half its final stored energy when the current has increased to this value. In general, we know that i is given by Eq. 30-41, and here i_∞ (see Eq. 30-51) is $\mathscr{E}/R$; so Eq. 30-52 becomes

$$\frac{\mathscr{E}}{R}(1 - e^{-t/\tau_L}) = \frac{\mathscr{E}}{\sqrt{2}R}.$$

By canceling $\mathscr{E}/R$ and rearranging, we can write this as

$$e^{-t/\tau_L} = 1 - \frac{1}{\sqrt{2}} = 0.293,$$

which yields

$$\frac{t}{\tau_L} = -\ln 0.293 = 1.23$$

or

$$t \approx 1.2\tau_L. \qquad \text{(Answer)}$$

Thus, the energy stored in the magnetic field of the coil by the current will reach half its equilibrium value 1.2 time constants after the emf is applied.