Chapter 43

Energy From The Nucleus

43.1: Nuclear Fission: The Basic Process:

Table 43-1

Energy Released by 1 kg of Matter

Form of Matter	Process	Time[a]
Water	A 50 m waterfall	5 s
Coal	Burning	8 h
Enriched UO_2	Fission in a reactor	690 y
^{235}U	Complete fission	3×10^4 y
Hot deuterium gas	Complete fusion	3×10^4 y
Matter and antimatter	Complete annihilation	3×10^7 y

[a] This column shows the time interval for which the generated energy could power a 100 W lightbulb.

In both atomic and nuclear burning, the release of energy is accompanied by a decrease in mass, according to the equation $Q=-\Delta m\, c^2$. The central difference between burning uranium and burning coal is that, in the former case, a much larger fraction of the available mass (by a factor of a few million) is consumed.

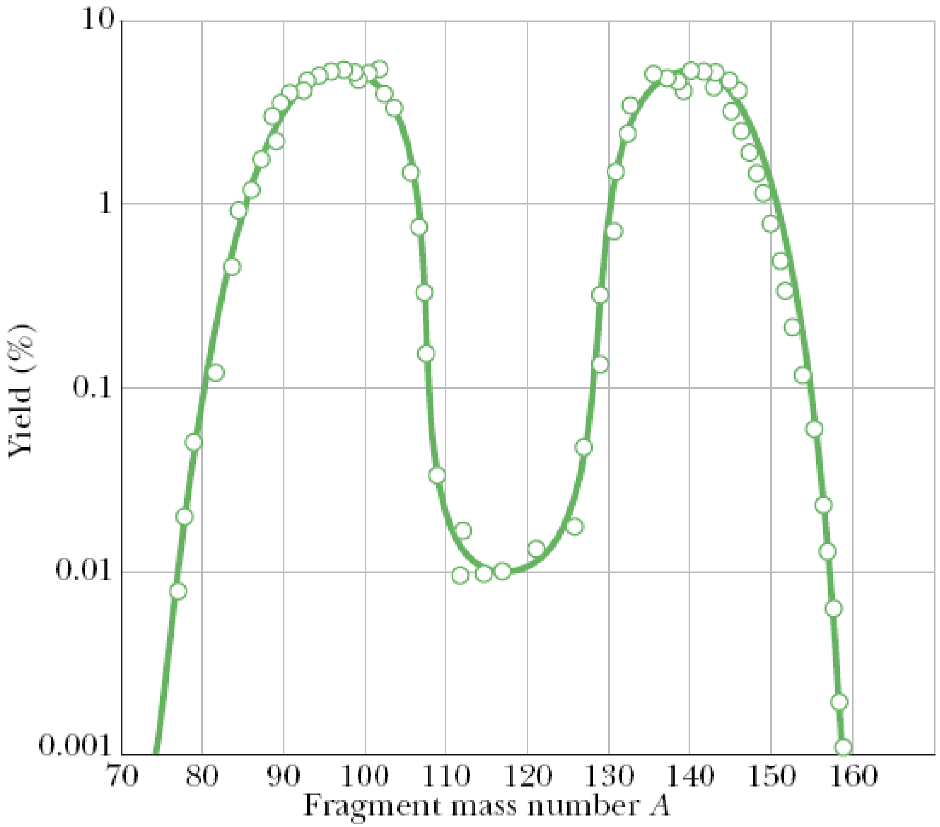


Fig. 43-1 The distribution by mass number of the fragments that are found when many fission events of ^{235}U are examined. Note that the vertical scale is logarithmic.

The most probable mass numbers, occurring in about 7% of the fission events, are centered around A ~95 and A~140.

An example: $^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{236}\mathrm{U} \rightarrow {}^{140}\mathrm{Xe} + {}^{94}\mathrm{Sr} + 2\mathrm{n}.$

Since the products are not stable, they undergo further fissions, such as:

$$^{140}\mathrm{Xe} \rightarrow {}^{140}\mathrm{Cs} \rightarrow {}^{140}\mathrm{Ba} \rightarrow {}^{140}\mathrm{La} \rightarrow {}^{140}\mathrm{Ce}$$

$T_{1/2}$	14 s	64 s	13 d	40 h	Stable
Z	54	55	56	57	58

$$^{94}\mathrm{Sr} \rightarrow {}^{94}\mathrm{Y} \rightarrow {}^{94}\mathrm{Zr}$$

$T_{1/2}$	75 s	19 min	Stable
Z	38	39	40

The energy released by the fission, *Q,* is:

$$Q = \begin{pmatrix} \text{total final} \\ \text{binding energy} \end{pmatrix} - \begin{pmatrix} \text{initial} \\ \text{binding energy} \end{pmatrix}.$$

$$Q = \begin{pmatrix} \text{final} \\ \Delta E_{\text{ben}} \end{pmatrix} \begin{pmatrix} \text{final number} \\ \text{of nucleons} \end{pmatrix} - \begin{pmatrix} \text{initial} \\ \Delta E_{\text{ben}} \end{pmatrix} \begin{pmatrix} \text{initial number} \\ \text{of nucleons} \end{pmatrix}.$$

For a high-mass nuclide (*A ~240),* the binding energy per nucleon is about 7.6 MeV/nucleon.
For middle-mass nuclides (*A~120),* it is about 8.5 MeV/nucleon.
Thus, the energy released by fission of a high-mass nuclide to two middle-mass nuclides is

$$Q = \left(8.5 \frac{\text{MeV}}{\text{nucleon}}\right)(2 \text{ nuclei})\left(120 \frac{\text{nucleons}}{\text{nucleus}}\right)$$

$$- \left(7.6 \frac{\text{MeV}}{\text{nucleon}}\right)(240 \text{ nucleons}) \approx 200 \text{ MeV}.$$

Checkpoint 1

A generic fission event is

$$^{235}U + n \rightarrow X + Y + 2n.$$

Which of the following pairs *cannot* represent X and Y: (a) ^{141}Xe and ^{93}Sr; (b) ^{139}Cs and ^{95}Rb; (c) ^{156}Nd and ^{79}Ge; (d) ^{121}In and ^{113}Ru?

c and d

Example, Q-value in a fission of U-235:

Find the disintegration energy Q for the fission event of Eq. 43-1, taking into account the decay of the fission fragments as displayed in Eqs. 43-2 and 43-3. Some needed atomic and particle masses are

^{235}U	235.0439 u	^{140}Ce	139.9054 u
n	1.008 66 u	^{94}Zr	93.9063 u

KEY IDEAS

(1) The disintegration energy Q is the energy transferred from mass energy to kinetic energy of the decay products. (2) $Q = -\Delta m\, c^2$, where Δm is the change in mass.

Calculations: Because we are to include the decay of the fission fragments, we combine Eqs. 43-1, 43-2, and 43-3 to write the overall transformation as

$$^{235}\mathrm{U} \rightarrow {}^{140}\mathrm{Ce} + {}^{94}\mathrm{Zr} + \mathrm{n}. \tag{43-7}$$

Only the single neutron appears here because the initiating neutron on the left side of Eq. 43-1 cancels one of the two neutrons on the right of that equation. The mass difference for the reaction of Eq. 43-7 is

$$\begin{aligned}\Delta m &= (139.9054\ \mathrm{u} + 93.9063\ \mathrm{u} + 1.008\,66\ \mathrm{u}) \\ &\quad - (235.0439\ \mathrm{u}) \\ &= -0.223\,54\ \mathrm{u},\end{aligned}$$

and the corresponding disintegration energy is

$$\begin{aligned}Q &= -\Delta m\, c^2 = -(-0.223\,54\ \mathrm{u})(931.494\,013\ \mathrm{MeV/u}) \\ &= 208\ \mathrm{MeV}, \qquad \text{(Answer)}\end{aligned}$$

which is in good agreement with our estimate of Eq. 43-6.

If the fission event takes place in a bulk solid, most of this disintegration energy, which first goes into kinetic energy of the decay products, appears eventually as an increase in the internal energy of that body, revealing itself as a rise in temperature. Five or six percent or so of the disintegration energy, however, is associated with neutrinos that are emitted during the beta decay of the primary fission fragments. This energy is carried out of the system and is lost.

43.3: A Model for Nuclear Fission:

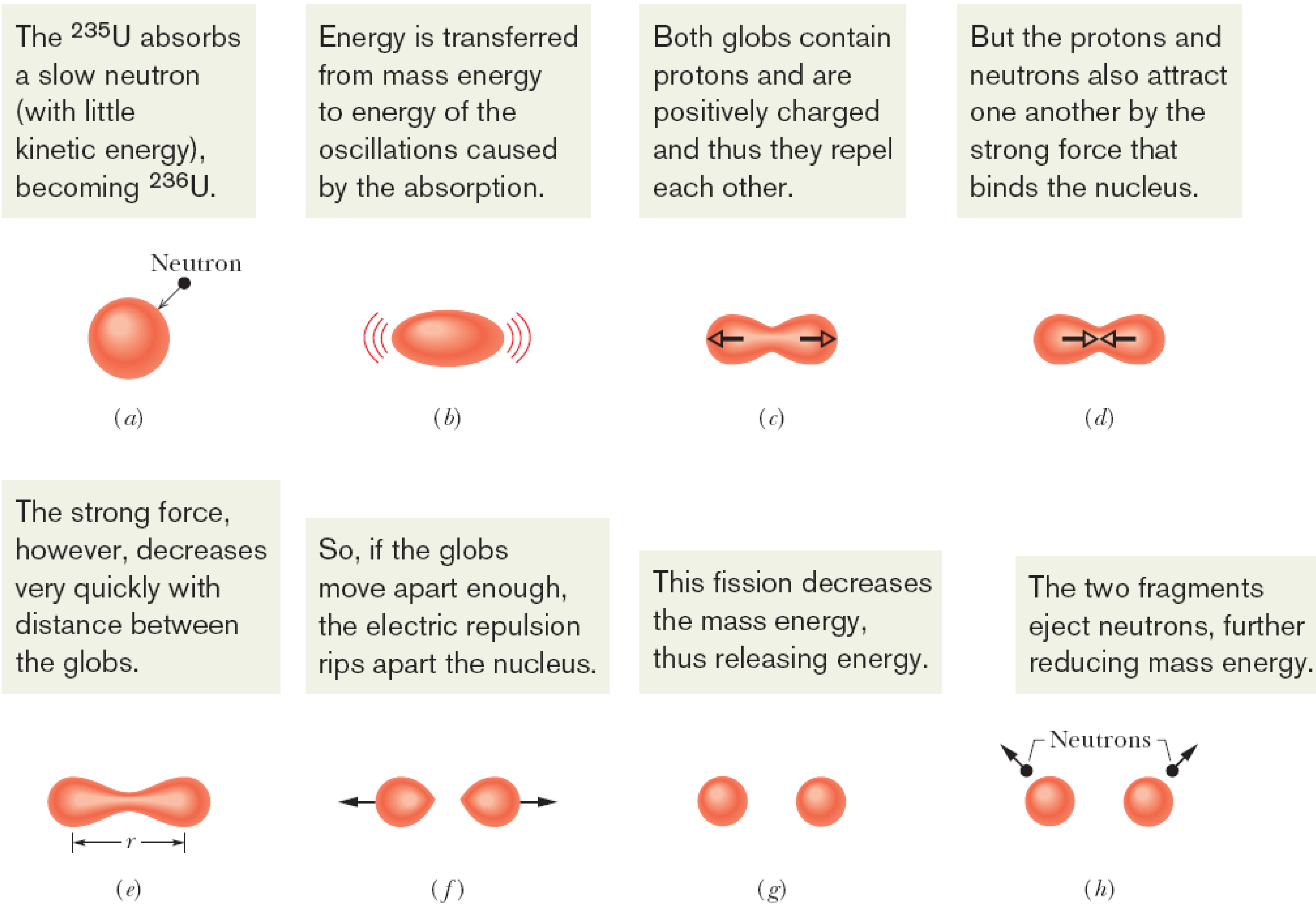
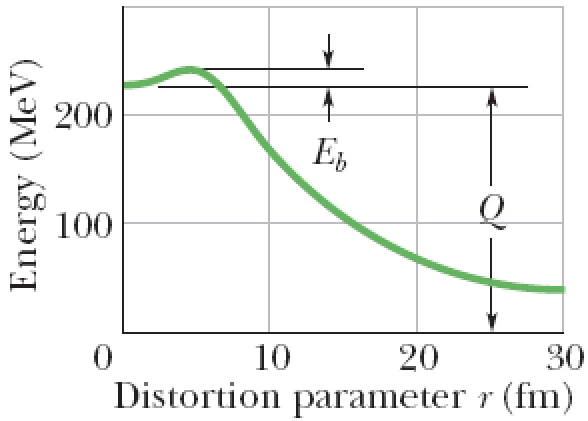


Fig. 43-2 The stages of a typical fission process, according to the collective model of Bohr and Wheeler.



E_b is an energy barrier that must be overcome.

Q is the energy that would then be released.

Fig. 43-3 The potential energy at various stages in the fission process, as predicted from the collective model of Bohr and Wheeler. The Q of the reaction (about 200 MeV) and the fission barrier height E_b are both indicated.

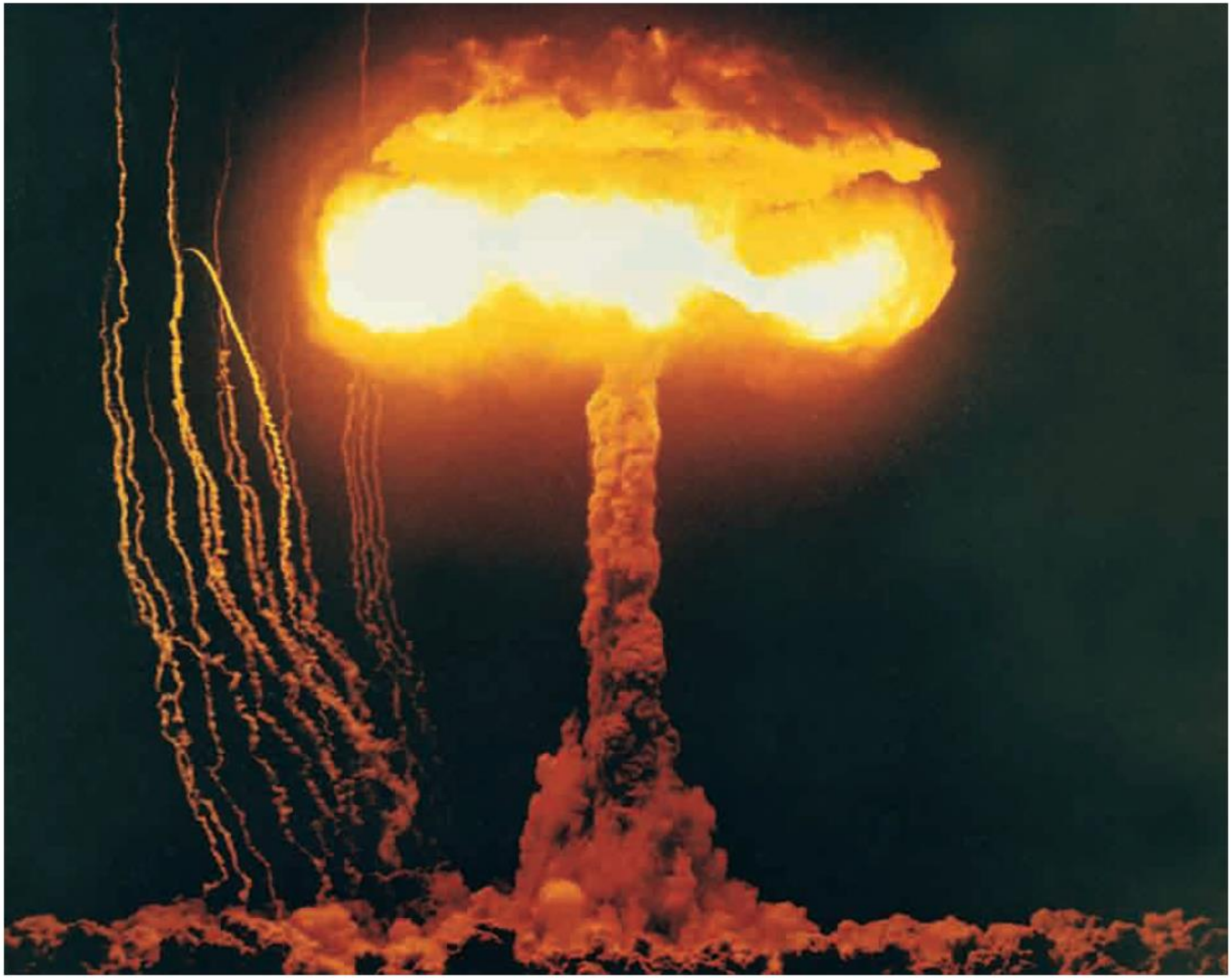
In the figure, the potential energy is plotted against the *distortion parameter r,* which is a rough measure of the extent to which the oscillating nucleus departs from a spherical shape. When the fragments are far apart, this parameter is simply the distance between their centers.

Table 43-2

Test of the Fissionability of Four Nuclides

Target Nuclide	Nuclide Being Fissioned	E_n (MeV)	E_b (MeV)	Fission by Thermal Neutrons?
^{235}U	^{236}U	6.5	5.2	Yes
^{238}U	^{239}U	4.8	5.7	No
^{239}Pu	^{240}Pu	6.4	4.8	Yes
^{243}Am	^{244}Am	5.5	5.8	No

Table 43-2 shows, for four high-mass nuclides, this test of whether capture of a thermal neutron can cause fissioning. For each nuclide, the table shows both the barrier height E_b of the nucleus that is formed by the neutron capture and the excitation energy E_n due to the capture.



Courtesy U.S. Department of Energy

Fig. 43-4 This image has transfixed the world since World War II.When Robert Oppenheimer, the head of the scientific team that developed the atomic bomb, witnessed the first atomic explosion, he quoted from a sacred Hindu text:“Now I am become Death, the destroyer of worlds.”

43.2: The Nuclear Reactor:

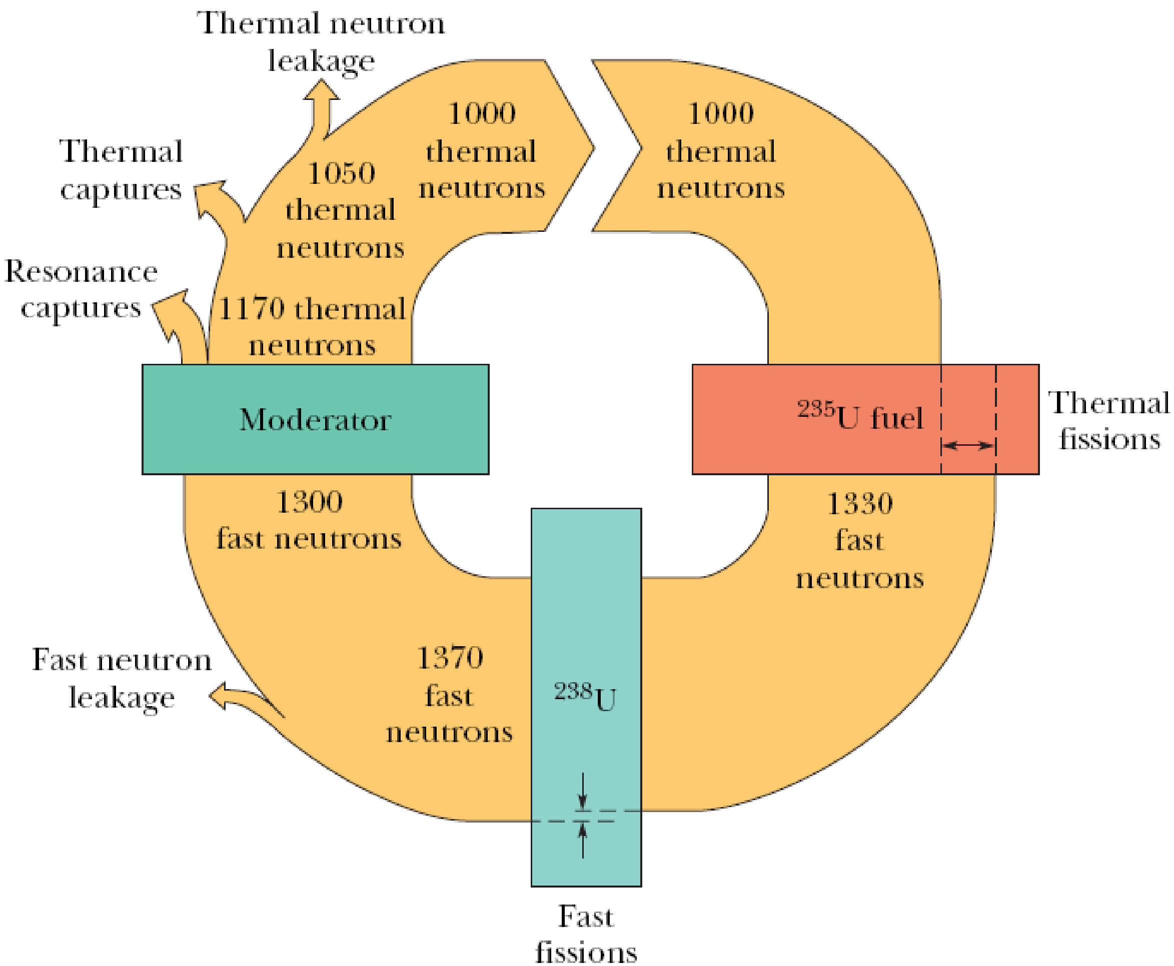


Fig. 43-5 Neutron bookkeeping in a reactor. A generation of 1000 thermal neutrons interacts with the ^{235}U fuel, the ^{238}U matrix, and the moderator. They produce 1370 neutrons by fission, but 370 of these are lost by nonfission capture or by leakage, meaning that 1000 thermal neutrons are left to form the next generation. The figure is drawn for a reactor running at a steady power level.

Three main difficulties stand in the way of a working reactor:

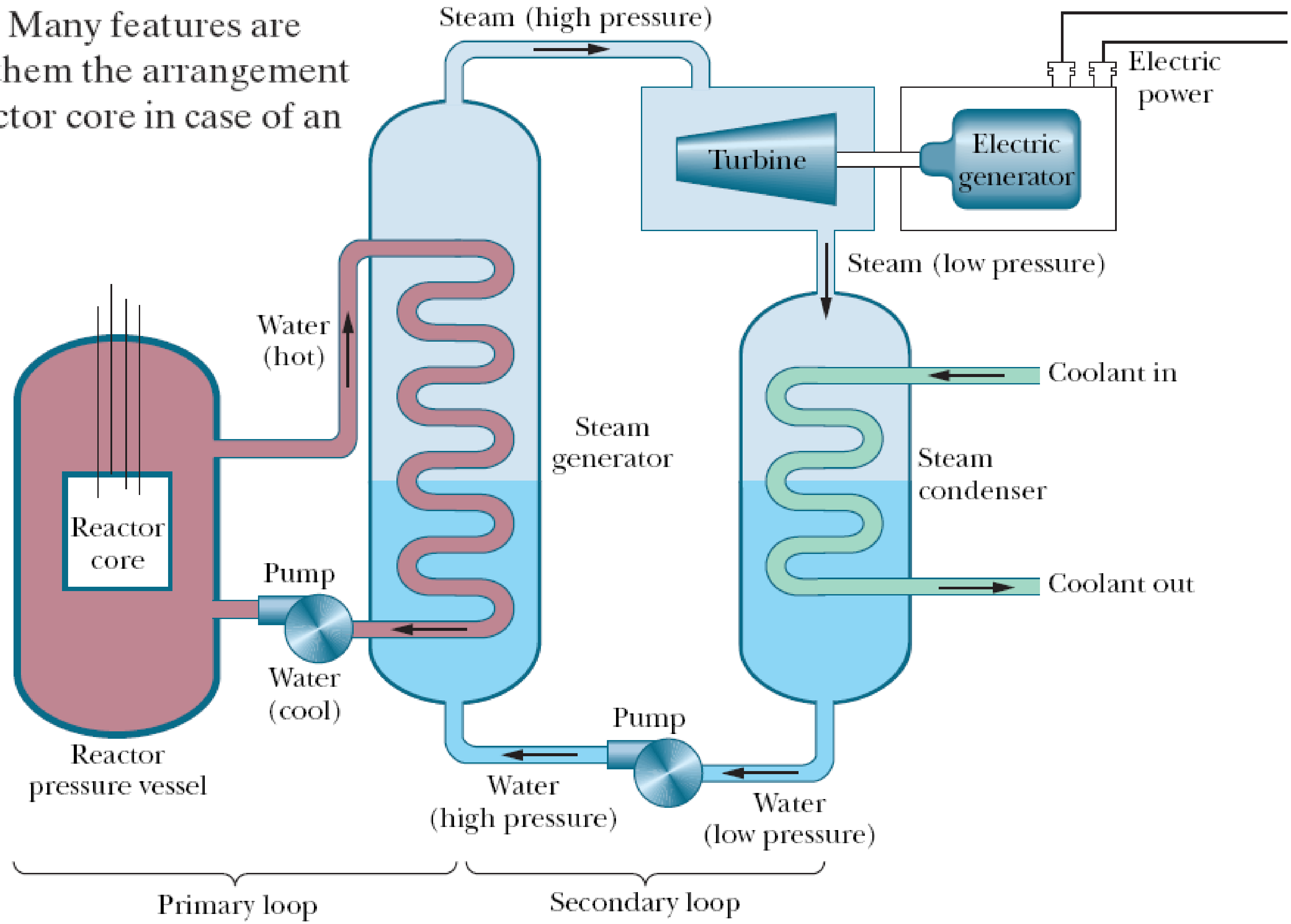
1.The Neutron Leakage Problem. (Some neutrons produced by fission leak out from the reactor).

2. The Neutron Energy Problem. (Fast neutrons are not as effective in producing fission as slower thermal neutrons).

3. The Neutron Capture Problem. (non-fission capture of neutrons)

43.2: The Nuclear Reactor:

Fig. 43-6 A simplified layout of a nuclear power plant, based on a pressurized-water reactor. Many features are omitted—among them the arrangement for cooling the reactor core in case of an emergency.



Example, Nuclear reactor:

A large electric generating station is powered by a pressurized-water nuclear reactor. The thermal power produced in the reactor core is 3400 MW, and 1100 MW of electricity is generated by the station. The *fuel charge* is 8.60×10^4 kg of uranium, in the form of uranium oxide, distributed among 5.70×10^4 fuel rods. The uranium is enriched to 3.0% ^{235}U.

(a) What is the station's efficiency?

Calculation: Here the efficiency (eff) is

$$\text{eff} = \frac{\text{useful output}}{\text{energy input}} = \frac{1100 \text{ MW (electric)}}{3400 \text{ MW (thermal)}}$$
$$= 0.32, \text{ or } 32\%. \qquad \text{(Answer)}$$

The efficiency—as for all power plants—is controlled by the second law of thermodynamics. To run this plant, energy at the rate of 3400 MW − 1100 MW, or 2300 MW, must be discharged as thermal energy to the environment.

(b) At what rate R do fission events occur in the reactor core?

Calculation: For steady-state operation (P is constant), we find

$$R = \frac{P}{Q} = \left(\frac{3.4 \times 10^9 \text{ J/s}}{200 \text{ MeV/fission}}\right)\left(\frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}}\right)$$
$$= 1.06 \times 10^{20} \text{ fissions/s}$$
$$\approx 1.1 \times 10^{20} \text{ fissions/s}. \qquad \text{(Answer)}$$

(c) At what rate (in kilograms per day) is the ^{235}U fuel disappearing? Assume conditions at start-up.

Calculations: The total rate at which the number of atoms of ^{235}U decreases is

$$(1 + 0.25)(1.06 \times 10^{20} \text{ atoms/s}) = 1.33 \times 10^{20} \text{ atoms/s}.$$
$$\frac{dM}{dt} = (1.33 \times 10^{20} \text{ atoms/s})(3.90 \times 10^{-25} \text{ kg/atom})$$
$$= 5.19 \times 10^{-5} \text{ kg/s} \approx 4.5 \text{ kg/d}. \qquad \text{(Answer)}$$

(d) At this rate of fuel consumption, how long would the fuel supply of ^{235}U last?

Calculation: At start-up, we know that the total mass of ^{235}U is 3.0% of the 8.60×10^4 kg of uranium oxide. So, the time T required to consume this total mass of ^{235}U at the steady rate of 4.5 kg/d is

$$T = \frac{(0.030)(8.60 \times 10^4 \text{ kg})}{4.5 \text{ kg/d}} \approx 570 \text{ d}. \qquad \text{(Answer)}$$

In practice, the fuel rods must be replaced (usually in batches) before their ^{235}U content is entirely consumed.

(e) At what rate is mass being converted to other forms of energy by the fission of ^{235}U in the reactor core?

Calculation: From Einstein's relation $E = mc^2$, we can write

$$\frac{dm}{dt} = \frac{dE/dt}{c^2} = \frac{3.4 \times 10^9 \text{ W}}{(3.00 \times 10^8 \text{ m/s})^2} \qquad (43\text{-}8)$$
$$= 3.8 \times 10^{-8} \text{ kg/s} = 3.3 \text{ g/d}. \qquad \text{(Answer)}$$