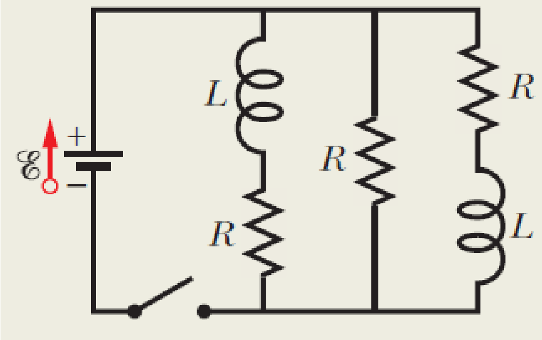
Induction and Inductance

Chapter 30

Figure 30-18*a* shows a circuit that contains three identical resistors with resistance $R = 9.0\ \Omega$, two identical inductors with inductance $L = 2.0$ mH, and an ideal battery with emf $\mathscr{E} = 18$ V.

(a) What is the current i through the battery just after the switch is closed?



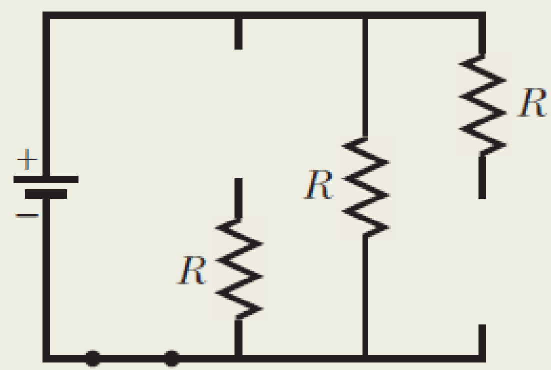
(*a*)

Calculations: Because the current through each inductor is zero before the switch is closed, it will also be zero just afterward. Thus, immediately after the switch is closed, the inductors act as broken wires, as indicated in Fig. 30-18*b*. We then have a single-loop circuit for which the loop rule gives us

$$\mathscr{E} - iR = 0.$$

Substituting given data, we find that

$$i = \frac{\mathscr{E}}{R} = \frac{18\ \text{V}}{9.0\ \Omega} = 2.0\ \text{A}. \qquad \text{(Answer)}$$



(*b*)

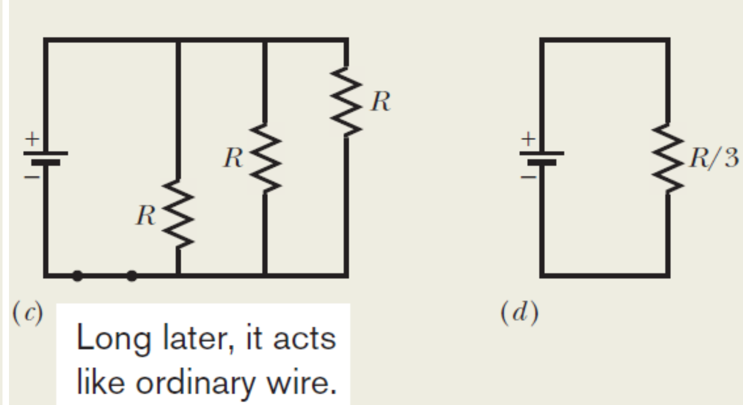
Initially, an inductor acts like broken wire.

(b) What is the current i through the battery long after the switch has been closed?

Long after the switch has been closed, the currents in the circuit have reached their equilibrium values, and the inductors act as simple connecting wires, as indicated in Fig. 30-18*c*.

Calculations: We now have a circuit with three identical resistors in parallel; from Eq. 27-23, their equivalent resistance is $R_{eq} = R/3 = (9.0\ \Omega)/3 = 3.0\ \Omega$. The equivalent circuit shown in Fig. 30-18*d* then yields the loop equation $\mathscr{E} - iR_{eq} = 0$, or

$$i = \frac{\mathscr{E}}{R_{eq}} = \frac{18\ \text{V}}{3.0\ \Omega} = 6.0\ \text{A}. \qquad \text{(Answer)}$$



30-8 Energy Density of a Magnetic Field

Consider a length l near the middle of a long solenoid of cross-sectional area A carrying current i; the volume associated with this length is Al. The energy U_B stored by the length l of the solenoid must lie entirely within this volume because the magnetic field outside such a solenoid is approximately zero. Moreover, the stored energy must be uniformly distributed within the solenoid because the magnetic field is (approximately) uniform everywhere inside. Thus, the energy stored per unit volume of the field is

$$u_B = \frac{U_B}{Al} \quad \Longrightarrow \quad U_B = \frac{1}{2} L i^2$$

30-8 Energy Density of a Magnetic Field

We have,

$$u_B = \frac{Li^2}{2Al} = \frac{L}{l}\frac{i^2}{2A}.$$

here L is the inductance of length l of the solenoid

Substituting for $\frac{L}{l}$ we get

$$\frac{L}{l} = \mu_0 n^2 A \quad \text{(solenoid)}.$$

$$u_B = \frac{1}{2}\mu_0 n^2 i^2$$

And we can write the **energy density** as

$$u_B = \frac{B^2}{2\mu_0}$$

Inductance—like capacitance—depends only on the geometry of the device.

Checkpoint 7

The table lists the number of turns per unit length, current, and cross-sectional area for three solenoids. Rank the solenoids according to the magnetic energy density within them, greatest first.

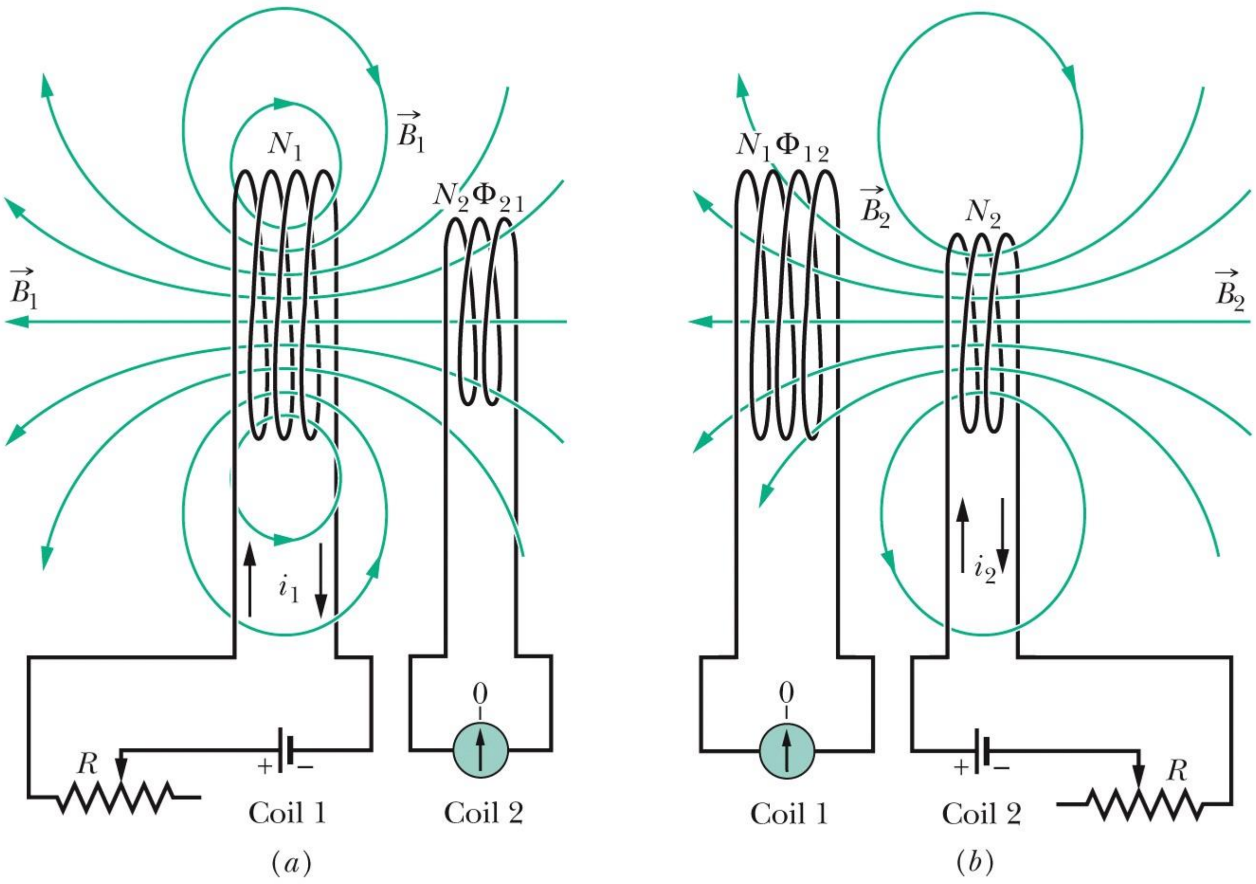
Solenoid	Turns per Unit Length	Current	Area
a	$2n_1$	i_1	$2A_1$
b	n_1	$2i_1$	A_1
c	n_1	i_1	$6A_1$

$$u_B = \frac{1}{2}\mu_0 n^2 i^2$$

a and *b* tie, then *c*

30-9 Mutual Induction

Mutual induction. (a) The magnetic field B_1 produced by current i_1 in coil 1 extends through coil 2. If i_1 is varied (by varying resistance R), an emf is induced in coil 2 and current registers on the meter connected to coil 2. (b) The roles of the coils interchanged.



30-9 Mutual Induction

If coils 1 and 2 are near each other, a changing current in either coil can induce an emf in the other. This mutual induction is described by

$$\mathscr{E}_2 = -M\frac{di_1}{dt}$$

and

$$\mathscr{E}_1 = -M\frac{di_2}{dt}.$$

We define the mutual inductance M_{21} of coil 2 with respect to coil 1 as

$$M_{21} = \frac{N_2\Phi_{21}}{i_1},$$

$$M_{21} = M_{12} = M,$$

where M (measured in henries) is the mutual inductance.

Summary

Magnetic Flux

- The magnetic flux through an area A in a magnetic field $\vec{B}$ is defined as

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$ **Equation 30-1**

- If $\vec{B}$ is perpendicular to the area and uniform over it, Equation 30-1 becomes

$$\Phi_B = BA \quad \left(\vec{B} \perp A, \vec{B} \text{ uniform}\right).$$ **Equation 30-2**

Summary

Faraday's Law of Induction

- The induced emf is,

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$ **Equation 30-4**

- If the loop is replaced by a closely packed coil of N turns, the induced emf is

$$\mathcal{E} = -N\frac{d\Phi_B}{dt}.$$ **Equation 30-5**

Summary

Lenz's Law

- An induced current has a direction such that the magnetic field due to this induced current opposes the change in the magnetic flux that induces the current.

emf and the Induced Magnetic Field

- The induced emf is related $\vec{E}$ by
  to

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{s}, \qquad \textbf{Equation 30-19}$$

- Faraday's law in its most general form,

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \textbf{Equation 30-20}$$

Summary

Inductor

- The inductance L of the inductor is

$$L = \frac{N\Phi_B}{i}$$ **Equation 30-28**

- The inductance per unit length near the middle of a long solenoid of cross-sectional area A and n turns per unit length is

$$\frac{L}{l} = \mu_0 n^2 A$$ **Equation 30-31**

Summary

Self-Induction

- This self-induced emf is,

$$\mathcal{E}_L = -L\frac{di}{dt}.$$

Equation 30-35

Series RL Circuit

- Rise of current,

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{\frac{-t}{\tau_L}}\right)$$

Equation 30-41

- Decay of current

$$i = i_0 e^{\frac{-t}{\tau_L}}$$

Equation 30-45

Summary

Magnetic Energy

- the inductor's magnetic field stores an energy given by

$$U_B = \frac{1}{2} L i^2$$

Equation 30-49

- The density of stored magnetic energy,

$$u_B = \frac{B^2}{2\mu_0}$$

Equation 30-55

Summary

Mutual Induction

- The mutual induction is described by,

$$\mathcal{E}_2 = -M\frac{di_1}{dt}$$ **Equation 30-64**

$$\mathcal{E}_1 = -M\frac{di_2}{dt}$$ **Equation 30-65**