31 Summary

LC Energy Transfer

- In an oscillating LC circuit, instantaneous values of the two forms of energy are

$$U_E = \frac{q^2}{2C} \quad \text{and} \quad U_B = \frac{Li^2}{2}$$ **Eq. 31-1&2**

LC Charge and Current Oscillations

- The principle of conservation of energy leads to

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$ **Eq. 31-11**

- The solution of Eq. 31-11 is

$$q = Q\cos(\omega t + \phi)$$ **Eq. 31-12**

- the angular frequency v of the oscillations is

$$\omega = \frac{1}{\sqrt{LC}}.$$ **Eq. 31-4**

Damped Oscillations

- Oscillations in an LC circuit are damped when a dissipative element R is also present in the circuit. Then

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0$$ **Eq. 31-24**

- The solution of this differential equation is

$$q = Qe^{-Rt/2L}\cos(\omega' t + \phi),$$ **Eq. 31-25**

Alternating Currents; Forced Oscillations

- A series RLC circuit may be set into forced oscillation at a driving angular frequency by an external alternating emf

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t.$$ **Eq. 31-28**

- The current driven in the circuit is

$$i = I\sin(\omega_d t - \phi)$$ **Eq. 31-29**

31 Summary

Series RLC Circuits

- For a series RLC circuit with an alternating external emf and a resulting alternating current,

$$I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathscr{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

Eq. 31-60&63

- and the phase constant is,

$$\tan\phi = \frac{X_L - X_C}{R}$$

Eq. 31-65

- The impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Eq. 31-61

Power

- In a series RLC circuit, the average power of the generator is,

$$P_{avg} = I_{rms}^2 R = \mathscr{E}_{rms} I_{rms} \cos\phi.$$

Eq. 31-71&76

Transformers

- Primary and secondary voltage in a transformer is related by

$$V_s = V_p \frac{N_s}{N_p}$$

Eq. 31-79

- The currents through the coils,

$$I_s = I_p \frac{N_p}{N_s}$$

Eq. 31-80

- The equivalent resistance of the secondary circuit, as seen by the generator, is

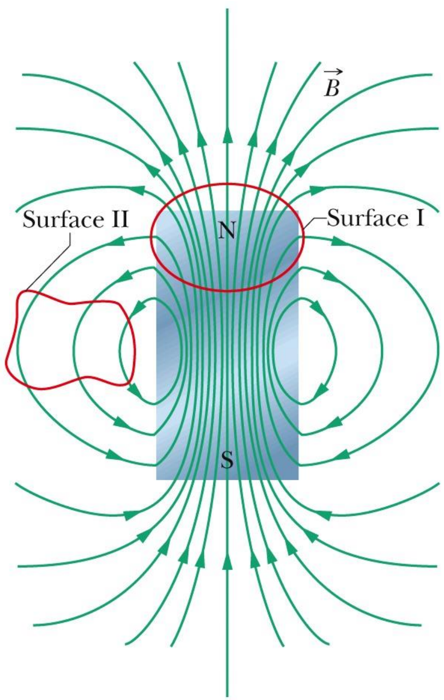
$$R_{eq} = \left(\frac{N_p}{N_s}\right)^2 R,$$

Eq. 31-82

Chapter 32

Maxwell Equations; Magnetism of Matter

32-1 Gauss' Law for Magnetic Fields



The field lines for the magnetic field ***B*** of a short bar magnet. The red curves represent cross sections of closed, three-dimensional Gaussian surfaces.

The simplest magnetic structure that can exist is a magnetic dipole. Magnetic monopoles do not exist (as far as we know).

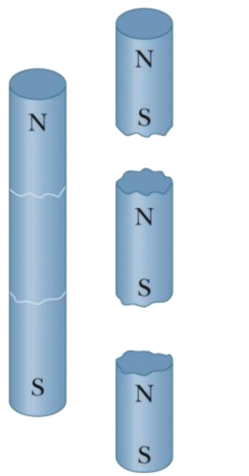
Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. The law asserts that the net magnetic flux Φ_B through <u>any</u> closed Gaussian surface is zero:

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Contrast this with Gauss' law for electric fields,

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

Gauss' law for magnetic fields says that there can be no net magnetic flux through the surface because there can be no net "magnetic charge" (individual magnetic poles) enclosed by the surface.

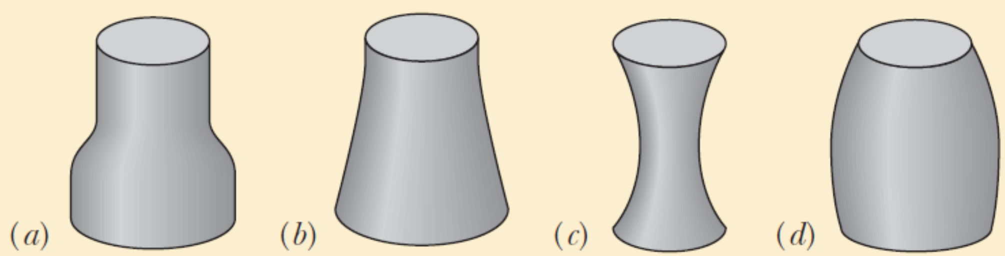


If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

Checkpoint 1

The figure here shows four closed surfaces with flat top and bottom faces and curved sides. The table gives the areas A of the faces and the magnitudes B of the uniform and perpendicular magnetic fields through those faces; the units of A and B are arbitrary but consistent. Rank the surfaces according to the magnitudes of the magnetic flux through their curved sides, greatest first.

Surface	A_{top}	B_{top}	A_{bot}	B_{bot}
a	2	6, outward	4	3, inward
b	2	1, inward	4	2, inward
c	2	6, inward	2	8, outward
d	2	3, outward	3	2, outward



d, b, c, a (zero)

32-2 Induced Magnetic Fields

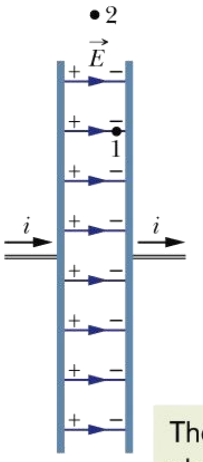
A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

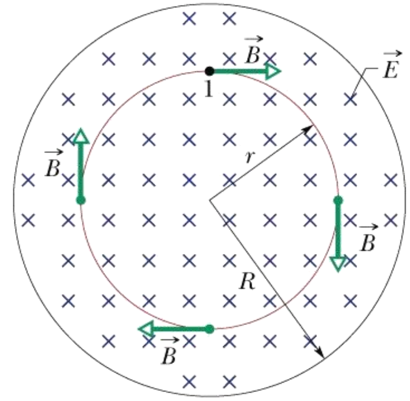
Charging a Capacitor.

As an example of this sort of induction, we consider the charging of a parallel-plate capacitor with circular plates. The charge on our capacitor is being increased at a steady rate by a constant current *i* in the connecting wires. Then the electric field magnitude between the plates must also be increasing at a steady rate.



(a)

The changing of the electric field between the plates creates a magnetic field.



(b)

32-2 Induced Magnetic Fields

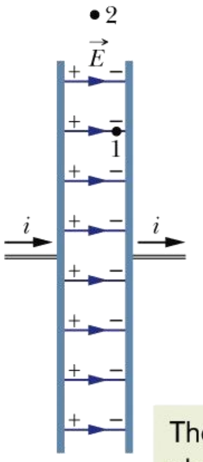
A changing electric flux induces a magnetic field ***B***. Maxwell's Law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Relates the magnetic field induced along a closed loop to the changing electric flux ϕ_E through the loop.

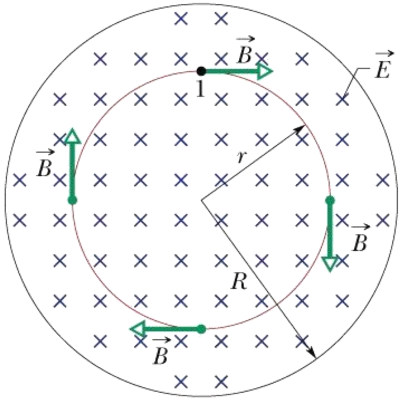
Charging a Capacitor (continued)

Figure (b) is a view of the right-hand plate of Fig. (a) from between the plates. The electric field is directed into the page. Let us consider a circular loop through point 1 in Figs.(a) and (b), a loop that is concentric with the capacitor plates and has a radius smaller than that of the plates. Because the electric field through the loop is changing, the electric flux through the loop must also be changing. According to the above equation, *this changing electric flux induces a magnetic field around the loop.*



(a)

The changing of the electric field between the plates creates a magnetic field.



(b)

32-2 Induced Magnetic Fields

Ampere-Maxwell Law

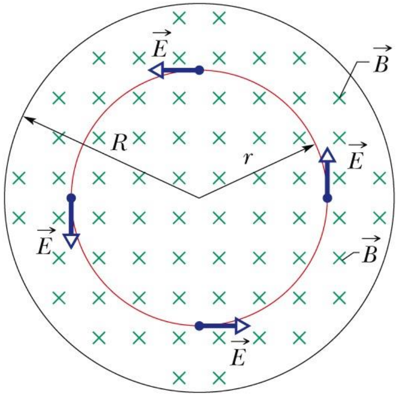
Ampere's law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

gives the magnetic field generated by a current i_{enc} encircled by a closed loop.

Thus, the two equations (the other being Maxwell's Law) that specify the magnetic field ***B*** produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$$

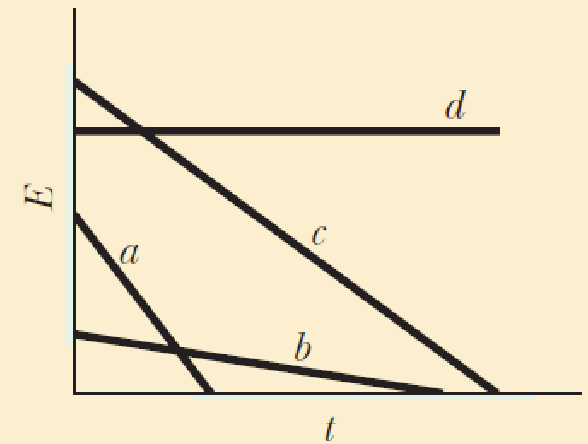


The induced $\vec{E}$ direction here is opposite the induced $\vec{B}$ direction in the preceding figure.

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of Eq. is zero, and so the Eq. reduces to Ampere's law. When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Eq. is zero, and so Eq. reduces to Maxwell's law of induction.

Checkpoint 2

The figure shows graphs of the electric field magnitude E versus time t for four uniform electric fields, all contained within identical circular regions as in Fig. 32-5*b*. Rank the fields according to the magnitudes of the magnetic fields they induce at the edge of the region, greatest first.



a, c, b, d (zero)

Sample Problem 32.01 Magnetic field induced by changing electric field

A parallel-plate capacitor with circular plates of radius R is being charged as in Fig. 32-5*a*.

(a) Derive an expression for the magnetic field at radius r for the case $r \leq R$.

KEY IDEAS

A magnetic field can be set up by a current and by induction due to a changing electric flux; both effects are included in Eq. 32-5. There is no current between the capacitor plates of Fig. 32-5, but the electric flux there is changing. Thus, Eq. 32-5 reduces to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}. \tag{32-6}$$

We shall separately evaluate the left and right sides of this equation.

Left side of Eq. 32-6: We choose a circular Amperian loop with a radius $r \leq R$ as shown in Fig. 32-5*b* because we want to evaluate the magnetic field for $r \leq R$—that is, inside the capacitor. The magnetic field $\vec{B}$ at all points along the loop is tangent to the loop, as is the path element $d\vec{s}$. Thus, $\vec{B}$ and $d\vec{s}$ are either parallel or antiparallel at each point of the loop. For simplicity, assume they are parallel (the choice does not alter our outcome here). Then

$$\oint \vec{B} \cdot d\vec{s} = \oint B\, ds \cos 0° = \oint B\, ds.$$

Due to the circular symmetry of the plates, we can also assume that $\vec{B}$ has the same magnitude at every point around the loop. Thus, B can be taken outside the integral on the right side of the above equation. The integral that remains is $\oint ds$, which simply gives the circumference $2\pi r$ of the loop. The left side of Eq. 32-6 is then $(B)(2\pi r)$.

Right side of Eq. 32-6: We assume that the electric field $\vec{E}$ is uniform between the capacitor plates and directed perpendicular to the plates. Then the electric flux Φ_E through the Amperian loop is EA, where A is the area encircled by the loop within the electric field. Thus, the right side of Eq. 32-6 is $\mu_0\varepsilon_0\, d(EA)/dt$.

Combining results: Substituting our results for the left and right sides into Eq. 32-6, we get

$$(B)(2\pi r) = \mu_0\varepsilon_0 \frac{d(EA)}{dt}.$$

Because A is a constant, we write $d(EA)$ as $A\, dE$; so we have

$$(B)(2\pi r) = \mu_0\varepsilon_0 A \frac{dE}{dt}. \tag{32-7}$$

The area A that is encircled by the Amperian loop within the electric field is the *full* area πr^2 of the loop because the loop's radius r is less than (or equal to) the plate radius R. Substituting πr^2 for A in Eq. 32-7 leads to, for $r \leq R$,

$$B = \frac{\mu_0\varepsilon_0 r}{2} \frac{dE}{dt}. \qquad \text{(Answer)} \tag{32-8}$$

This equation tells us that, inside the capacitor, B increases linearly with increased radial distance r, from 0 at the central axis to a maximum value at plate radius R.

(b) Evaluate the field magnitude B for $r = R/5 = 11.0$ mm and $dE/dt = 1.50 \times 10^{12}$ V/m·s.

Calculation: From the answer to (a), we have

$$\begin{aligned} B &= \frac{1}{2}\mu_0\varepsilon_0 r\frac{dE}{dt} \\ &= \tfrac{1}{2}(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(8.85 \times 10^{-12}\,\mathrm{C^2/N\cdot m^2}) \\ &\quad \times (11.0 \times 10^{-3}\,\mathrm{m})(1.50 \times 10^{12}\,\mathrm{V/m\cdot s}) \\ &= 9.18 \times 10^{-8}\,\mathrm{T}. \qquad \text{(Answer)} \end{aligned}$$

(c) Derive an expression for the induced magnetic field for the case $r \geq R$.

Calculation: Our procedure is the same as in (a) except we now use an Amperian loop with a radius r that is greater than the plate radius R, to evaluate B outside the capacitor. Evaluating the left and right sides of Eq. 32-6 again leads to Eq. 32-7. However, we then need this subtle point: The electric field exists only between the plates, not outside the plates. Thus, the area A that is encircled by the Amperian loop in the electric field is *not* the full area πr^2 of the loop. Rather, A is only the plate area πR^2.

Substituting πR^2 for A in Eq. 32-7 and solving the result for B give us, for $r \geq R$,

$$B = \frac{\mu_0\varepsilon_0 R^2}{2r}\frac{dE}{dt}. \qquad \text{(Answer)} \qquad (32\text{-}9)$$

This equation tells us that, outside the capacitor, B decreases with increased radial distance r, from a maximum value at the plate edges (where $r = R$). By substituting $r = R$ into Eqs. 32-8 and 32-9, you can show that these equations are consistent; that is, they give the same maximum value of B at the plate radius.

The magnitude of the induced magnetic field calculated in (b) is so small that it can scarcely be measured with simple apparatus. This is in sharp contrast to the magnitudes of induced electric fields (Faraday's law), which can be measured easily. This experimental difference exists partly because induced emfs can easily be multiplied by using a coil of many turns. No technique of comparable simplicity exists for multiplying induced magnetic fields. In any case, the experiment suggested by this sample problem has been done, and the presence of the induced magnetic fields has been verified quantitatively.