Chapter 32

Maxwell Equations; Magnetism of Matter

32-3 Displacement Current

If you compare the two terms on the right side of Eq. (Ampere-Maxwell Law), you will see that the product $\varepsilon_0(d\phi_E/dt)$ must have the dimension of a current. In fact, that product has been treated as being a fictitious current called the **displacement current** i_d:

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$$

Ampere-Maxwell Law then becomes,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}} + \mu_0 i_{\text{enc}}$$

where $i_{d,enc}$ is the displacement current encircled by the integration loop.

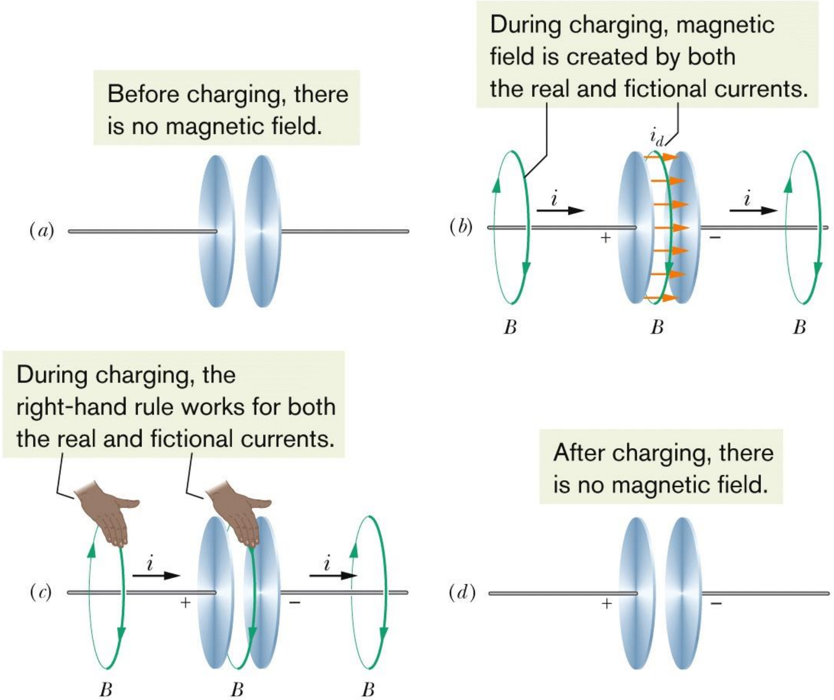
32-3 Displacement Current

Finding the Induced Magnetic Field:
In Chapter 29 we found the direction of the magnetic field produced by a real current i by using the right-hand rule. We can apply the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current i_d, as is shown in the center of Fig. (c) for a capacitor.
Then, as done previously, the magnitude of the magnetic field at a point inside the capacitor at radius r from the center is

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right)r$$

the magnitude of the magnetic field at a point outside the capacitor at radius r is

$$B = \frac{\mu_0 i_d}{2\pi r}$$



(a) Before and (d) after the plates are charged, there is no magnetic field. (b) During the charging, magnetic field is created by both the real current and the (fictional) (c) displacement current. (c) The same right-hand rule works for both currents to give the direction of the magnetic field.

32-3 Displacement Current

The four fundamental equations of electromagnetism, called Maxwell's equations and are displayed in Table 32-1.

Table 32-1 Maxwell's Equations[a]

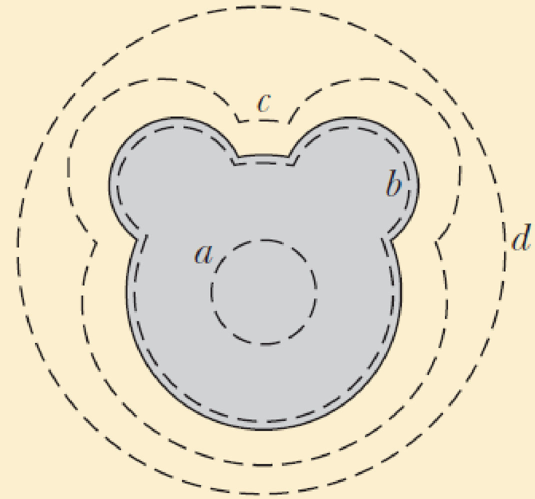
Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\varepsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

[a]Written on the assumption that no dielectric or magnetic materials are present.

These four equations explain a diverse range of phenomena, from why a compass needle points north to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, television transmitters and receivers, telephones, scanners, radar, and microwave ovens.

Checkpoint 3

The figure is a view of one plate of a parallel-plate capacitor from within the capacitor. The dashed lines show four integration paths (path b follows the edge of the plate). Rank the paths according to the magnitude of $\oint \vec{B} \cdot d\vec{s}$ along the paths during the discharging of the capacitor, greatest first.



Sample Problem 32.02 Treating a changing electric field as a displacement current

A circular parallel-plate capacitor with plate radius R is being charged with a current i.

(a) Between the plates, what is the magnitude of $\oint \vec{B} \cdot d\vec{s}$, in terms of μ_0 and i, at a radius $r = R/5$ from their center?

KEY IDEA

A magnetic field can be set up by a current and by induction due to a changing electric flux (Eq. 32-5). Between the plates in Fig. 32-5, the current is zero and we can account for the changing electric flux with a fictitious displacement current i_d. Then integral $\oint \vec{B} \cdot d\vec{s}$ is given by Eq. 32-11, but because there is no real current i between the capacitor plates, the equation reduces to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d,\text{enc}}. \tag{32-18}$$

Calculations: Because we want to evaluate $\oint \vec{B} \cdot d\vec{s}$ at radius $r = R/5$ (within the capacitor), the integration loop encircles only a portion $i_{d,\text{enc}}$ of the total displacement current i_d. Let's assume that i_d is uniformly spread over the full plate area. Then the portion of the displacement current encircled by the loop is proportional to the area encircled by the loop:

$$\frac{\begin{pmatrix}\text{encircled displacement} \\ \text{current } i_{d,\text{enc}}\end{pmatrix}}{\begin{pmatrix}\text{total displacement} \\ \text{current } i_d\end{pmatrix}} = \frac{\text{encircled area } \pi r^2}{\text{full plate area } \pi R^2}.$$

This gives us

$$i_{d,\text{enc}} = i_d \frac{\pi r^2}{\pi R^2}.$$

Substituting this into Eq. 32-18, we obtain

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_d \frac{\pi r^2}{\pi R^2}. \tag{32-19}$$

Now substituting $i_d = i$ (from Eq. 32-15) and $r = R/5$ into Eq. 32-19 leads to

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i \frac{(R/5)^2}{R^2} = \frac{\mu_0 i}{25}. \qquad \text{(Answer)}$$

(b) In terms of the maximum induced magnetic field, what is the magnitude of the magnetic field induced at $r = R/5$, inside the capacitor?

KEY IDEA

Because the capacitor has parallel circular plates, we can treat the space between the plates as an imaginary wire of radius R carrying the imaginary current i_d. Then we can use Eq. 32-16 to find the induced magnetic field magnitude B at any point inside the capacitor.

Calculations: At $r = R/5$, Eq. 32-16 yields

$$B = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) r = \frac{\mu_0 i_d (R/5)}{2\pi R^2} = \frac{\mu_0 i_d}{10\pi R}. \tag{32-20}$$

From Eq. 32-16, the maximum field magnitude $B_{\max}$ within the capacitor occurs at $r = R$. It is

$$B_{\max} = \left(\frac{\mu_0 i_d}{2\pi R^2}\right) R = \frac{\mu_0 i_d}{2\pi R}. \tag{32-21}$$

Dividing Eq. 32-20 by Eq. 32-21 and rearranging the result, we find that the field magnitude at $r = R/5$ is

$$B = \tfrac{1}{5} B_{\max}. \qquad \text{(Answer)}$$

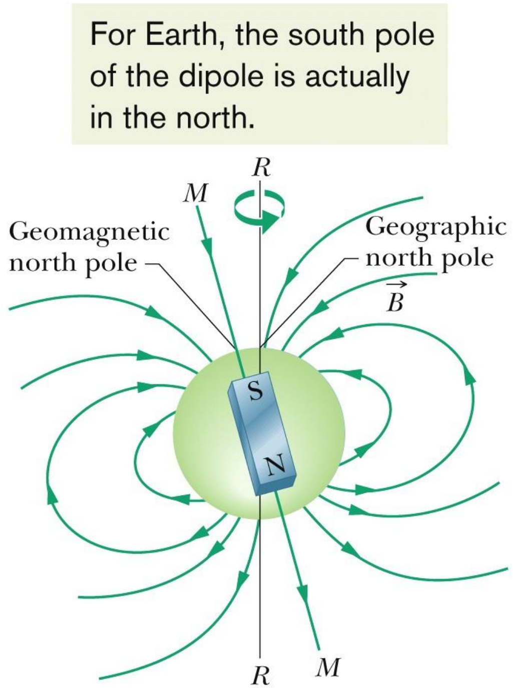
We should be able to obtain this result with a little reasoning and less work. Equation 32-16 tells us that inside the capacitor, B increases linearly with r. Therefore, a point $\frac{1}{5}$ the distance out to the full radius R of the plates, where $B_{\max}$ occurs, should have a field B that is $\frac{1}{5}B_{\max}$.

32-4 Magnets

The Magnetism of Earth

Earth is a huge magnet; for points near Earth's surface, its magnetic field can be approximated as the field of a huge bar magnet — a magnetic dipole — that straddles the center of the planet. Figure shown here is an idealized symmetric depiction of the dipole field, without the distortion caused by passing charged particles from the Sun.

The direction of the magnetic field at any location on Earth's surface is commonly specified in terms of two angles. The **field declination** is the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field. The **field inclination** is the angle (up or down) between a horizontal plane and the field's direction.



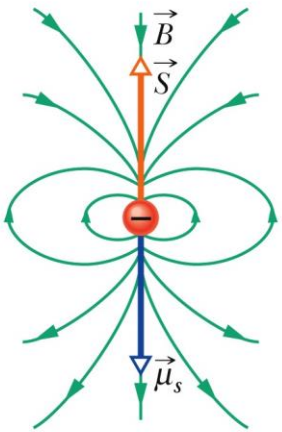
Earth's magnetic field represented as a dipole field. The dipole axis MM makes an angle of 11.5° with Earth's rotational axis RR. The south pole of the dipole is in Earth's Northern Hemisphere.

32-5 Magnetism and Electrons

Spin Magnetic Dipole Moment. An electron has an intrinsic angular momentum called its spin angular momentum (or just spin) ***S***; associated with this spin is an intrinsic spin magnetic dipole moment $\boldsymbol{\mu_s}$. (By intrinsic, we mean that ***S*** and $\boldsymbol{\mu_s}$ are basic characteristics of an electron, like its mass and electric charge.) Vectors ***S*** and $\boldsymbol{\mu_s}$ are related by

$$\vec{\mu}_s = -\frac{e}{m}\vec{S},$$

For an electron, the spin is opposite the magnetic dipole moment.



in which *e* is the elementary charge (1.60×10^{-19} C) and *m* is the mass of an electron (9.11×10^{-31} kg). The minus sign means that $\boldsymbol{\mu_s}$ and ***S*** are oppositely directed.
For a measurement along a z axis, the component $\boldsymbol{S_z}$ can have only the values given by

$$S_z = m_s \frac{h}{2\pi}, \quad \text{for } m_s = \pm ½$$

Similarly, $$\mu_{s,z} = \pm \frac{eh}{4\pi m} = \pm\mu_B,$$ where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = 9.27 \times 10^{-24}\ \mathrm{J/T}.$$

32-5 Magnetism and Electrons

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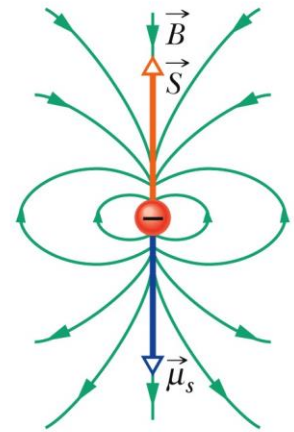


Energy. When an electron is placed in an external magnetic field $\boldsymbol{B}_{ext}$, an energy U can be associated with the orientation of the electron's spin magnetic dipole moment $\boldsymbol{\mu_s}$ just as an energy can be associated with the orientation of the magnetic dipole moment $\boldsymbol{\mu}$ of a current loop placed in $\boldsymbol{B}$. The orientation energy for the electron is

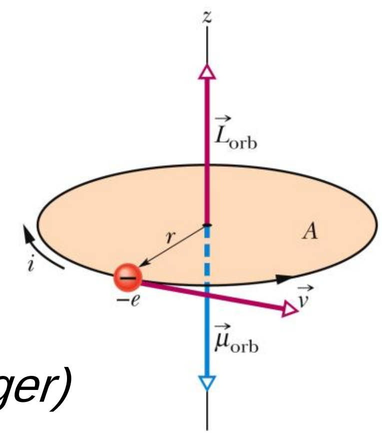
$$U = -\vec{\mu}_s \cdot \vec{B}_{ext} = -\mu_{s,z}B_{ext},$$

where the z axis is taken to be in the direction of $\boldsymbol{B}_{ext}$.

32-5 Magnetism and Electrons

Orbital Magnetic Dipole Moment. When it is in an atom, an electron has an additional angular momentum called its orbital angular momentum $\boldsymbol{L}_{orb}$. Associated with $\boldsymbol{L}_{orb}$ is an orbital magnetic dipole moment $\boldsymbol{\mu}_{orb}$ the two are related by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m}\vec{L}_{\text{orb}}.$$



The minus sign means that $\boldsymbol{\mu}_{orb}$ and $\boldsymbol{L}_{orb}$ have opposite directions. Orbital angular momentum is quantized and can have only measured values given by

$$L_{\text{orb},z} = m_\ell \frac{h}{2\pi},$$ for m_l=0, ±1, ±2, …, ±(limit integer)

The associated magnetic dipole moment is given by

$$\mu_{\text{orb},z} = -m_\ell \frac{eh}{4\pi m} = -m_\ell \mu_{\text{B}}.$$

An electron moving at constant speed v in a circular path of radius r that encloses an area A.

The energy U associated with the orientation of the orbital magnetic dipole moment in an external magnetic field B_{ext} is

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z} B_{\text{ext}}.$$

32-6 Diamagnetism

A diamagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a magnetic dipole moment directed opposite $\vec{B}_{ext}$. If the field is nonuniform, the diamagnetic material is repelled *from* a region of greater magnetic field *toward* a region of lesser field.

Levitating Frog: The frog in the figure is diamagnetic (as is any other animal). When the frog was placed in the diverging magnetic field near the top end of a vertical current- carrying solenoid, every atom in the frog was repelled upward, away from the region of stronger magnetic field at that end of the solenoid. The frog moved upward into weaker and weaker magnetic field until the upward magnetic force balanced the gravitational force on it, and there it hung in midair. The frog is not in discomfort because every atom is subject to the same forces and thus there is no force variation within the frog.

Courtesy A.K. Geim, University of Manchester, UK