4. RC CIRCUITS

OBJECT: To study the frequency response and phase shift in (i) a CR circuit (high-pass)

(ii) an RC circuit (low-pass)

APPARATUS: Oscillator, Oscilloscope, various combinations of resistances and capacitors, connecting wires.

THEORETICAL BACKGROUND:

Filters are devices used to smooth out or eliminate some undesirable time-varying voltages/signals in electronic circuits. An RC series circuit constitutes one of the simplest filters. We shall investigate the characteristics of two distinct RC filters i.e. (a) high-pass and (b) low-pass RC filters. In particular we shall study their frequency response and phase shifts.

(a) High-Pass RC Filter: Consider the simple RC series circuit shown in Fig.1, where a source of alternating emf (constant voltage amplitude but variable frequency)supplies

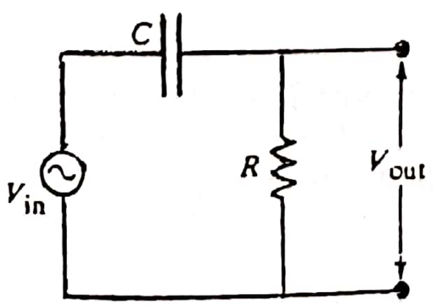

Figure 1: A simple RC high-pass filter.

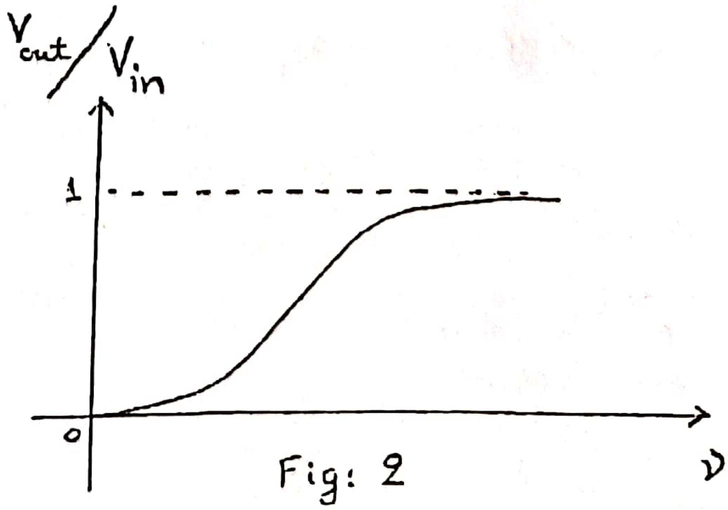

Fig: 2

Scanned with CamSca

an input voltage "V" to the RC circuit. Suppose that "V" is of the form

$$V = V_m \operatorname{Sin} \omega t \qquad - (1)$$

This creates an alternating current in the circuit given by

$$I = I_m \operatorname{Sin} (\omega t - \phi) \qquad - (2)$$

where (i) I_m and V_m are the maximum values of current and input voltage respectively

(ii) $\omega = 2\pi\nu$ is the angular frequency of the input voltage

(iii) $\phi = \tan^{-1} \frac{X_C}{R} = \tan^{-1}\left(\frac{1}{\omega CR}\right) \qquad - (3)$

is the phase difference between "I" and "V_{in}"

We will be interested only in the maximum values of "I" and "V". The maximum input voltage $V_{in}(=V_m)$ is related to the maximum current "I_m" through

$$I_m = \frac{V_{in}}{Z} \quad \text{or} \quad V_{in} = I_m Z \qquad - (4)$$

But $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ = Total impedance of the circuit

Therefore $$V_{in} = I_m \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad - (5)$$

The input voltage appears partly as a potential difference "V_C" across the capacitor and partly as a potential difference "V_R" across the resistor

$$V_{in} = V_C + V_R \qquad - (6)$$

Scanned with CamSca

If we take the potential difference across "R" as our output "V_{out}", then from Ohm's Law,

$$V_{out} = V_R = I_{in}R \qquad - \quad (7)$$

Equations (5) and (7) yield the ratio of output voltage to input voltage as:

$$\frac{V_{out}}{V_{in}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \qquad - \quad (8)$$

From this equation we notice that at low frequencies ($\omega = 2\pi\nu$), the capacitive reactance $\left(\frac{1}{\omega C}\right) > R$

and "V_{out}" is small compared with "V_{in}". On the other hand at high frequencies $X^2 = \left(\frac{1}{\omega C}\right)^2 << R^2$

and $\frac{V_{out}}{V_{in}} \simeq 1$ or $V_{out} \simeq V_{in}$

Thus the circuit will pass signals of high frequency with relatively little or no attenuation at all. However, the signals of low frequency are heavily attenuated and therefore "filtered out". This circuit is appropriately named "RC high-pass filter". Physically this is a result of the blocking action of the capacitor to low frequencies or direct current. Fig.2 gives the variation of "V_{out}/V_{in}" as a function of frequency in an RC high-pass circuit.

Scanned with CamScanner

Lower Cut-off Frequency

The frequency "ω_1", at which $X_C = R$, is sometimes called the lower cut-off frequency for a high pass filter. Thus

$$X_C = \frac{1}{\omega_1 C} = \frac{1}{2\pi\nu_1 C} = R$$

$$\text{yields } \nu_1 = \frac{1}{2\pi RC} \qquad - (9)$$

The circuit may be used to remove signal components having frequency below ν_1.

(b) Low-Pass RC circuit

Now consider the RC series circuit shown in Fig.3. In this case, the "V_{out}" is taken across the capacitor.

The capacitive reactance $= X_C = \frac{1}{\omega C}$

$$\therefore \; V_{out} = I_m \, X_C = \frac{I_m}{\omega C} \qquad - (10)$$

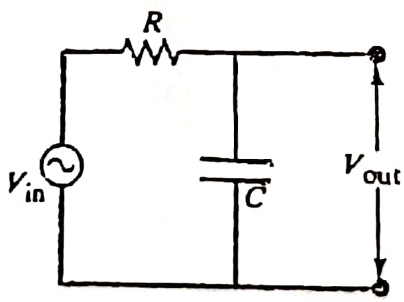

Figure 3: A simple RC low-pass filter.

The ratio of the output voltage to input voltage is (using equations 5 and 10)

$$\frac{V_{out}}{V_{in}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}} \qquad - (11)$$

Scanned with CamSca

This equation shows that when "ω" is small (low frequencies)

$R^2 \ll \left(\frac{1}{\omega C}\right)^2$ or $\omega^2 C^2 R^2 \ll 1$ and $V_{out} \simeq V_{in}$

On the other hand when "ω" is large (high frequencies)

$$R^2 \gg \left(\frac{1}{\omega C}\right)^2 \quad \text{or} \quad \omega^2 C^2 R^2 \gg 1 \quad \text{and}$$

$V_{out} \ll V_{in}$. Thus, in this case, high frequency signals are heavily attanuated or"filtered out" while the low frequency signal pass with little or no attenuation(Fig. 4).

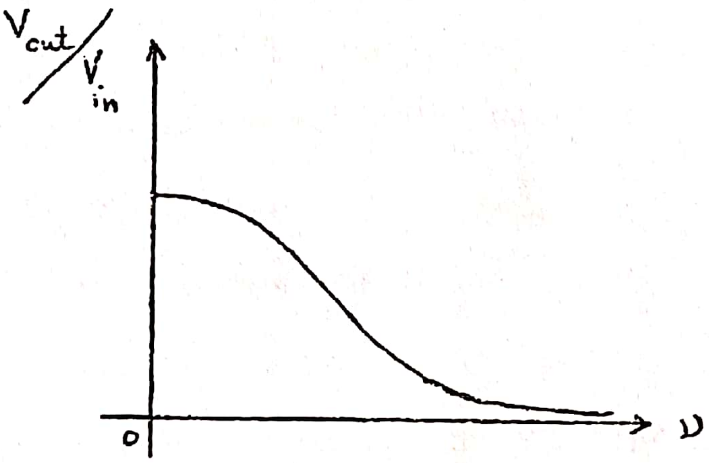

Fig. 4

<u>Upper Cut-off Frequency</u>

The frequency "ω_2" at which $X_C = R$ is sometimes called the upper cut-off frequency for a low-pass filter.

Thus $$X_C = \frac{1}{\omega_2 C} = \frac{1}{2\pi\nu_2 C} = R$$

yields $$\nu_2 = \frac{1}{2\pi RC} \qquad - (12)$$

The low-pass circuit may be used to remove signal components of frequency above ν_2.

Scanned with CamSca

Procedure: (a) High-Pass RC Filter

(1) The circuit is connected as shown in Fig.5. Your instructor might like to see the connections before you proceed any further.

(2) The oscillator and oscilloscope are switched on and some time is allowed for these devices to "warm up".

(3) A suitable low frequency (say 50 Hz) is selected on the oscillator and the waveform-selector knob of the oscillator is turned to simusoidal output.

(4) The voltage output potentiometer of the oscillator is opened up fully for maximum "V_{in}" to the RC circuit.

(5) The V_{in} and its frequency are seen on the oscilloscope by connecting the terminal marked o in Fig.5 to the vertical deflection on the oscilloscope. This is recorded. Note that the oscilloscope will show the peak to peak voltage.

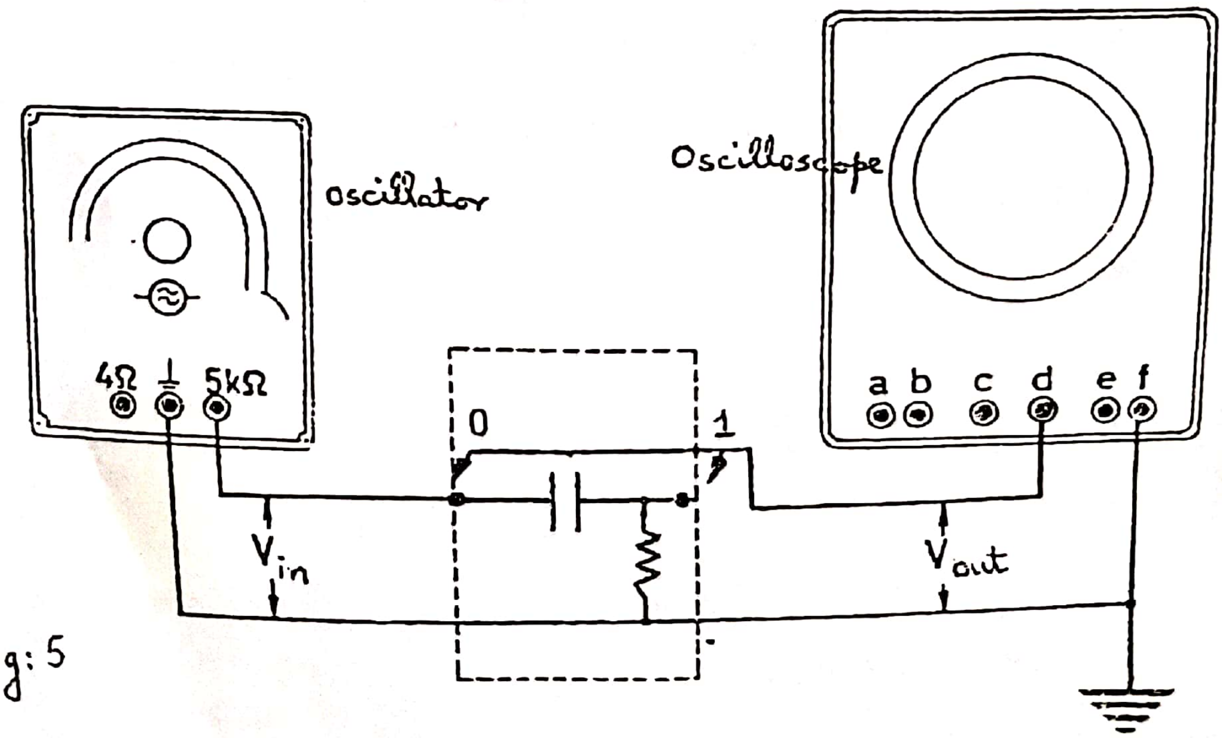

Fig: 5

Scanned with CamSca

(7) The ratio V_{out}/V_{in} is calculated for this frequency and recorded. Note that V_{in} may also change with frequency.

(8) The frequency of the source is readjusted and proceeding in steps of 50 Hz (say), a series of values of V_{out}/V_{in} is obtained corresponding to each frequency chosen.

The frequency is varied over a range 50 - 250,000 Hz. However, the frequency step may be increased or decreased to see some significant changes in V_{out}/V_{in}

(9) The data are tabulated and a graph showing the variation of ν against V_{out}/V_{in} is plotted.

This graph should indicate that low frequencies are heavily attenuated while high frequencies pass with little or no attenuation (Fig.2)

(10) Calculate the lower cut off frequency for this circuit and compare with the value computed from equation (9).

(b) <u>Low-Pass Filter</u>

The circuit is connected as shown in Fig.6 and the experiment is conducted in the same manner as described for High-Pass Filter. A graph is plotted showing the ratio of V_{out}/V_{in} as a function of frequency. The upper cut-off frequency for this circuit is measured and compared with the value calculated from equation (12).

Scanned with CamScanner