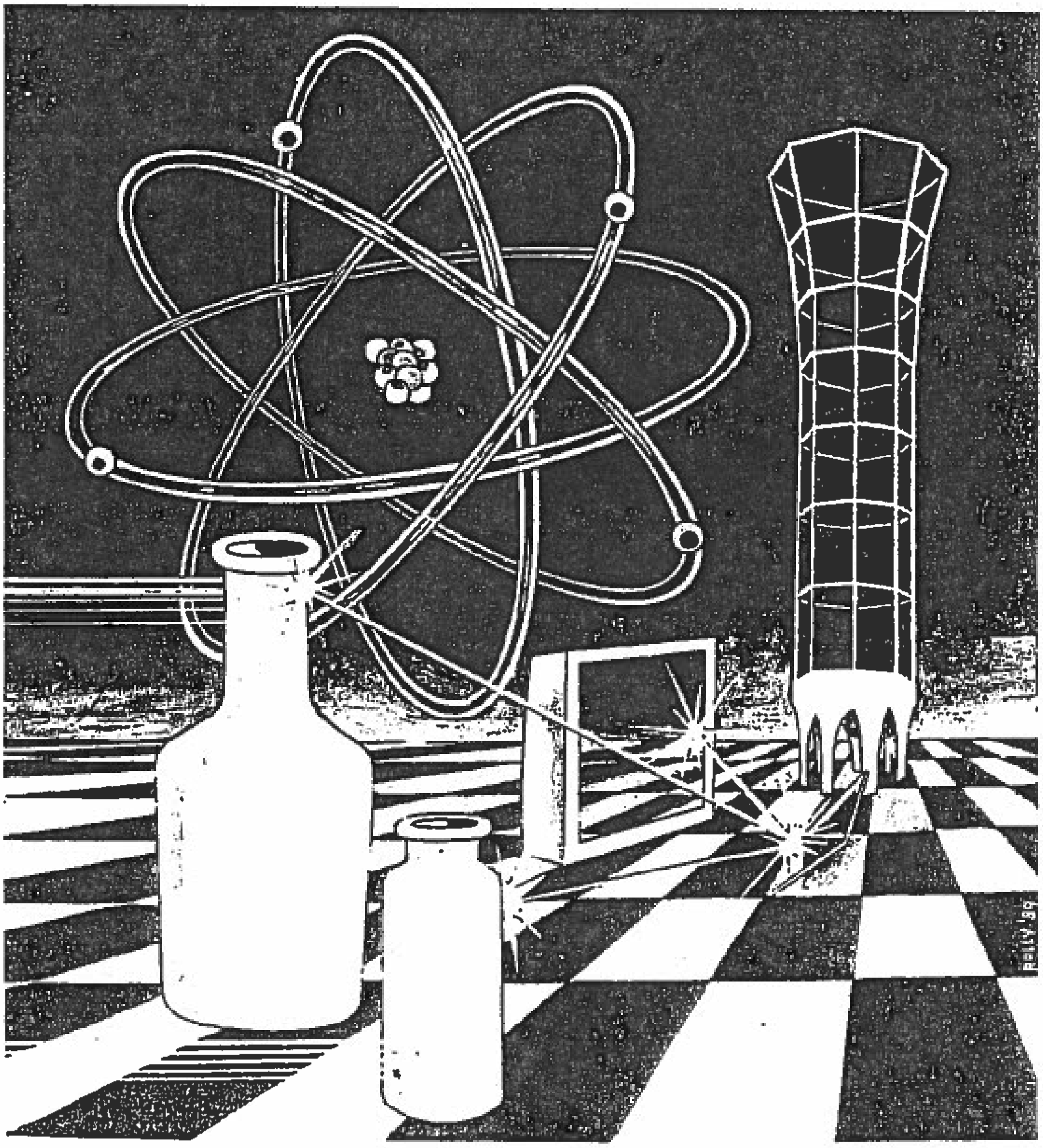
DEPARTMENT OF PHYSICS

PHYSICS 201



LABORATORY EXPERIMENTS

Makader

PHYS 201

lab manual

1995 Edition

Department of Physics
King Fahd University of Petroleum & Minerals
Dhahran 31261
Saudi Arabia

PREFACE

This lab manual contains laboratory experiments for "General Physics III" (Physics 201). These experiments have been designed to further the sophomore students' knowledge of the fundamentals of apparatus manipulation, physical measurements, data recording and analysis aimed at verifying known laws.

This manual is the result of successive efforts by phys 201 instructors over the past ten years or so. The very first edition was prepared by Dr. S. El-Kateb. Further contributions were made by Drs. M. Al-Jarallah, A. Khan, Ph. Martin, and A. AL-Shukri.

It is hoped that both instructors and students will find the present manual useful, for it was written with this express motivation in mind. Also comments and suggestions from both students and instructors will help bring up new experiments and improve existing ones.

Dhahran
June 1995

LABORATORY POLICY

Supplies

1. Students must bring the supplies they need, such as ruled paper, pencils, ruler, eraser and calculator, as none of these items will be provided to them in the lab.

2. Students will only be provided with the required graph paper and such instruments as protractor and stopwatch as and when needed.

Attendance

1. Attending the lab session is compulsory.

2. If and when the total number of absences (excused and/or unexcused) reaches three (3) a grade of "WF" will be assigned for the course.

3. A student absent from a lab session and who submits an official excuse will be allowed to make up the associated experiment, if possible. The case of a student who has no official excuse will be dealt with at the discretion of the instructor.

4. A student cannot make up a lab with an other section without a written request to that effect from his own lab instructor.

Lab Grade

1. The lab work comprises 20% of the total score for the course. The final lab grade will be calculated according to the prevailing policy (section average: 14/20, section standard deviation: 1/20 or greater).

2. Laboratory instructor are free to use a combination of techniques to assess the performance of the students, such as: reports, quizzes and/or lab exams.

Preparation, Lab work, etc...

1. Students should read the write-up of each experiment before coming to the lab.

2. All experiments have been designed so that they can be completed within the allotted time (3 hours). A students does not need to do any work at home.

3. Students should arrive in time for their lab. Late arrival will be dealt with at the discretion of the instructor.

4. A student found in possession of an old lab report during a lab session will get a zero for that lab irrespective of whether he used that lab report or not.

5. Students are required to leave the equipment in a proper state after they finish an experiment. Electrical appliances, if used, should be switched off and disconnected, etc... Failure to do so might result in a penalty, at the discretion of the instructor.

TABLE OF CONTENTS

1. DATA AND ERROR ANALYSIS

OBJECT:

To review various aspects such as the meaning of the word "error", absolute and relative uncertainty, the propagation of errors, mean value, standard deviation, and proper graph, linear and logarithmic scales.

APPARATUS:

No measurement to be done. The various measurements are given in the text. Students are to analyse these results in order to exercise the various aspects of error analysis.

THEORY:

There is always some uncertainty in a measurement basically for two reasons:

(1) Statistical fluctuations in measured quantity, and
(2) Inaccuracies in our measurements.

In this text the word "error" refers to such uncertainty. For example, the height of a water column coming out of a water hose fluctuates with time due to changes in the water pressure in the hose. Measurement of the height, no matter how accurate, will have a statistical distribution about a mean value. On the other hand, if we want to measure the length of a fish swimming in the water, measurements will be inaccurate due to the motion of the fish, even though the length of the fish remains unchanged.

Here, let us name the quantity to be measured x and its mean value x_o. Absolute uncertainty Δx and relative uncertainty $\Delta x/x_o$ are commonly used as a measure of the accuracy of our measurements.

Absolute uncertainty, Δx, in this text will have a loose meaning, in that it will sometimes be termed "standard deviation", "absolute error", "root mean square", "r.m.s", etc. The value of Δx is assigned according to the circumstances. For example, if the statistical distribution of x_i $(i=1,2,\ldots n)$ is given, then x_o and Δx are defined as:

$$x_o = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \text{(mean value, or average value)}$$

and

$$\Delta x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - x_o)^2}{n - 1}} \qquad \text{(standard deviation)}$$

However, in some cases, there are not enough data to find x_o and Δx from the above formulas. We may only have a single reading: for example, measuring the time it takes for the sound of lightning to reach the observer after the lightning strikes. Since no similar event exists, the uncertainties are assigned on the basis of the measuring devices and methods employed. If we used an ordinary wrist watch the accuracy could not be better than a second, therefore, $\Delta t \approx 1$ sec.

PROPAGATION OF UNCERTAINTIES:

If the result we seek, say z, is a function of several measurable quantities, such as x,y,t, etc., then we adopt with caution, the following rule to find the absolute uncertainty, Δz, in terms of Δx, Δy, Δt, etc.:

$$\Delta z = \left| \frac{\partial z}{\partial x} \right|_o \Delta x + \left| \frac{\partial z}{\partial y} \right|_o \Delta y + \left| \frac{\partial z}{\partial t} \right|_o \Delta t + \ldots$$

where the absolute values of the partial derivatives are evaluated at the mean values x_o, y_o, t_o, etc., and Δx, Δy, Δt, etc. are the absolute uncertainties for the corresponding measurable variables.

Example: Centripetal force F is given by:

$$F = \frac{mv^2}{r}$$

Here we assume m, v, and r are measurable variables. Then, ΔF and $\Delta F/F_o$ are found as follows:

$$\Delta F = \left| \frac{\partial F}{\partial m} \right|_o \Delta m + \left| \frac{\partial F}{\partial v} \right|_o \Delta v + \left| \frac{\partial F}{\partial r} \right|_o \Delta r$$

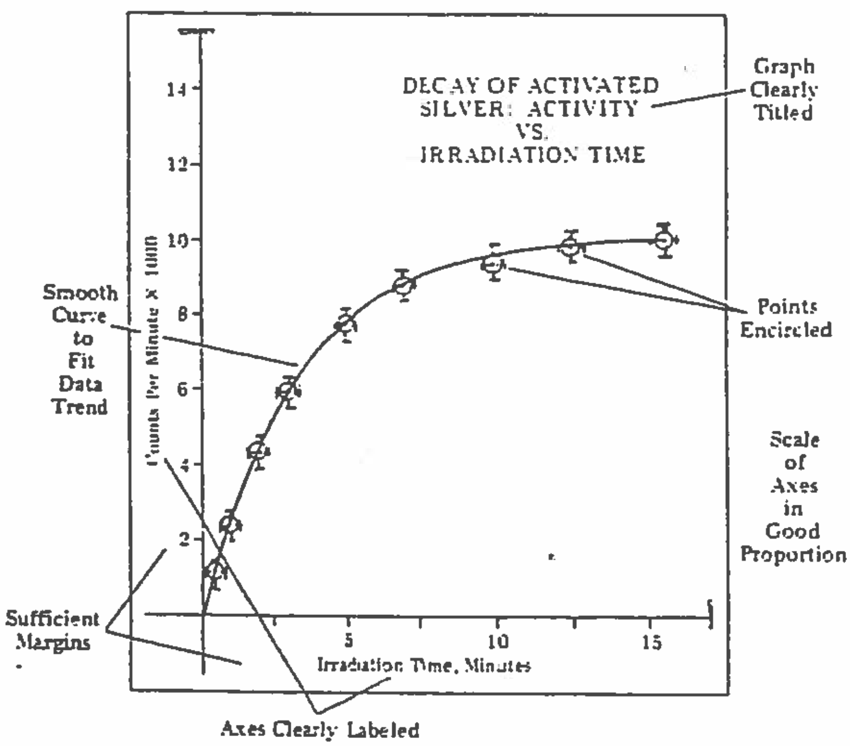
$$\Delta F = \frac{v_o^2}{r_o} \Delta m + \frac{2m_o v_o}{r_o} \Delta v + \frac{m_o v_o^2}{r_o^2} \Delta r, \quad \text{and}$$

$$\frac{\Delta F}{F_o} = \frac{\Delta F}{\left(\frac{m_o v_o^2}{r_o}\right)} = \frac{\Delta m}{m_o} + 2\frac{\Delta v}{v_o} + \frac{\Delta r}{r_o}$$

Here m_o, v_o, r_o, and F_o are the mean values, and Δm, Δv, Δr and ΔF are the absolute uncertainties. Also notice that the last term is taken to be positive because of the absolute partial derivatives.

THE PROPER GRAPH:

When plotting a graph, the following rules are recommended:



A Proper Graph.

The figure given above is an example of a graph plotted on a linear scale. In some instances, instead of numbers their logarithms are used on a linear scale, or, equivalently, the numbers themselves are used on a logarithmic scale. Exercise 4 demonstrates this point nicely. There are basically two reasons for using a logarithmic scale:

(1) If the numbers used span too large of a region, such as from 1 to 10^6, then by using their logarithms (or by using a logarithmic scale) the range is transferred from 0 to 6. This allows us to see the small as well as the large variations in the numbers on a single scale.

(2) It allows us to obtain a linear graph between y and x connected to each other through

$$y = \beta\, x^{\alpha}.$$

Since $\log y = \log \beta + \alpha \log x$, $\log y$ vs $\log x$ will yield a linear graph, in that the origin corresponds to $y = x = 1$ ($\log 1=0$), $\log \beta$ is the intersection, and α is the slope of the line. Refer to exercise 4 for more detail. In the following, five exercises are given to practice the various aspects of error analysis described above.

Exercise 1

In a calorimetry experiment to measure the latent heat of fusion of water, L, a known mass of ice, m_i, is added to a known mass of water m_w, contained in an insulated beaker. The initial and final temperatures of the water are recorded and L is computed from the equation

$m_i L + m_i T_2 = m_w(T_1 - T_2)$ calories, where

m_i = mass of ice = 14.2 ± 0.1 g

m_w = mass of water = 72.3 ± 0.1 g

T_1 = initial temperature of water = $(25.4 \pm 0.1)^{\circ}C$

T_2 = final temperature of water = $(7.8 \pm 0.1)^{\circ}C$

Calculate L (in cal/g), the absolute uncertainty in L, and the percentage uncertainty in L.

Exercise 2

The index of refraction, n, of a material can be calculated by measuring the angle of minimum deviation, D, of a parallel beam of light incident on a prism of apex angle, A, made of the material, and making use of the formula

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If the angle A is measured to be $60^{o} \pm 2'$, and the angle D is measured to be $23^{o}14' \pm 2'$, calculate values of n and δn.

Exercise 3

The fundamental frequency of oscillation, f, of a sonometer wire of length ℓ under a tensile force T is given by

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\ \text{Hz}.$$

where μ is the mass per unit length of the wire. The following table gives the results of an experiment in which the tension in the wire was changed and the length of the wire was varied until, for each value of T, the sonometer vibrated at a fundamental frequency f = 100 Hz (exactly).

(T±0.01) Nt	2.00	4.00	6.00	8.00	10.00	12.00	14.00
(ℓ±0.05) m	0.39	0.52	0.64	0.73	0.85	0.93	1.00

Plot an appropriate graph of the data to indicate a linear relationship. From the graph obtain a maximum, a minimum, and a best slope, and calculate values for μ and $\delta\mu$.

Exercise 4

Certain vacuum tube diodes exhibit current-voltage characteristics which may be represented by the equation

$I = A\ V^n$ where

I = current in amperes

V = voltage in volts

A and n = constants.

In order to determine the values of the constants A and n for a particular diode, the current through the diode was measured for various voltages across the diode. The following table lists the results.

V(volts) ± 10%	5	10	15	20	30	40	60
I (milliamp.)	6±1	16±2	33±3	47±4	85±5	148±10	270±20

Plot an appropriate graph of the data to indicate a linear relationship. From the graph obtain values for n, A, and δn using the techniques employed in exercise 3. Comment on the difficulties in determining δA.

Exercise 5

When measuring the radius 'a' of the capillary of a viscosimeter, the following measurements are obtained:

a = 0.085; 0.087; 0.085; 0.086; 0.085; 0.087; 0.086; 0.085; 0.086; 0.085 cm
Calculate the average $\bar{a}$ and the r.m.s. error e.

2. DATA ANALYSIS USING LEAST SQUARES

DISCUSSION OF PRINCIPLES

An experiment performed in a lab often results in a set of points which appear to lie on a straight line. For example, one might measure the velocity of a falling ball dropped from a height y_0. The ball's velocity will be $v(t) = v_0 - gt$ and its position will be $y = y_0 + v_0t - gt^2/2$, as shown in Figure 1. If we graph v vs. t we expect a straight line.

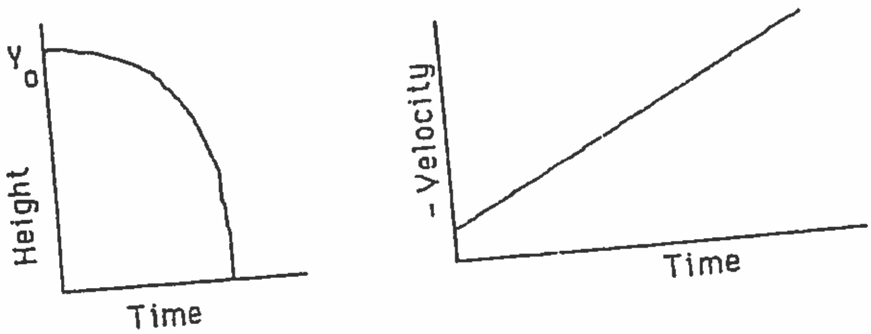


Figure 1.
Height vs time and velocity vs time for a falling object.

When measurements are collected in an experiment, the data will not generally lie exactly straight line. One expects a spread of points which might look like this:

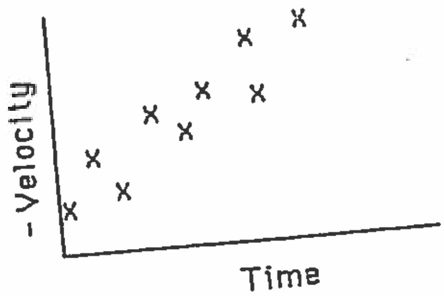


Figure 2.
Measured velocity vs. time for a falling ball. The points are spread around a straight line.

The points in figure two appear to lie on a straight line. The student or experimenter draw the "best" straight line through the data points, and this line is then the result. O quite difficult to decide what is the best straight line. Yet the choice of a line is very since results are obtained from it. For example, we might do the experiment described order to determine the initial velocity and measure g. Remember that in the real world we

v_0 exactly, and g is also a measured quantity which does, in fact, vary slightly from place to place on Earth's surface. Once we have our best straight line, v_0 is the intercept on the v axis and g is the slope. Our experimentally determined values of v_0 and g can be no more accurate that our choice of best straight line.

Fortunately, there is a straightforward technique for finding the best straight line every time. This technique is called the Method of Least Squares. Least squares is possibly the most commonly used technique for analyzing data in scientific research, engineering, and many other areas. Moreover, least squares can also be used to fit data which is not on a straight line. You will be shown how to do this later.

PROCEDURE A: How Does Least Squares Work?

The development of least squares is relatively straightforward, but the algebra is a bit tedious at some points. You should be aware at the outset that many hand calculators have built in routines to do the actual number crunching for you. Even so, least squares will be more meaningful to you if you understand the principles of its operation.

To begin the development of least squares, suppose we have a set of points (x_1,y_1), (x_2,y_2), ...,(x_n,y_n) which lie approximately on a straight line. We will simply write that the equation for the best straight line through these points is

$$y = mx + b.$$

Notice that we have not really done anything yet, since we have not said what m and b are.

Now consider a particular data point (x_1,y_1). This point says that we measured a variable y when we knew the value of x. We found that $x = x_1$ implies $y = y_1$. The value of y predicted by our best straight line is

$$y_{1p} = mx_1 + b,$$

where the subscript p stands for predicted. Since the experimental value of y is y_1, our best straight line is in error by

$$\text{Error} = E_1 = | y_{1p} - y_1|.$$

We will look at the error squared, or

$$E_1{}^2=(y_{1p} - y_1)^2.$$

Similarly, y_2 is the experimental value of y when $x = x_2$ and y_{2p} is the value of y predicted by our best straight line when $x = x_2$. Thus $E_2{}^2$ is the error at point two squared.

$$E_2{}^2 = (y_{2p} - y_2)^2.$$

In general, for the ith point, we have

$$E_i{}^2 = (y_{ip} - y_i)^2.$$

Now we will look at the sum of the squares of all the errors we have made.

$$E^2 = E_1{}^2 + E_2{}^2 + E_3{}^2 + ... + E_N{}^2 = \sum_{i=1}^{N} E_i{}^2$$

We can now plug in what $E_i{}^2$ is.

$$E^2 = \sum_{i=1}^{N} (y_{ip} - y_i)^2 = \sum_{i=1}^{N} (mx_i + b - y_i)^2$$

Notice that we still have not said anything about what m and b are. But we do have an expression for E^2 which is a measure of how far off our best straight line is. you should be able to see that E^2 measures how closely the best straight line $y = mx + b$ fits the data points. Notice that the expression for E^2 contains only m, b, and the given data points.

A good line would result in a small value of E^2, and it follows that the best line would result in a minimum value of E^2. How can we make E^2 a minimum? More precisely, we seek the values of m and b which make E^2 a minimum. There is a mathematical technique to do this. Those students who have taken advanced math courses will know that one takes the partial derivatives of E^2 with respect to m and b and sets these derivatives equal to zero. Those students who are not familiar with partial deivatives do not need to be concerned. The student only needs to understand that we have found expressions for values of m and b which make E^2 as small as possible. These expressions are:

$$m = \frac{N\sum_{i=1}^{N} x_i y_i - \left(\sum_{i=1}^{N} x_i\right)\left(\sum_{i=1}^{N} y_i\right)}{N\sum_{i=1}^{N} x_i{}^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \qquad b = y - mx$$

Notice that we are now finished. All we have to do is plug our measured data (x_1,y_1), (x_2, y_2), ... (x_n,y_n) into the expressions above and we will have found m and b. We then have our best straight line.

As was said before, many calculators will do much of the work for us. Usually one just enters the x's and y's and the calculator will display m and b. To determine whether your calculator does least squares, consult your calculator's instruction book. You might look in the index under least squares, linear regression, or curve fitting. The student should learn how to use the least squares routine in his or her calculator (if it has one).

PROCEDURE B: The Need for a Graph.

One common mistake is to apply least squares without making a graph first. This is not a good idea. One should graph data before applying least squares for at least two reasons. First, one should verify that the data does fall approximately on a straight line. Second, if one data point is way out of line as in the example graph below, the values of m and b will be off quite a bit. This can happen if a mistake is made in a measurement, for example.

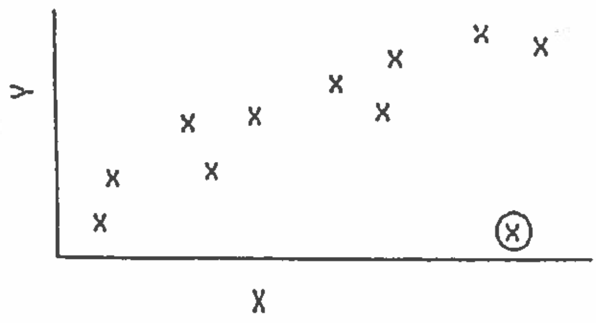


Figure 3.
In the example drawn above the bad data point would be discarded. However, be aware that it must be <u>obvious</u> that a data point is a mistake before it can be discarded.

EXERCISE 1

A ball is thrown upward with some initial velocity v_0. We know from mechanics that its velocity as a function of time is given by

$$v = v_0 - gt,$$

where v_0 is the initial velocity and g is the gravitational acceleration. An experiment was performed which resulted in the data below. Using the graphing instructions you have been given, graph the data and obtain v_0 and g from your graph. Next apply least squares to find the best value of v_0 and g. Compare the values of v_0 and g obtained from your graph to those obtained from least squares.

V	t
96	0.0
86	1.8
78	2.5
61	3.6
10	4.1
43	6.1

Constants from graph:

V_0 = ________

g = ________

Constants from least squares:

V_0 = ________

g = ________

PROCEDURE C: Data for a Non-linear Relationship

Earlier we said that least squares could be applied to data which does not fall on a straight line. This is done by making a change of variables such that the new variables lie on a straight line. The procedure is best illustrated by an example.

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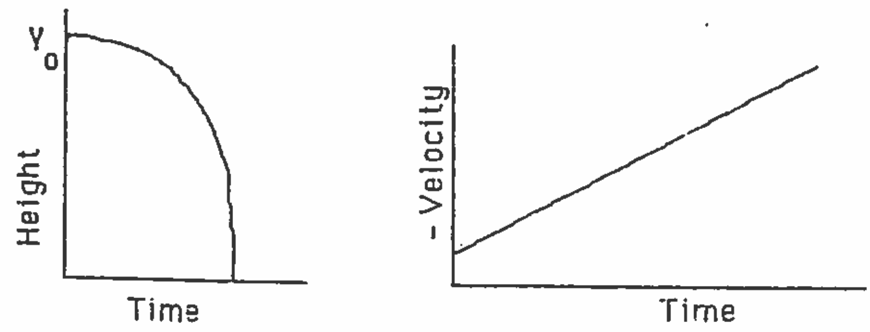

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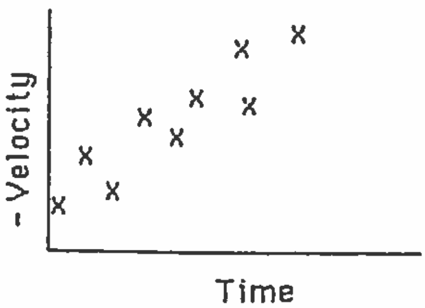

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Measured velocity vs. time for a falling ball. The points are spread around a straight line.

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$$E_2{}^2 = (y_{2p} - y_2)^2.$$

In general, for the ith point, we have

$$E_i{}^2 = (y_{ip} - y_i)^2.$$

Imagine that you are a space traveler in a not too nearby galaxy no time soon. You have landed on a strange (very) planet named llih lepahc. Your mission is to measure the gravitational constant g on the surface of llih lepahc. You notice that one can drop a ball from a set height and the time it takes to fall is related to g. If one drops the ball from a height y_0 at time t=0 with no initial velocity, its height at time t is

$$y = y_0 - gt^2/2$$

Suppose you were to set up an experiment to measure the ball's height at various times after its release. As explained in the measurements section of the lab manual, we would not simply measure y and t once and then find g. We need to find y and t at several points. A graph of y vs. t might look like the one drawn in figure 4.

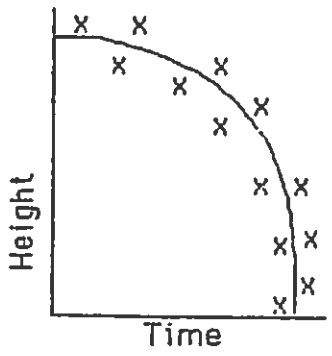

Figure 4.
Height vs. time for a falling object. The x's are data and the line is our estimate of the best smooth curve through the data.

The parabola is our estimate of the best curve through the data. But how can we measure g from this curve? It is possible to do so. (Can you think of a way? hint: think about tangent lines to the curve at various points.) However, such a procedure would be clumsy and difficult. One would like to have a procedure like least squares for a parabola. We could then find the best values for g and y_0. To this end, consider what happens if we graph y vs. t^2. We can think of a new variable x where $x = t^2$. We then have two equivalent equations for the parabola.

$$(1) \quad y = y_0 - gt^2/2$$

is equivalent to

$$(2) \quad y = y_0 - gx/2.$$

Notice that the second form is that of a straight line. See figure 5.

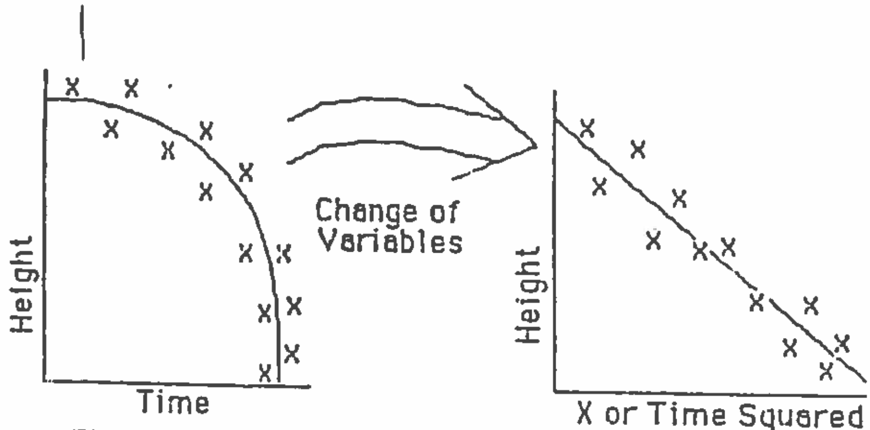

Fig 5a

Fig 5b

Figure 5.
The same data is graphed two different ways. Figure 5a shows height vs time while figure 5b shows height vs time squared. Least squares can now be applied to the data in 5b.

We would want to make a data table which looks like the one below.

Y (meters)	t (sec)	X (sec^2)

Next we would make a graph of y vs x and we expect a straight line. We could then apply least squares to the line y vs. x. Recall that least squares will give us the best slope and the best intercept for our data. Notice that the slope of the line

$$y = y_0 - gx/2$$

is just g/2. Thus least squares has provided us with the best value of g according to our data.

This same procedure of changing variables can be used over and over again to apply least squares to a wide variety of experiments. You will be called upon to make a change of variables and apply least squares several times this semester.

EXERCISE 2

It is found that an experiment yields data of the form $Y = Y_0 \exp(-t/t)$ where Y_0 and t are constants to be evaluated from the data. By making a suitable change of variables, apply least squares to the data below to find the best values of Y_0 and t according to our data. As always, you must first make a graph of your data. To help illustrate the procedure, graph Y vs t and your new variable Z vs t. Hint concerning the change of variables: Begin by taking the natural log of the given equation.

Z	Y	t
	181	1.1
	134	2.5
	60	3.2
	13	7.2
	2.0	10.70

Change of Variables:
What is Z as a function of Y?

Constants determined from least squares fit:

Y_0 = ________

τ = ________

SUMMARY

(1) Least squares provides an accurate way to get the best straight line through a set of data points.

(2) The basic idea behind least squares is to minimize the sum of the squares of the difference between predicted and observed values of data.

(3) Many calculators have least squares routines to quickly evaluate data.

(4) Even when least squares is used, a graph should be made <u>first</u> so one can see if the data is approximately on a straight line, and to identify points which are obviously mistakes.

(5) Least squares can be used for data which does not fall on a straight line if a change of variables can be found such that the new variables will lie on a straight line. There are also more advanced techniques which apply least squares directly to data which is not on a straight line. These more advanced techniques, which will not be covered in this course, can be used even if no suitable change of variables can be found.

QUESTIONS FOR DISCUSSION

1. Suppose you were to apply least squares to only two data points. What do you think would happen? Would the results be meaningful?

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least-square equations (case of straight line)

$$y_i = a + bx_i$$

$$a = \frac{1}{\Delta}(\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i) \quad \text{and} \quad b = \frac{1}{\Delta}(N \Sigma x_i y_i - \Sigma x_i \Sigma y_i)$$

$$\Delta = N \Sigma x_i^2 - (\Sigma x_i)^2$$

least-square equations (case of straight line)

$$y_i = a + bx_i$$

$$a = \frac{1}{\Delta}\left(\Sigma x_i^2 \, \Sigma y_i - \Sigma x_i \Sigma x_i y_i \right)$$

$$b = \frac{1}{\Delta}\left(N \, \Sigma x_i y_i - \Sigma x_i \, \Sigma y_i \right)$$

$$\Delta = N \Sigma x_i^2 - (\Sigma x_i)^2$$

3. CURRENT BALANCE

OBJECT

-- To investigate the magnitude and direction of the magnetic force experienced by parallel current carrying conductors.

-- To determine the value of the permeability of free space μ_o.

THEORY

A conductor carrying a current establishes a magnetic field about itself. For an infinitely long and straight cylindrical conductor (e.g. a wire), the field lines are in the form of concentric circles with the conductor at the center. At a perpendicular distance 'r' from the conductor, the field has a magnitude given by:

$$B = \frac{\mu_o I}{2\pi r} \qquad 3.2 \text{ mm} \tag{1}$$

where B is the magnetic field, I is the current and μ_o is the permeability of free space. (It is assumed in the equation that the conductor is situated in air, for which the permeability is approximately equal to that of vacuum).

When a current carrying conductor is located within a magnetic field established by some other source, it will experience a force and will move if not constrained. The force is given by:

$$\vec{F} = \int_0^L I\vec{dL} \times \vec{B} \tag{2}$$

where $I\vec{dL}$ is a current element of the conductor having the direction of the current. Note that at all points along the conductor, the force is perpendicular to both current direction and field direction.

If the conductor in question is long and straight and if the external magnetic field is uniform (having same direction and magnitude) along the length of the conductor and is directed perpendicular to I, then the force experienced is:

$$F = ILB \tag{3}$$

When two current carrying conductors are in proximity, an interaction will occur : each conductor sets up a magnetic field which produces a force on the other. If the conductors are again, long and straight, and if they carry the same current, then the force experienced per unit length by each can be readily shown to be:

$$F = \frac{\mu_o}{2\pi} \frac{L}{r} I^2 \tag{4}$$

where r is the separation between their centers. The conductors will either be attracted to each other or repelled by each other depending upon the relative directions of the current through them.

Note that in all of the equations indicated, the units are MKS.

In this experiment, a current balance will be used to examine the relationship between the direction of the force experienced by parallel straight conductors and the relative direction of the current through them. Then, the variation of F as a function of I^2 will be investigated for the particular apparatus supplied, where the conductors are in fact not infinitely long.

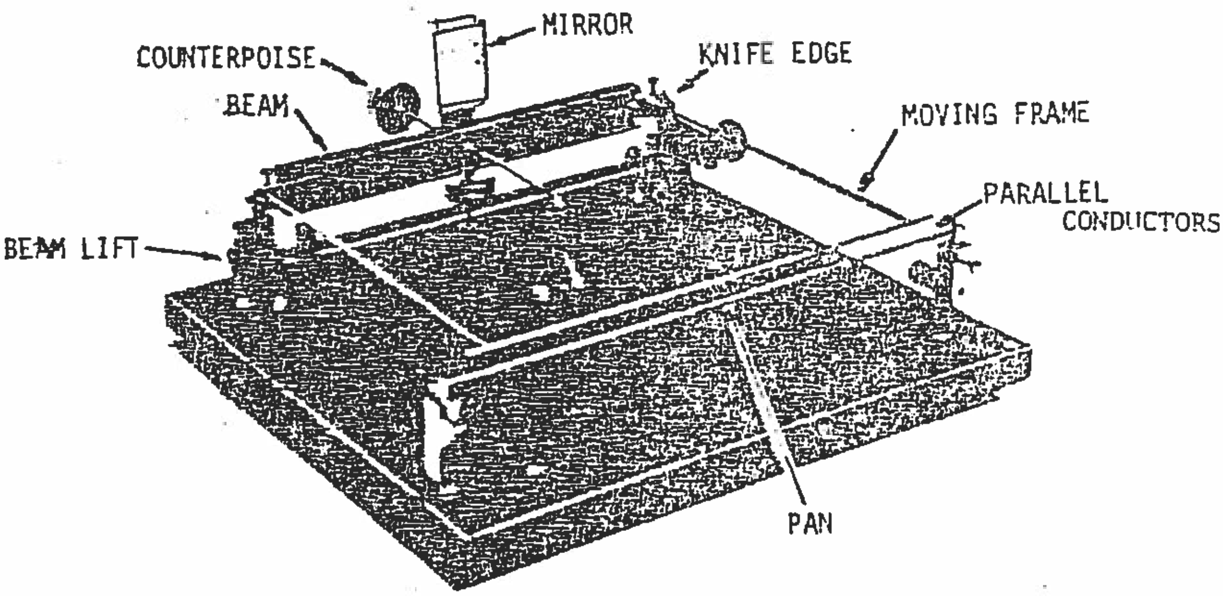


Figure (1)

EXPERIMENTAL PROCEDURE AND ANALYSIS:

STUDY 1 : DIRECTION OF THE MAGNETIC FORCE:

(1) Level the base of the current balance.

(2) Position the beam via the attached knife-edges on the back support and adjust the positioning of both counterpoise weights until the center of the upper conductor is approximately 0.5 cm above the center of the lower, fixed conductor. Ensure that the clamping vane is not in contact with the magnets on the base of the apparatus.

(3) Connect the circuit shown in Figure (2). Ensure that the lead wires connected to the binding posts on the balance leave the posts at right angles to the rods of the moving frame.

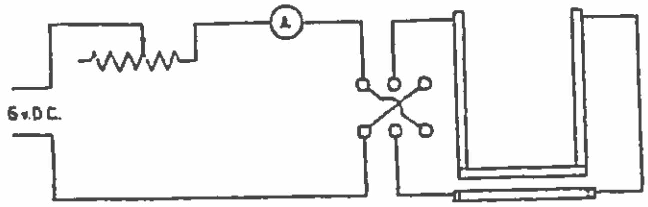


Figure (2)

(4) Pass a 5A current through the balance and note the direction of motion of the upper conductor. Relate the direction of the force between the conductors to the relative direction of the current through them.

(5) Change the lead connections on the balance so as to "reverse" the relative direction of the current through the conductors. Repeat step (4).

NOTE: Do not leave the upper conductor suspended when not in use for any length of time. Rest the moving frame on the beam lift.

STUDY 2 : MAGNITUDE OF THE MAGNETIC FORCE

(1) Measure the length L of the upper conductor. Note that this length is the distance between the supporting bars.

(2) Align the two bars. By careful adjustment the two bars should be aligned as accurately as can be determined by the unaided eye when viewed from the front and from the top.

(3) Adjust the counter-poise until the upper bar (conductor) is few millimeters above the lower bar.

(4) Set up the telescope and scale 1 to 3 meters from the mirror. Adjust the telescope until you can see the scale clearly. Record the equilibrium point indicated by the cross hairs on telescope scale.

(5) In increments of 5 mg, place weights in the pan. Adjust the current until the scale reading returns to its equilibrium value and record the current. Note that the magnetic force experienced by the conductor is, therefore, equal to mg where m is the mass added to the pan and g is the gravitational acceleration.

Note:

For each mass increment reverse the direction of the current through the balance by means of the switch. Adjust the current until the scale reading returns to its equilibrium and record the current. Calculate the average value of I and use this average value in the rest of the experiment. Can you explain the importance of reversing the direction of current?

(6) Find the center to center distance r between the bars at equilibrium. This is determined as follows: the scale reading at equilibrium is noted. Then the upper bar is depressed by placing a small coin on the scale pan until it is in contact with the lower bar and the new scale reading is noted. The distance, r_o, between the two bars is given by:

$$r_o = \frac{Da}{2b}$$

where D is the difference in readings,

a is the mean distance from knife-edge to the bar, and

b is the distance from mirror to scale.

The center to center distance (r) is obtained by adding the diameter of either rods to r_o.

(7) Plot the force F as a function of I^2 and determine the slope of the resultant line.

(8) From the slope and using equation 4 determine the value of the permeability of free space μ_o.

(9) The accepted value of μ_o is $4\pi \times 10^{-7}$ tesla. meter/amp. Calculate the percent error between the experimental and the accepted values and discuss any discrepancy.

4. RC CIRCUITS

OBJECT: To study the frequency response and phase shift in (i) a CR circuit (high-pass)

(ii) an RC circuit (low-pass)

APPARATUS: Oscillator, Oscilloscope, various combinations of resistances and capacitors, connecting wires.

THEORETICAL BACKGROUND:

Filters are devices used to smooth out or eliminate some undesirable time-varying voltages/signals in electronic circuits. An RC series circuit constitutes one of the simplest filters. We shall investigate the characteristics of two distinct RC filters i.e. (a) high-pass and (b) low-pass RC filters. In particular we shall study their frequency response and phase shifts.

(a) High-Pass RC Filter: Consider the simple RC series circuit shown in Fig.1, where a source of alternating emf (constant voltage amplitude but variable frequency) supplies

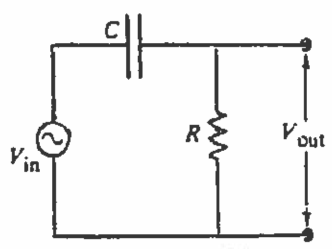


Figure 1: A simple RC high-pass filter.

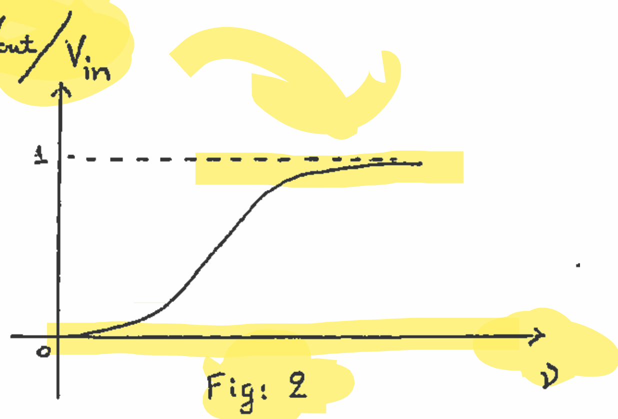


Fig: 2

an input voltage "V" to the RC circuit. Suppose that "V" is of the form

$$V = V_m \sin \omega t \qquad - (1)$$

This creates an alternating current in the circuit given by

$$I = I_m \sin (\omega t - \phi) \qquad - (2)$$

where (i) I_m and V_m are the maximum values of current and input voltage respectively

(ii) $\omega = 2\pi\nu$ is the angular frequency of the input voltage

(iii) $\phi = \tan^{-1} \frac{X_C}{R} = \tan^{-1}\left(\frac{1}{\omega CR}\right)$ - (3)

is the phase difference between "I" and "V_{in}"

We will be interested only in the maximum values of "I" and "V". The maximum input voltage $V_{in}(=V_m)$ is related to the maximum current "I_m" through

$$I_m = \frac{V_{in}}{Z} \quad \text{or} \quad V_{in} = I_m Z \qquad - (4)$$

But $Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ = Total impedance of the circuit

Therefore $$V_{in} = I_m \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} \qquad - (5)$$

The input voltage appears partly as a potential difference "V_C" across the capacitor and partly as a potential difference "V_R" across the resistor

$$V_{in} = V_C + V_R \qquad - (6)$$

If we take the potential difference across "R" as our output "V_{out}" then from Ohm's Law,

$$V_{out} = V_R = I_m R \qquad - (7)$$

Equations (5) and (7) yield the ratio of output voltage to input voltage as:

$$\frac{V_{out}}{V_{in}} = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \qquad - (8)$$

From this equation we notice that at low frequencies ($\omega = 2\pi\nu$), the capacitive reactance $\left(\frac{1}{\omega C}\right) > R$

and "V_{out}" is small compared with "V_{in}". On the other hand at high frequencies $X^2 = \left(\frac{1}{\omega C}\right)^2 << R^2$

and $\frac{V_{out}}{V_{in}} \simeq 1$ or $V_{out} \simeq V_{in}$

Thus the circuit will pass signals of high frequency with relatively little or no attenuation at all. However, the signals of low frequency are heavily attenuated and therefore "filtered out". This circuit is appropriately named "RC high-pass filter". Physically this is a result of the blocking action of the capacitor to low frequencies or direct current. Fig.2 gives the variation of "V_{out}/V_{in}" as a function of frequency in an RC high-pass circuit.

Lower Cut-off Frequency

The frequency "ω_1", at which $X_C = R$, is sometimes called the lower cut-off frequency for a high pass filter. Thus

$$X_C = \frac{1}{\omega_1 C} = \frac{1}{2\pi\nu_1 C} = R$$

yields $$\nu_1 = \frac{1}{2\pi RC} \qquad - (9)$$

The circuit may be used to remove signal components having frequency below ν_1.

(b) Low-Pass RC circuit

Now consider the RC series circuit shown in Fig.3. In this case, the "V_{out}" is taken across the capacitor.

The capacitive reactance $= X_C = \frac{1}{\omega C}$

$$\therefore \quad V_{out} = I_m X_C = \frac{I_m}{\omega C} \qquad - (10)$$

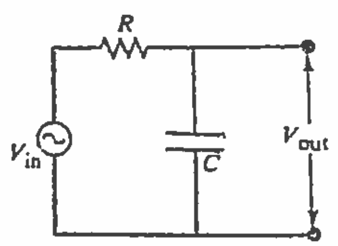


Figure 3: A simple RC low-pass filter.

The ratio of the output voltage to input voltage is (using equations 5 and 10)

$$\frac{V_{out}}{V_{in}} = \frac{1/\omega C}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}} \qquad - (11)$$

This equation shows that when "ω" is small (low frequencies) $R^2 \ll \left(\frac{1}{\omega C}\right)^2$ or $\omega^2C^2R^2 \ll 1$ and $V_{out} \approx V_{in}$

On the other hand when "ω" is large (high frequencies)

$$R^2 \gg \left(\frac{1}{\omega C}\right)^2 \quad \text{or} \quad \omega^2C^2R^2 \gg 1 \quad \text{and}$$

$V_{out} \ll V_{in}$. Thus, in this case, high frequency signals are heavily attanuated or"filtered out" while the low frequency signal pass with little or no attenuation(Fig. 4).

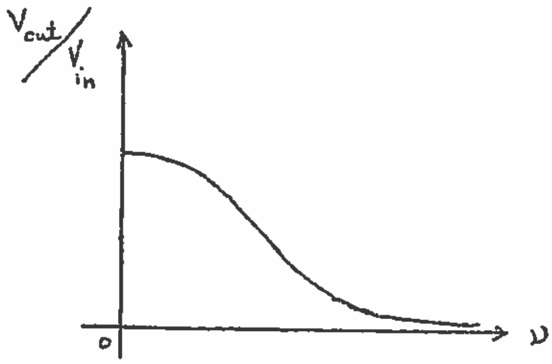


Fig. 4

Upper Cut-off Frequency

The frequency "ω_2" at which $X_C = R$ is sometimes called the upper cut-off frequency for a low-pass filter.

Thus $X_C = \frac{1}{\omega_2 C} = \frac{1}{2\pi\nu_2 C} = R$

yields $$\nu_2 = \frac{1}{2\pi RC} \qquad - (12)$$

The low-pass circuit may be used to remove signal components of frequency above ν_2.

Procedure: (a) High-Pass RC Filter

(1) The circuit is connected as shown in Fig.5. Your instructor might like to see the connections before you proceed any further.

(2) The oscillator and oscilloscope are switched on and some time is allowed for these devices to "warm up".

(3) A suitable low frequency (say 50 Hz) is selected on the oscillator and the waveform-selector knob of the oscillator is turned to simusoidal output.

(4) The voltage output potentiometer of the oscillator is opened up fully for maximum "V_{in}" to the RC circuit.

(5) The V_{in} and its frequency are seen on the oscilloscope by connecting the terminal marked o in Fig.5 to the vertical deflection on the oscilloscope. This is recorded. Note that the oscilloscope will show the peak to peak voltage.

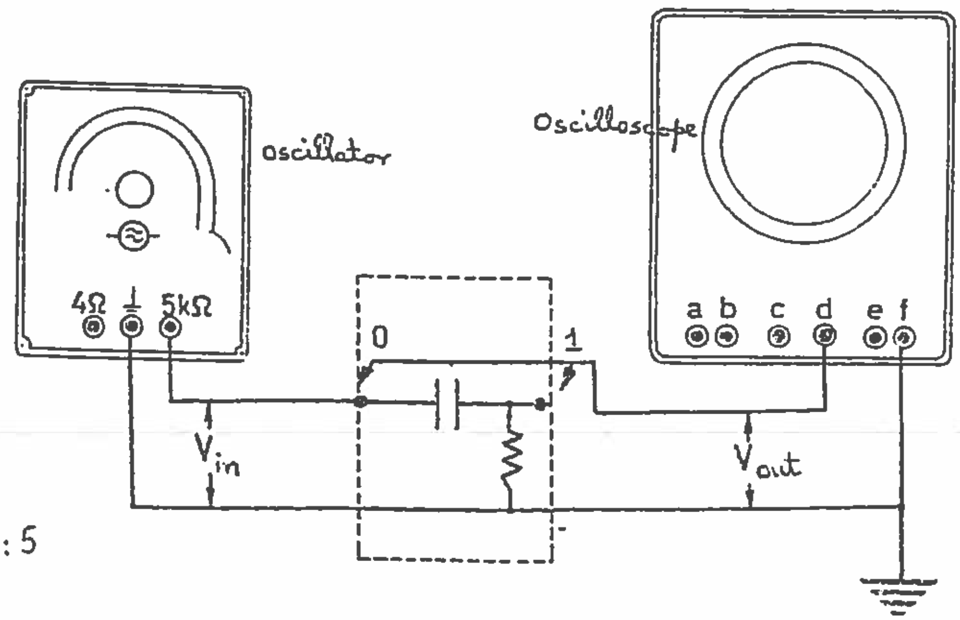


Fig: 5

(7) The ratio V_{out}/V_{in} is calculated for this frequency and recorded. Note that V_{in} may also change with frequency.

(8) The frequency of the source is readjusted and proceeding in steps of 50 Hz (say), a series of values of V_{out}/V_{in} is obtained corresponding to each frequency chosen. The frequency is varied over a range 50 - 250,000 Hz. However, the frequency step may be increased or decreased to see some significant changes in V_{out}/V_{in}

(9) The data are tabulated and a graph showing the variation of ν against V_{out}/V_{in} is plotted. This graph should indicate that low frequencies are heavily attenuated while high frequencies pass with little or no attenuation (Fig.2)

(10) Calculate the lower cut off frequency for this circuit and compare with the value computed from equation (9).

(b) Low-Pass Filter

The circuit is connected as shown in Fig.6 and the experiment is conducted in the same manner as described for High-Pass Filter. A graph is plotted showing the ratio of V_{out}/V_{in} as a function of frequency. The upper cut-off frequency for this circuit is measured and compared with the value calculated from equation (12).

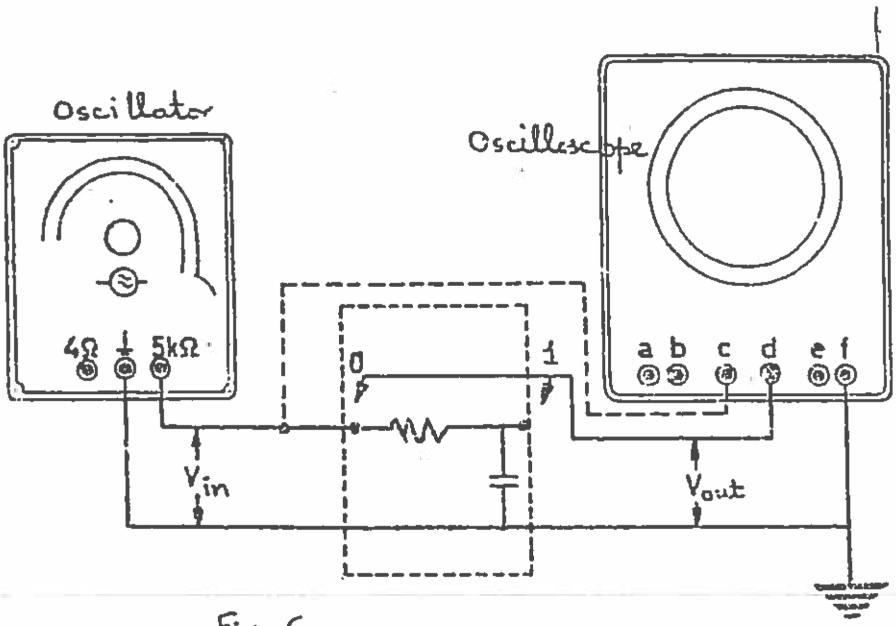


Fig: 6

Phase Shift of Output Voltage - A qualitative study:

This is a study of superposition of two sinusoidal waves of the same frequency propagating in the same direction. Here, we will consider the amplitudes of the two waves to be at right angles to each other and we will suppose that the two waves are out of phase by an angle Φ.

e.g.

$$V_x(t) = V_{xo} \sin \omega t$$

$$V_y(t) = V_{yo} \sin (\omega t \pm \phi)$$

The superposition of these two waves will in general create an elliptical wave. However, if $\phi = 0, \pi, 2\pi, \ldots\ldots$ the result is a straight line, and if $\phi = \pi/2, 3\pi/2, \ldots\ldots$ & $V_{xo} = V_{yo}$ the result is a circle.

The capacitor in the RC circuit introduces a phase shift which is frequency dependent. Thus in Fig.6 if we apply the output from the oscillator (from terminal 0) to the horizontal deflection on the oscilloscope and the output from terminal 1 to the vertical deflection on the oscilloscope, we should see an ellipse on the oscilloscope screen. Here, the horizontal and vertical deflections take place with the same frequency but not in the same phase. The phase angle can be determined from the shape and position of the ellipse. The effect of variation of frequency on the phase shift and the shape of ellipse can be observed. Try to observe a straight line or circle on the screen in this experiment.

Express your observations regarding the effect of change of frequency on phase shift.

5. RLC CIRCUITS

Object: (i) To study the response of an RLC series AC circuit as a function of the frequency of the applied voltage.

(ii) To determine the resonant frequency of a given RLC series circuit.

(iii) To study the quality factor of RLC circuits as a function of "R" in the circuit.

Apparatus: Audio oscillator, inductor, capacitor, resistor, precision AC ammeter, oscilloscope (or precision AC voltmeter), connecting wires.

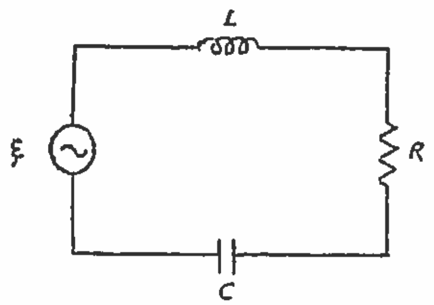
If a source of alternating emf (constant voltage amplitude "ξ_m" but variable angular frequency "ω") given by:

$$\xi = \xi_m \operatorname{Sin} \omega t \qquad - \quad (1)$$

is connected across an RLC circuit (Fig.1), an alternating current "i" flows through the circuit which is given by:

$$i = i_m \operatorname{Sin} (\omega t - \Phi) \qquad - \quad (2)$$

Fig. 1



In the above equations:

- $\omega = 2\pi\nu$ = angular frequency of the source
- "ξ" and "i" are instantaneous values of emf and the current respectively.
- "ξ_m" and "i_m" are maximum values of emf and current respectively.
- Φ = Phase angle between "i" and "ξ".

Since this is a single loop series circuit, we have:

$$i_m = \frac{\xi_m}{Z} = \frac{\xi_m}{\sqrt{R^2 + X^2}} = \frac{\xi_m}{\sqrt{R^2 + (X_L - X_C)^2}} \qquad - \quad (3)$$

Also $\tan\Phi = \frac{X}{R} = \frac{X_L - X_C}{R}$ $\qquad - \quad (4)$

where: Z = total impedance of the RLC circuit

R = resistance in the circuit

$X_L = \omega L = 2\pi\nu L$ $\qquad - \quad (5)$ is the inductive reactance

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$ $\qquad - \quad (6)$ is the capacitive reactance

It is important to note that most AC meters (ammeters/voltmeters) are calibrated to read root mean squared (rms) values and not maximum values. Therefore, it may be convenient to rewrite equation (3) using rms values i.e.

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \qquad - \quad (3.a)$$

where $I = i_{rms} = \frac{i_m}{\sqrt{2}}$ and $V = \xi_{rms} = \frac{\xi_m}{\sqrt{2}}$ $\qquad - \quad (7)$

From equations (5) and (6) above, we notice that the inductive reactance "X_L" is directly proportional to the frequency ω of the source whereas the capacitvie reactance "X_C" is inversely proportional to the frequency. In other words, at low frequencies capacitivie reactance "X_C" predominates and at high frequencies inductive reactance predominates. Fig.2a illustrates this dependance as a function of frequency. A logarithmic frequency scale (log ω) has been used in Fig.2 because of the wide range of frequencies covered. The variation of total impedance "Z" is also illustrated in Fig.2 as a function of frequency.

It is apparent from Fig.2a that there is one particular frequency at which "X_L" and "X_C" are numerically equal. At this frequency $X = X_L - X_C$ = Zero and the impedance is minimum i.e. $Z_{Min} = R$ (the resistance in the circuit).

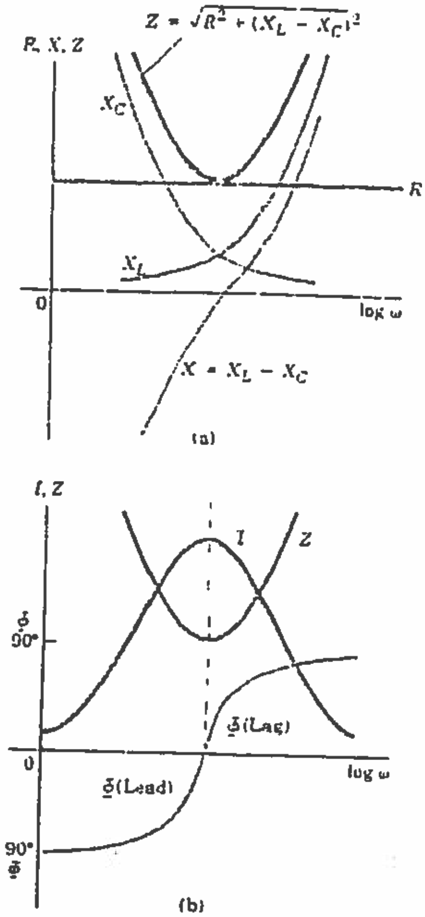


Fig. 2

From equation (3.a) it is clear that as "Z" changes "I" also changes because "V" is constant. Fig. 2.b gives a sketch of variation of "I" as a function of frequency. Variation of "Z" and phase angle "ϕ" is also indicated in Fig.2.b as a function of frequency. Obviously when "Z" is minimum, "I" is maximum. Furthermore, at that point ($Z=Z_{min}$), the current and voltage are in phase i.e. $\phi = 0^o$.

Resonance: An LC circuit (a special case of RLC circuit with R = 0) has a natural frequency of oscillations given by

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad - \quad (8)$$

The presence of R provides damping of these oscillations. When the angular frequency of the alternating source of emf equals the natural angular frequency "ω_o" of the LC circuit, we have the condition of resonance,

$$\omega = \omega_o$$

At this frequency we have the maximum response i.e. maximum current in the case of RLC circuit.

$$\text{Since} \quad I_{max} = \frac{V}{R} \qquad - \quad (9)$$

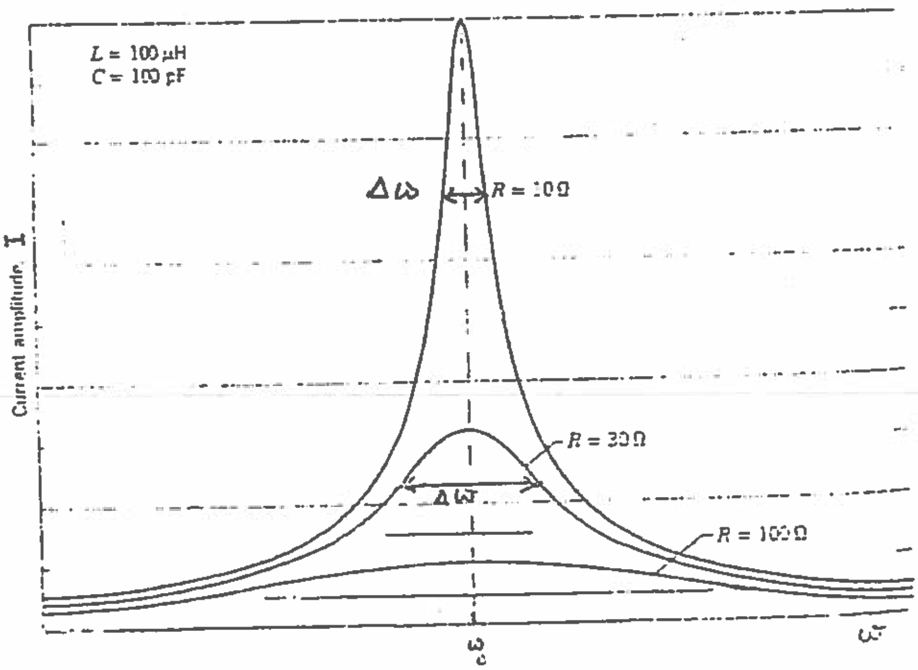
$$\text{when } R \to o \qquad I_{max} \to \infty$$

Real circuits always have some resistance even though no "resistors" are present. This is because of the internal resistance of the inductors and connecting wires etc. etc. Thus "I_{max}" would be limited to some finite, though large, value.

Effect of increasing "R"

When "R" is increased in the RLC circuit, the maximum current for $\omega=\omega_o$ is reduced and the resonance curve "I" versus "ω" is broadened (Fig.3)

Fig. 3



Quality Factor "Q"

The sharpness of the resonance curve is usually described by a dimensionless parameter known as the quality factor "Q". A convenient expression for the Q factor of the RLC series circuit is

$$Q = \frac{\omega_o}{\Delta\omega} \qquad - \qquad (10)$$

where ω_o is the resonance frequency and $\Delta\omega$ is the total width of the resonance curve between the points where the current $I = I_{max}/\sqrt{2} = 0.707\ I_{max}$ - (11)

Other expressions for defining Q-factor are discussed in various text books.

Procedure:

In this experiment we shall use an audio oscillator as a source of variable frequency alternating emf. An oscilloscope or AC voltmeter connected across the audio oscillator will help in checking and maintaining a constant voltage amplitude for the oscillator output. The oscilloscope, if used, may be further useful in calibrating the frequency scale of the audio oscillator.

The following step-wise procedure may be adopted.

1) The circuit is connected as shown in Fig.4 but without any resistance. Your instructor might like to see the connections before you proceed any further.

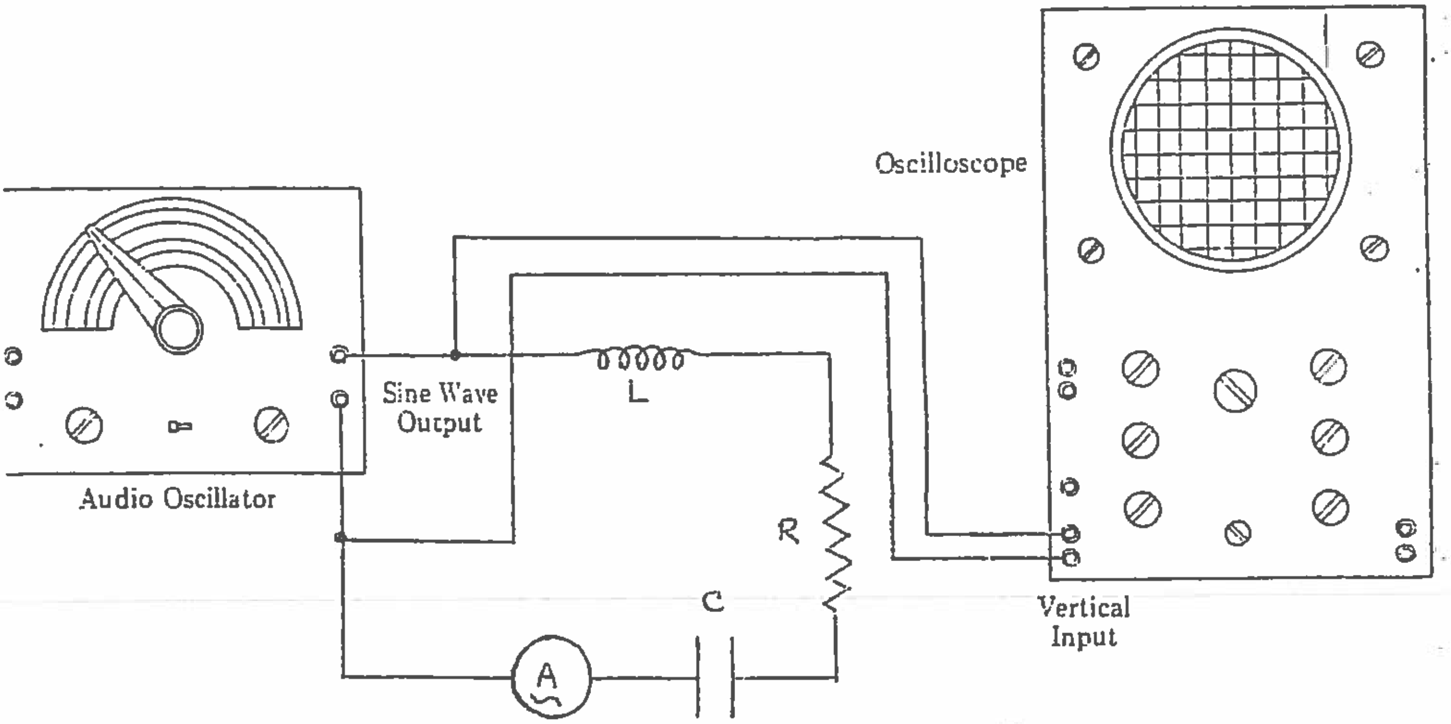


Fig. 4

AC ammeter

2) The audio oscillator and the oscilloscope (if used) are switched on and some time is allowed for these devices to "warm up" and stabilize.

3) The wave-form selector knob of the oscillator is set on si: usoidal output.

4) The output voltage "V" of the oscillator is adjusted to some suitable value (e.g. 10 volts rms). This is read on the oscilloscope or AC voltmeter. This value is to be kept constant throughout the experiment by suitable adjustments of the oscillator output-voltage potentiometer. Note that the oscilloscope will show peak to peak voltage and not rms value.

5) A low value of frequency "ν" (say ν= 50HZ) is selected on the frequency dial of the oscillator. There may be a multiplier knob (X1; X10, x 100 etc.) which should be adjusted for the desired range of frequencies. The input frequency may be checked on the oscilloscope (if used) and in this way the frequency dial of the oscillator may be calibrated.

6) The reading of the ammeter corresponding to this frequency is recorded. This is "I".

7) The frequency "ν" is readjusted and proceeding in steps of (say) 50 HZ the corresponding values of I are recorded, in a tabular form, over a wide range of frequencies through resonance and beyond.

It is important to check and readjust, if necessary, the output voltage "V" from the oscillator (maintaining it constant) after every change of frequency. Furthermore, it is advisable to take several readings with smaller steps of frequency in the region of resonance.

8) The Oscillator and the oscilloscope are switched off.

9) A resistance of 30 Ω (say) is connected in series with the circuit as in Fig.1. The oscillator and oscilloscope are switched on once again.

10) Keeping V the output voltage from the oscillator the same as in step 4, a new set of data for I as a function of ν are obtained as detailed in steps 5 - 7 above. The oscillator and oscilloscope are switched off and the circuit is disconnected.

11) A graph showing the variation of I with angular frequency $\omega = 2\pi\nu$ is plotted as shown in fig.3 for the two cases (a) with R=o and (b) with R≈30Ω.

12) The precise value of resonance frequency $\omega_o = 2\pi\nu_o$ is determined from the graph. This is compared with the value calculated from equation (8) using the values of "L" and "C" used in the experiment.

If the value of "L" is not given, it can be calculated from an alternate expression for Q-factor

i.e. $Q = \frac{\omega_o L}{R} \quad \therefore \quad L = \frac{Q\,R}{\omega_o}$

In this case you may not be required to compare the experimental value of ω_o with the calculated value.

13) The width $\Delta\omega$ of the resonance curve between the points where $I = I_m/\sqrt{2} = 0.707\, I_m$ is calculated for both cases ($R \approx o\Omega$ and $R \approx 30\Omega$)

14) The Q-factor of the circuit is calculated for both cases.

Questions:

1) What is the wavelength of the oscillator voltage at the resonant frequency $\omega = \omega_o$?

2) What is the difference between radio freqency and audio frequency?

3) At resonance, what will be the potential difference across the resistor if the voltage applied to the RLC circuit from the oscillator is 10 volts (rms)?

4) If the value of the resistance "R" is made very large, what will happen to the Q of the circuit?

5) Mention some possible applications of RLC series resonance circuit.

6. THIN LENSES AND SPHERICAL MIRRORS

OBJECT

-- The object of this experiment is to investigate the image formation by spherical mirrors and thin lenses and to determine focal lengths for these mirrors and lenses.

INTRODUCTION

1) Lenses

For both converging or convex lenses (thicker at the center than at the edges) and diverging or concave lenses (thicker at the edges) an object and its image have locations which are related by the lens equation:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \qquad (1)$$

In this equation p is the object distance (measured from the centre of the lens), q is the image distance, and f is the focal length of the lens, i.e. image distance when object is at infinity.

The following sign convention applies to the distances indicated in equation (1)

a) f is positive for converging lenses and negative for diverging lenses.
b) p is positive for real objects and negative for virtual objects.
c) q is positive for real images and negative for virtual images.

A real image is one which can be "caught" on a screen whereas a virtual image is one which must be viewed by looking back through the lens towards the object such as one does when using a microscope. A virtual object can come about when the image formed by one lens serves as the object for a second lens. Virtual images are created by a diverging lens for all real objects positions and by a converging lens when p is less than f in magnitude.

For any object-image configuration, the magnification, M, produced can be determined by comparing image size to object size and also by using the following equation:

$$M = q/p \tag{2}$$

When two lenses are used in combination, the "net" magnification is the product of the two individual magnifications:

$$M_{net} = M_1 \times M_2 \tag{3}$$

2) Mirrors:

A spherical mirror is a small section of a spherical shell. If the reflecting surface is on the same side as the centre of curvature of the sphere, the mirror is termed concave; if the reflecting surface is on the other side, the mirror is termed convex.

For both types of mirrors, the object and image locations are related by the equation:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{r} \tag{4}$$

In this equation, p is the object distance, q is the image distance, f is the focal length, and r is the radius of curvature of the reflecting surface.

The same sign convention as that for lenses applies to p and q. Both f and r are positive for a concave mirror and negative for a convex mirror. Virtual images are created by a convex mirror for all real object positions and by a concave mirror when p is less than f in magnitude. The magnification produced by a spherical mirror for any object-image configuration can be determined in the same manner as that produced by a thin lenses.

APPARATUS

Optical bench, light source, arrow object screen, image screen biconcave lens, biconvex lens, concave mirror and ruler.

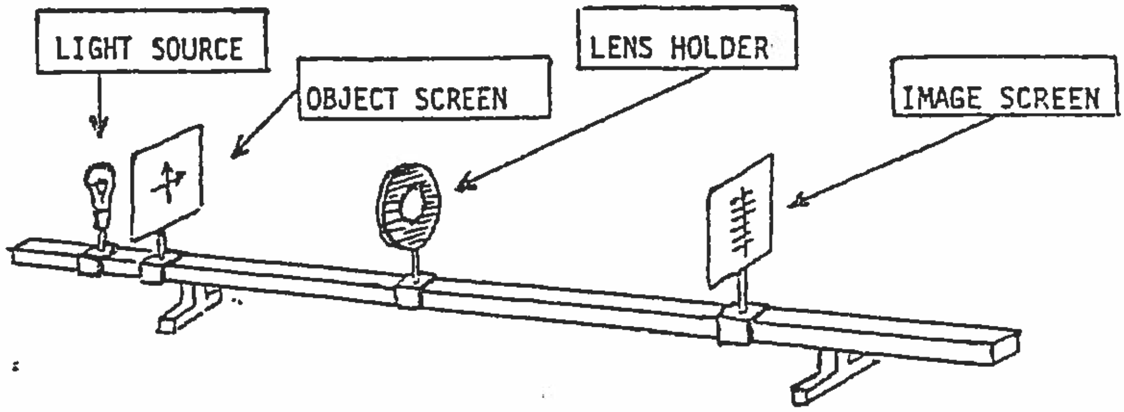


Figure 1

EXPERIMENTAL PROCEDURE

Pare I - Convex Lens:

1. Set up apparatus as shown in Figure 1 with light source, object screen, image screen and convex lens mounted on the bench.
2. With an object distance of 40 cm, slide the image screen along the bench until a sharp image is formed. Note image distance, image size, image orientation (erect or inverted), and object size.
3. Calculate a value of f using equation (1) and value for the magnification by use of equation (2).
4. With an object distance of 70 cm, repeat steps 2 and 3.
5. Calculate an average value for f from the results of steps 3 and 4 above. Then set the object at $p = 2\ f_{av}$ on the bench and find the image distance and image size.

Part II - Concave Lens:

1. Set the light source, object screen, and image screen on the bench.

2. Determine the focal length of the concave lens by the virtual object method: (i) set the convex lens on the bench so that it is at a distance of twice its focal length from the object; (ii) adjust the screen position so that the real image created is in sharp focus; (iii) place the concave lens so that it comes between the convex lens and the image screen. (The real image created by the convex lens now becomes a virtual object for the concave lens); (iv) re-adjust the screen position so that a sharp image is acquired; (v) note the distance between the concave lens and the first image screen location (i.e. the object distance) and the distance between this lens and the second screen location (i.e. the image distance); (vi) note also the second image size and the second image orientation, and calculate a value for the focal length of the concave lens. Finally calculate the net magnification for the two lens combination.

Part III - Concave Mirror

1. Set light source, object screen and concave mirror on the bench. Insert image screen between object and mirror and set it as low as possible in its mounting.
2. With an object distance of 80 cm, slide the image screen along the bench until a sharp image is formed. Note image distance, image size, and image orientation.
3. Calculate a value for f using equation (4) and value for the magnification by use of equation (2).
4. Remove the image screen and find the focal length by the coincidence method: (1) adjust mirror position until sharp image of the object is formed on the object screen (image distance and object distance are then equal); (ii) calculate a value for f.

QUESTION:

For the double convex lens if $\Delta P = 0.2$ cm and $\Delta q = 0.6$ cm. Calculate the fractional error $\frac{\Delta f}{f}$ for this lens.

7. REFRACTIVE INDEX AND COLOUR

OBJECT

-- Meeting the prism spectrometer, the concinuous light spectrum and a line spectrum.

-- Refractive index and dispersion.

-- More on errors and the use of calculus.

-- Absorption spectra.

INTRODUCTION

A hot filament will produce a continuous spectrum of colours (i.e. wavelengths) because of the almost continuous set of energy levels that exists in the solid. In the case of a gas, the atoms are more or less independent of each other and can only radiate fixed wavelengths corresponding to transitions between the quantum energy states of the isolated atoms; this leads to a characteristic line spectrum, and in this experiment you will be using the line spectrum of helium gas in order to take measurements at known wavelengths. You will study line spectra in much greater detail in Experiment (8). At the end of the present experiment you will look at some band spectra of molecules in which the electronic energy transitions produce a band in the spectrum rather than a line because the molecule can exist in different (quantised) energy states of vibration or rotation before and after the light is emitted (or absorbed).

The prism spectrometer can be used to split white light into its component colours from violet to red (and beyond), (as was done by Newton), or to spread out the line spectrum from a gas discharge tube. The ability of a prism to do this is because the refractive index varies with the wavelength, so that different wavelengths are refracted through different angles from Snell's Law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \qquad (1)$$

The refractive index of a given medium, n_1 is a measure of the speed of light in the medium c_1,

$$n_1 = c_0/c_1 \tag{2}$$

where c_0 is the speed of light in vacuum (=299.79 Mm/s); a variation of c_1 with wavelength and frequency (that produces the variation in n_1), is referred to as dispersion, a term that can be used for waves in any medium, e.g. water surface waves. Many wave media are non-dispersive (velocity independent of frequency) e.g. electromagnetic waves in a vacuum from γ-rays to radio waves. Non-dispersive waves met earlier in the laboratory were waves on a string, and sound waves. However, it is important to remember that in general wave speeds are not constant for a given medium.

MINIMUM DEVIATION

With the aid of Snell's Law (equation 1) one can calculate the path of a ray of light through a prism as in Figure 1.

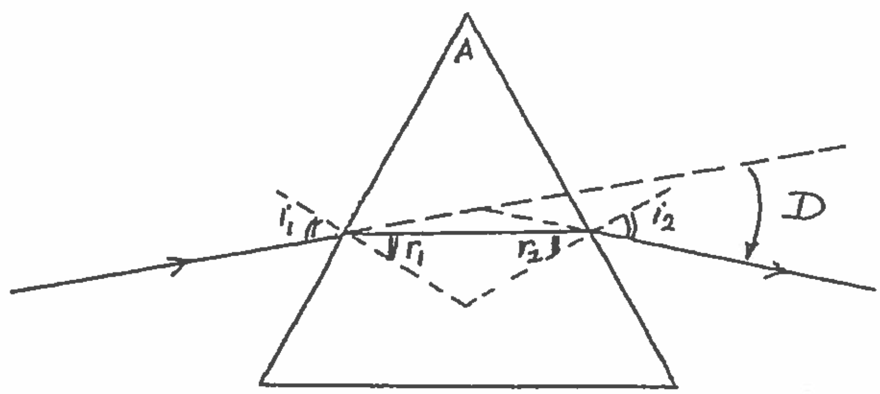


Figure 1

The angle measured by the prism spectrometer is the angle D between the incident ray and the emergent ray,

$$D = (i_1 - r_1) + (i_2 - r_2)$$

As the prism is rotated, the angle D goes through a minimum value, D_m (for a given wavelength); at this minimum value the ray of light passes symmetrically through the prism and a particularly simple relationship holds between the angles. For the minimum deviation condition (and assuming that for air $n_0 = 1$ it can be shown that,

$$n_1 = \sin \frac{(A+D_m)}{2} / \sin \frac{A}{2} \qquad (3)$$

<u>FOCUSSING THE SPECTROMETER</u>

N.B. Under no circumstances adjust the three screws that control the tilt of the spectrometer table.

1. Remove the prism from the turntable; DO NOT touch the flat faces of the prism - handle it by the edges. (N.B. In Experiment (8) <u>DO NOT</u> remove the grating!) Clamp the telescope and carry the spectrometer to a bench beside a window. Please hold the spectrometer by its base and not by the telescope and/or collimator. Focus the telescope on a distant object, by rotating the mounting of the objective (farther) lens. When doing this, keep both eyes open and cover the unused eye with your hand - this helps to ensure that the eye is relaxed and focussed for distant objects. Focus the crosswires with the eyepiece, and arrange them vertically and horizontally. When the image is properly focussed in the plane of the crosswires there should be no <u>parallax</u>, i.e. when your head is moved slightly from side to side, the image of the distant object should not move relative to the cross wires.

2. Put the spectrometer on the bench with the light bulb in front of the slit. The second arm of the spectrometer is the <u>collimator</u> whose function is to generate a parallel beam of light by having the slit at the focus of the collimator lens. View the slit with the telescope in line with the collimator; adjust slit width if necessary. Rotate the mounting of the collimator lens until the image of the slit falls on the cross wires; if the telescope is properly focussed on finity then this will occur when the collimator generates a parallel beam of light. In this condition there should be no parallax between the cross wires and the image of the slit, ar you should check the focus by this means.

PROCEDURE

Now with the white light source in front of the slit, clamp the 60° (=A) prism on the table with one side flat against the base of the clamp and the opposite corner at the centre of the table, so that light from the collimator is divided as shown in the diagram. (See Fig.2) The angles ϕ refer to the scale on the spectrometer table.

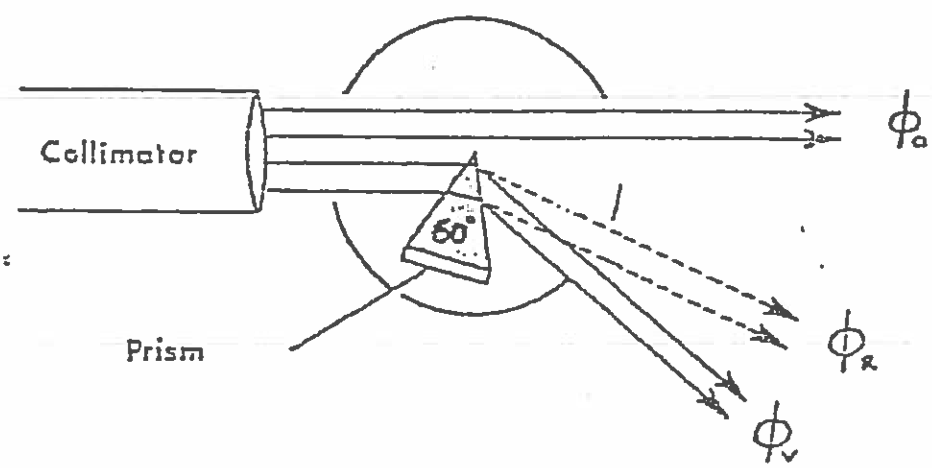


Figure 2

The following is the method for finding the position of minimum deviation. Read each step carefully, before attempting to do the step.

1. First locate the spectrum image at ϕ by eye; swing the telescope out of the way. You should see the spectrum within the ring of the collimator lens housing; if not, then you are looking at a spurious image that will disappear when you drape the black cloth over the apparatus. Convince yourself that the violet end of the spectrum is deviated through a greater angle than the red end. (Where do you place your eye to see violet in the centre of the collimator lens..... and where for red?).

2. Unlock the table, by loosening the knob situated under the table the prism sits on. If you are not sure which one to adjust, ask someone in charge before adjusting any knobs.

3. Rotate the table so that the red end of the spectrum appears to move towar the collimator when you view the spectrum by eye. In actual fact, the spectrum is moving toward ϕ_o (See Fig.2), even though it appears to be movi toward the collimator. You will observe a point where it has moved as far as possible toward the collimator, and further rotation only causes it to move away again. (Since your eye follows the image, this may take some practice to see clearly). This is the position of minimum deviation which you will now find more accurately with the telescope.

4. Clamp the table temporarily in its new position, and unclamp the telescope The knob to unclamp the telescope is the one directly under the telescope and in line with the direction of the telescope.

5. Move the telescope around to the yellow part of the spectrum, then clamp the telescope.

6. The spectrometer is now set approximately for the minimum deviation angle of the prism at the red of the spectrum. Replace the lamp bulb with the helium gas discharge tube, without moving the spectrometer: the helium source should be as close as possible to the collimator. The bright yellow spectrum line should now be near the centre of the field of view. Partially cover the spectrometer with the black cloth to improve visibility but allow enough background light to enter so that the cross wires remain visible: illumination of the cross wires may be helped by placing the lamp bulb behind the helium source. It may be necessary to adjust the slit width.

7. Unclamp the table and locate the minimum deviation position for the yellow line by alternately rotating the table and then setting the cross wires on the line at the minimum position. The telescope can be clamped, and then moved with the fine adjustment screw. Clamp the table. Record the angle in the Table described below, taking measurements to an accuracy of 30 sec. with the aid of the vernier. Keeping the table clamped move the telescope around to measure and record the angle of the direct image, ϕ_o.

8. Measure each of the visible lines of the helium spectrum. Strictly speaking, the minimum deviation procedure should be followed for each line. However, leave the table clamped at the minimum deviation position for the yellow line, then there will be systematic but small errors in the n-values that increase as one moves farther from the yellow line. When you have measured all the lines, repeat for the strong red and the strong violet lines, but this time setting for minimum deviation as in instruction (7) : don't forget to measure ϕ_o this time because the table will be moved.

Enter your results in a table like the following:

1

Spectrum line	Wavelength (nm)*	ϕ (degrees)	ϕ_o (degrees)	D_m (degrees)	$\frac{A+D_m}{2}$	n_1	c_1 (Mm/s)**
Red	706.5						
Red-strong	667.8						
Yellow-strong	587.6						
Green	504.8						
Green-strong	501.6						
Blue-green	492.2						
Blue	471.3						
Violet-strong	447.1						
Violet	438.8						
Red(repeat)	667.8						
Violet(repeat)	447.1						

* nm = nanometre ≡ 10^{-9} m ** Mm ≡ Megametre ≡ 10^{6} m

Calculate n and c to 4 significant figures from equations (2) and (3) respectively; the prisms have accurate 60^{o} angles, so that sine A/2 = 1/2 exactly.

2

9. Plot a graph of the speed of light against wavelength.

3

10. Plot a graph of the refractive index n_1 against wavelength and compare with Fig.3 to determine the material of the prism.

11. Estimate the error in the measurements of $\underline{\underline{c_1}}$ as indicated below.

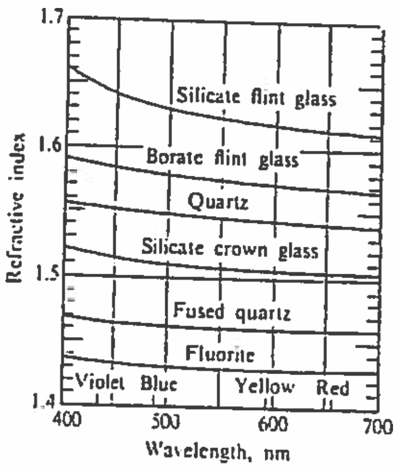
Combine equations (2) and (3) to give an equation for c_1 as a function of the measured angle D_m. Differentiate this equation to find the error Δc_1 in terms of ΔD_m (where it is assumed that c_0 and A are constants having negligible error).

Note that there are several contributions to the error D_m, including a systematic error because of setting for minimum deviation for the yellow line only. <u>Explain carefully what the different errors are, and how you have estimated them.</u>

<u>ABSORPTION SPECTRA</u>

This part of the experiment is <u>qualitative</u> and takes negligible time t complete; it is not necessary to record anything in your notebook.

Using the spectrometer to examine various types of absorption spectra. Such a spectrum is obtained by passing white light through a colored substance (usually a filter) and analyzing the colors which emerge on the other side. For example, if a red filter is used, one expects red light to be transmitted and other colors to be absorbed.



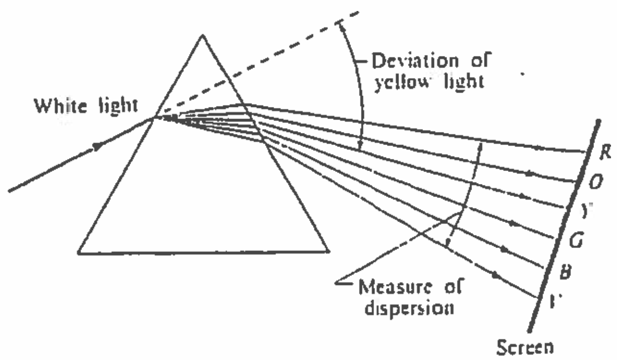


Figure 3

8. THIN FILM INTERFERENCE

OBJECT

-- To demonstrate the wave nature of light, by the study of interference.

-- To use the phenomenon of interference for the measurement of the radius of curvature of a lens, and the thickness of a hair.

BACKGROUND

Coherent & Incoherent Light Sources:

Everybody knows that a dark room which is illuminated by only one light source becomes brighter particularly everywhere, if a second light is switched on. Fig. 1.

If however one illuminates an area with light of the same source, first separating out two beams and then combining them in a certain fashion, one can observe a modification of the intensity with bright and dark "fringes." This effect is called "interference" and it is an illustration of the fact that light is a wave phenomenon.

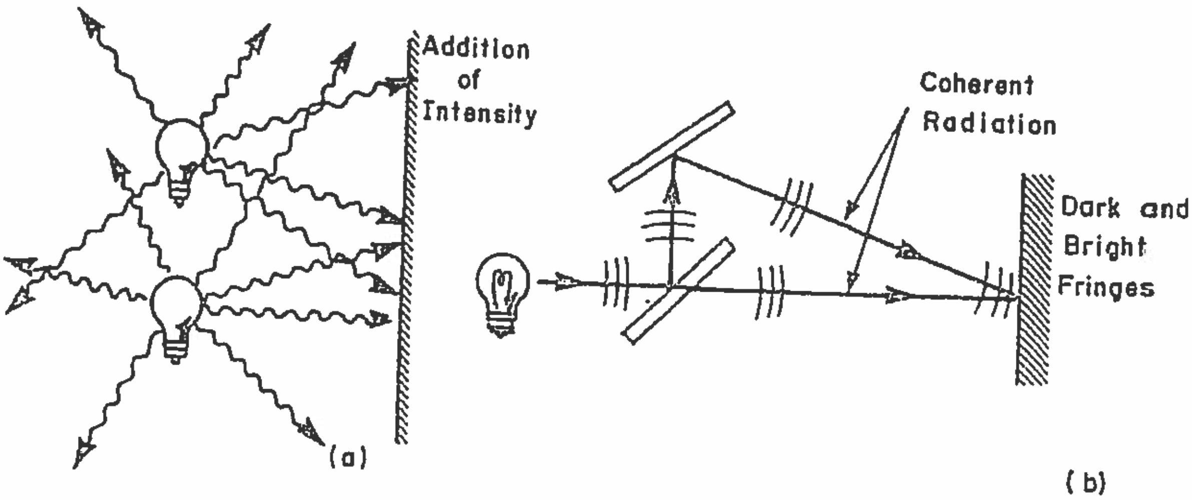


Figure 1 Incoherent Radiation

Interference only occurs, if the two light beams are "coherent" i.e. if there are fixed relations between the phases in various parts of the wave-fronts. The light emitted by two lamps, Fig. 1a is completely "incoherent", so that no interference is produced.

There are many different methods to produce interference. One can only produce bright or dark fringes of one colour, if monochromatic light is used. White light will give rise to coloured interference fringes ... why?

NEWTONS RINGS

One of the simplest devices to produce interference simply requires a thin air film, as for instance contained between a flat plate and a thin lens, Fig. 2a. For the observer at 0, the interference occurs between the light reflected from the surface S_1 and the light reflected from the surface S_2. With monochromatic illumination a dark spot will be seen where the extra path length for the light reflected at S_2 is $m\lambda$, where m is an integer and λ the wavelength of the light. Such points lie on circles around the contact point between lens and plate. Hence one observes circular fringes in this experiment. The radius of the fringes is related to the radius of curvature R, and the wavelength of the light, λ, as follows:

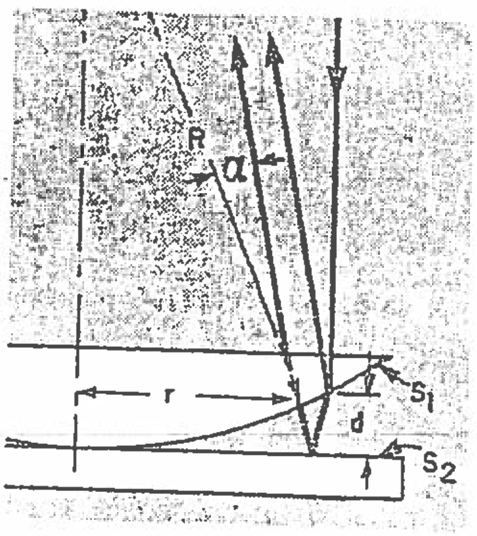


Figure 2a Geometry of Newton's Rings

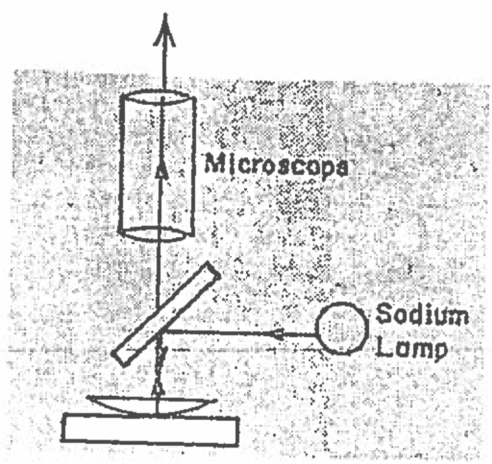


Figure 2b Newton's Rings Apparatus

Consider a radius r, where the air film has the thickness d, then by Pythagoras we obtain:

$$R^2 = (R - d)^2 + r^2$$
$$= R^2 - 2Rd + d^2 + r^2$$
$$d = \frac{r^2}{2R} + \frac{d^2}{2R}$$

Since R, and r are much greater than d, the last term is negligible, so that

$$d = r^2/2R \qquad (1)$$

We do not have to worry about the refraction which takes place when the beam passes from the glass into the air film and back again since the incident angle α is close to zero.

For the two beams reflected at the radial position r we have the following situation:

- O a path difference: 2d
- O an optical path difference n.2d (n = index of refraction of the film
- O a phase difference $(2\,\pi/\lambda)$ n.2d + π

The extra phase shift π occurs since one of the beams is reflected on the optically thin side of the surface S_2. Due to this extra phase shift, which corresponds to a path difference of $\lambda/2$, destructive interference is obtained if

$$n2.d = m.\lambda$$

Further using eqn. (1), one finds:

$$d = \frac{m\lambda}{2n} = \frac{r^2}{2R}, \quad \text{or} \quad r^2 = \frac{R.m\lambda}{n} \qquad \text{(dark fringe of radius r)} \qquad (2)$$

and constructive interference is obtained if

$$r^2 = \frac{R(m + \tfrac{1}{2})\ \lambda}{n} \qquad \text{(bright fringe)} \qquad (3)$$

The radii can be measured with the travelling microscope, and one ca use relation (2) or (3) to determine d, n, λ, r or R.

Experiment:

Set up the apparatus as shown in Fig.2b, with the microscope focussed on the top of the glass plate. Measure the diameters of a series of rings. In order to avoid errors due to distortions and uneven contact at the center it is preferable to measure radii well removed from the center such as the 5th, 10th, 15th, ... from the first clearly defined ring r_x. Now squaring the radii and taking the differences one obtains, the difference of the squares, F.

$$F_5 = r_{x+5}^2 - r_x^2 = \frac{R(x+5)\lambda}{n} - \frac{Rx\lambda}{n} = \frac{R\lambda}{n} 5$$

$$F_{10} = r_{x+10}^2 - r_x^2 = \frac{R(x+10)\lambda}{n} - \frac{Rx\lambda}{n} = \frac{R\lambda}{n} 10$$

in general:

$$F_j = r_{x+j}^2 - r_x^2 = \frac{r(x+j)\lambda}{n} - \frac{Rx\lambda}{n} = \frac{R\lambda}{n} . j$$

The left side is hence a linear function of the difference of the order of interference, j.

Plot $F_j = (r_{x+j}^2 - r_x^2)$ as function of j.

Enter the error bars of the points. How did you get them? Draw a straight line through the points and measure the slope. Calculate from the slope the radius R of curvature of the lens, assuming $n_{air} = 1$ and taking for the wavelength of the sodium light λ=5893 Å . Draw the maximum and the minimum slope through the error flags to obtain the smallest and the largest experimental value for the radius R.

THIN AIR WEDGE. EXPERIMENT.

A thin air wedge is also provided. It is formed between two glass plates which are separated on one side by a hair, see Fig. 3. If this object is placed under the microscope, a system of straight fringes is observed, which can be used to measure the width of the hair, T.

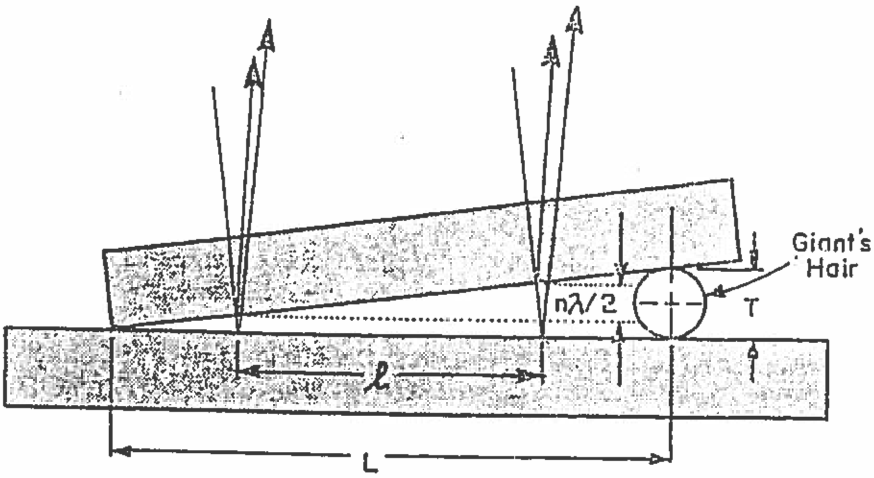


Fig. 3

If ℓ is the lateral distance between n fringes, and L the distance between the hair and the edge of the wedge, then the thickness of the hair is given by

$$T = L\, n\, \lambda/2\, \ell$$

Measure the width of the hair using this relation. Measure the width of the hair under the microscope, and compare both results.

Discuss the error of both measurements.

9. MICHELSON INTERFEROMETER

<u>OBJECT:</u> A) To set up Michelson interferometer for observing interference of light.

B) *To measure the wavelength of light from a given source (He-Ne Laser, Na or Hg lamp).

C) To measure the refractive index of air.

<u>APPARATUS:</u> Michelson interferometer assembly, source of light. Na lamp or Hg lamp (with green filter) <u>or</u> a helium-neon laser , diffuser, pin pointer, cell with optical windows, a small vacuum pump, screen.

<u>REFERENCES:</u> See, for example,

i) Physics Part <u>2</u> by Halliday & Resnick (Third Edition) Pages 1011-13;

ii) Experimental Physics by R.M.Whittle and J.Yarwood Pages 49-54.

<u>INTRODUCTION:</u> In Michelson interferometer, light from an extended source is divided into two parts (beams) by a beam splitter "B" which is a half-silvered mirror

---

* This will be possible if a movable carriage for one of the mirrors is provided e.g. Ealing-Beck interferometer, but not with Sargent-Welch Model 3559 Michelson Interferometer.

The two beams of light travel along different paths and are then reflected by two plane mirrors "M_1" and "M_2" and are finally brought together again to form an interference pattern (see fig.1). Note that beam 1 passes through "B" three times while beam 2 only once. A compensator plate "C" is usually introduced in the path of beam 2 in order to compensate for the additional path through glass.

One of the mirror M_1 is usually mounted on a carriage and can be moved backward and forward through small distances by means of a micrometer screw drive.

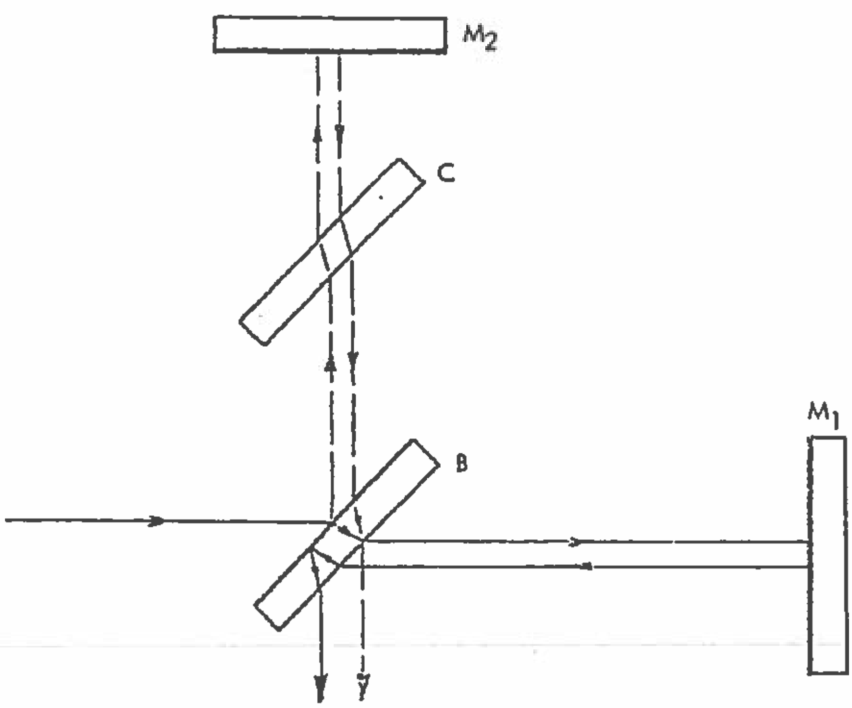


Fig.1 Schematic diagram of the Michelson Interferometer.

PART A : SETTING-UP THE INTERGEROMETER

An interference pattern will not normally be seen unless mirrors M_1 and M_2 are in planes at 90^o to one another. To adjust them to be so, a large pin on a stand is set up vertically between the extended source and B. Multiple images of this pin will be seen by the observer looking along O B. The screws behind the mirrors M_1 and M_2 are carefully adjusted until the images of the pin coincide. As the images are made to coincide, interference fringes become visible. A final delicate adjustment of the mirrors M_1 and M_2 makes these fringes concentric circles. A further slight adjustment of one of the mirrors will introduce a small angle of departure from 90^o producing curved or straight parallel fringes.

The micrometer screw drive can alter the distance of M_1 from B. In concentric fringe pattern new interference fringes will appear to be born at the center of the field of view if M_1B is increased and interference fringes will disappear at the center if M_1B is decreased. If straight parallel fringes are observed, movement of M_1 will cause the fringes to traverse across the field of view.

PART B : MEASURING THE WAVELENGTH OF LIGHT

i) The interferometer is adjusted and a suitable interference pattern is obtained as outlined in part A above.

ii) The micrometer position "d_1" is read on the scale.

iii) By turning the micrometer head, the carriage is moved slowly in either direction and the number of fringes "N" appearing or disappearing in the center of the concentric fringe pattern is counted.Otherwise, the number of fringes "N" moving past a reference point may be counted. For a satisfactory precision, some two hundred fringes should be counted.

iv) The new reading of the micrometer head "d_2" is noted.

v) The distance (d_1-d_2) is related to the wavelength through

$$(d_1-d_2) = N\frac{\lambda}{2} \qquad \ldots\ldots \qquad (1)$$

The value of λ is calculated.*

vi) Steps (ii) - (v) are repeated two to three times and an average value of λ is obtained.

vii) A comparison is made with the known values of .

λ_{Na} = 589.3 nm , λ_{Hg} (green light) = 546.1 nm.

WAVELENGTH OF HE - NE LASER

The adjustments of the interferometer are much easier to make when a laser is used as the source of light. The pattern can be projected on a screen or a wall. The screws at the back of mirrors M_1 and M_2 are adjusted until the two spots defining the two reflected beams are made to coincide. The interference pattern becomes visible after some very delicate adjustments.

CAUTION: DO NOT LOOK DIRECTLY INTO THE LASER BEAM!!

It is necessary to enlarge the laser beam so that a major portion of the beam splitter B and the mirrors M_1 and M_2, is filled with the laser light. A lens system may be used for this purpose.

The number of fringes "N" moving past a reference line may be counted as the mirror M_1 is moved backwards or forwards through a distance (d_1-d_2). Then using equation (1) above the wavelength of laser light may be determined. The value obtained is compared with the standard value:

λ = 632.8 nm

---

* The micrometer reading may not correspond to the exact displacement of the mirror M_1. For the Ealing-Beck interferometer in use in our Labs, Actual Displacement of the mirror = (1/5) X Micrometer Reading

PART C : INDEX OF REFRACTION FOR AIR

A chamber with glass windows is placed in the path M_2B and air is pumped out of the chamber with a small pump. Then the air is leaked back into the chamber very slowly so that the number of fringes passing a given point can be counted.

When light from a monochromatic source travels through the evacuated chamber, its speed is $c = 3 \times 10^8$ m/s. After air has leaked into the chamber, the light will travel at a lesser speed "v" through it. Since the frequency "ν" of the light is the same whether in air or in vacuum, the wavelength $(\frac{v}{\nu})$ will be shorter when the light is travelling through air than it would be in vacuum $(\frac{c}{\nu})$.

If "ℓ" is the length of the chamber, the path of light through the chamber = 2ℓ

In the evacuated chamber, $2\ell = N_o \lambda_o$ .... (2)

where N_o in the number of vacuum wavelengths λ_o contained in the path.

But $\lambda_o = \frac{c}{\nu}$

$$\therefore N_o = \frac{2\ell\nu}{c} \qquad \text{........} \qquad (3)$$

When air is leaked back into the chamber, a greater number of wavelengths will be contained in the path "2ℓ" (since "λ_{air}" is shorter than "λ_o").

Let this number be N_1 i.e. $N_1 = \frac{2\ell}{\lambda_{air}}$

where $\lambda_{air} = \frac{v}{\nu}$

$$\therefore N_1 = \frac{2\ell\nu}{v} \qquad \text{......} \qquad (4)$$

From equations (3) and (4) we get

$$N_1 = \frac{c}{v} N_o \qquad \text{......} \qquad (5)$$

The fringe pattern shifts because of the increase in the number of wavelengths through the path "2ℓ" that the light beam travels. The numberof fringes "N" counted as they cross a reference point while the chamber is brought back to atmospheric pressure gives the change in the number of wavelengths through the path "2ℓ".

$$N = N_1 - N_o = N_o \frac{c}{v} - N_o = N_o (\frac{c}{v} - 1)$$

$$\text{or} \quad N = \frac{2\ell}{\lambda_o} (\frac{c}{v} - 1) \quad \ldots\ldots\ldots\ldots \quad (6)$$

But $\frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in air}}$ = refractive index "n" of air.

$$\therefore N = \frac{2\ell}{\lambda_o} (n-1)$$

$$\text{or} \quad n = 1 + \frac{N \lambda_o}{2\ell} \quad \ldots\ldots\ldots\ldots \quad (7)$$

PROCEDURE FOR MEASURING THE REFRACTIVE INDEX OF AIR

i) The Michelson interferometer is adjusted for observing interference pattern as in part A above. A reference line is marked on the screen.

ii) The chamber is fixed in the path as shown in Fig.2

iii) The end of the tubing on the chamber assembly is connected to a vacuum pump.

iv) The control valve is opened and the chamber is evacuated. The interference pattern will be observed to change rapidly.

v) When the interference pattern becomes stationary the control valve is closed and the pump is disconnected.

vi) Now keeping an eye on the interference pattern, the control valve is opened very slightly so that air is leaked into the chamber very slowly. As this is done, the number of fringes "N" moving past the reference line is counted. The counting of fringes is continued until the chamber is at atmospheric pressure and the pattern becomes stationary again. A trial run may be necessary in order to familiarize yourself with the procedure.

vii) The steps (iii) - (vi) are repeated until consistent results are obtained.

viii) The length "ℓ" of the chamber is measured. (The thickness of the glass windows may be neglected).

ix) Using the values of "N", "ℓ" and "λ_o", the index of refraction for air is calculated with the help of equation (7).

x) A comparison is made with the known valve of n_{air}.

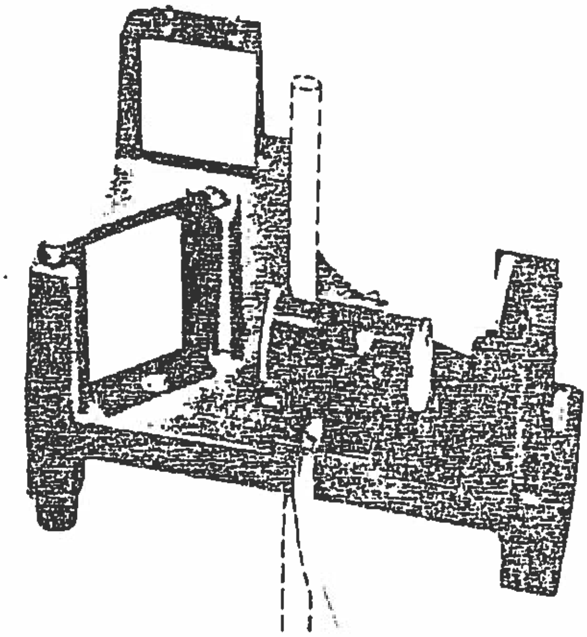


Fig.2. Michelson Interferometer with a vacuum cell.

QUESTIONS:

1. Discuss possible sources of error in this experiment.
2. Is it necessary to use a monochromatic source of light in this experiment?
3. Sodium yellow light consists of a doublet i.e. two closely spaced wavelengths at 589.0 and 589.6 nm. Discuss what you would observe as you move one mirror through a considerable distance backwards or forwards.
4. Why is it easier to display the fringes on a screen when we use laser light instead of any other source of light?
5. Discuss the use of Michelson Interferometer for calibrating a meter bar as a standard of length in terms of wavelengths of Krypton 86 .

OBTAINING INTERFERENCE FRINGES

MICHELSON INTERFEROMETER.

The Michelson interferometer is assembled as shown in Figure 3. Refer to the foregoing descriptions of each component for specific details. Remember that the mirror, unit B, should be located on the quadrant approximately perpendicular to mirror, unit A.

Switch-on the mercury lamp and clip the metal pointer onto the diffusing screen at the 12 o'clock position.

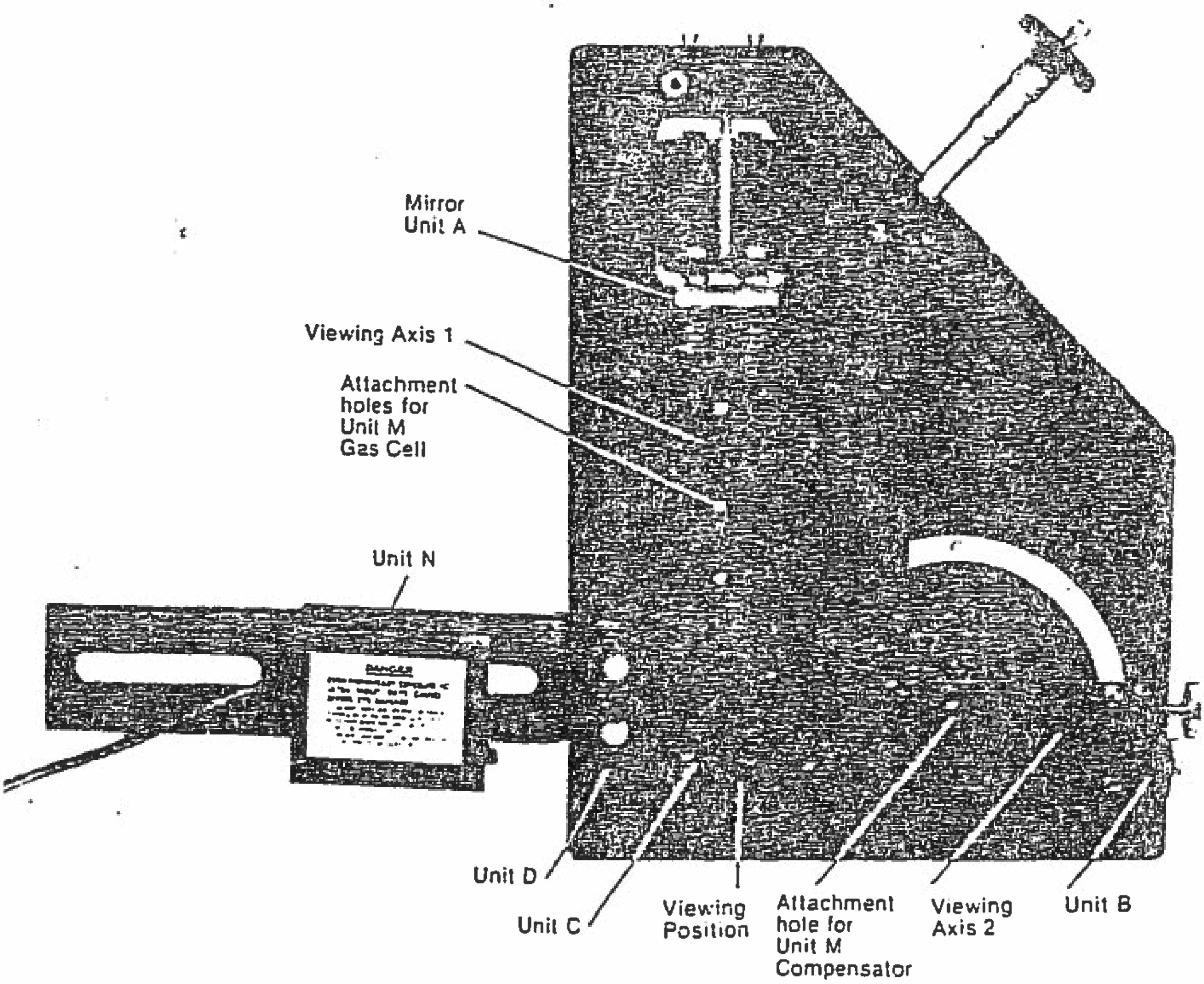


Figure 3. Plan of the Michelson Interferometer

Align the eye with the viewing axis, and about 25 cm from the instrument. Three reflected images of the pointer will then be visible in the beam splitting head (Fig. 4). Slight adjustment of the tilt controls on unit B will indicate which of these can be moved. The initial adjustment consists of superimposing this image on the right-hand image of the stationary pair of images (Fig. 5).

Where the path lengths are unequal and the mirrors not parallel, hyperbolic fringes are obtained, composed of curved sections surrounding a centre outside the field of view. By making careful adjustments to the tilting controls on mirror B, the mirrors can be set exactly parallel to each other to produce a pattern of circular fringes, with the centre in the field of view. They are sited at infinity and some practice in relaxing the eye, as if looking at a distant object, may be necessary to view them. If there is little difference in the path lengths, few fringes will be present (Fig. 6).

To obtain zero path difference, turn the path length control in the direction which causes the circular fringes to collapse to the centre; as the paths become equal the pattern will decrease until only a single fringe will be visible.

When the path lengths are equal to within ±5 fringes, it is possible to detect interference fringes by means of the tungsten-filament light source ("white-light" fringes).

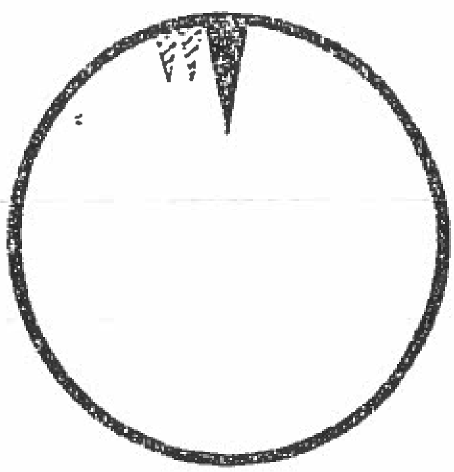


Figure 4. Three Reflected Images of the Pointers

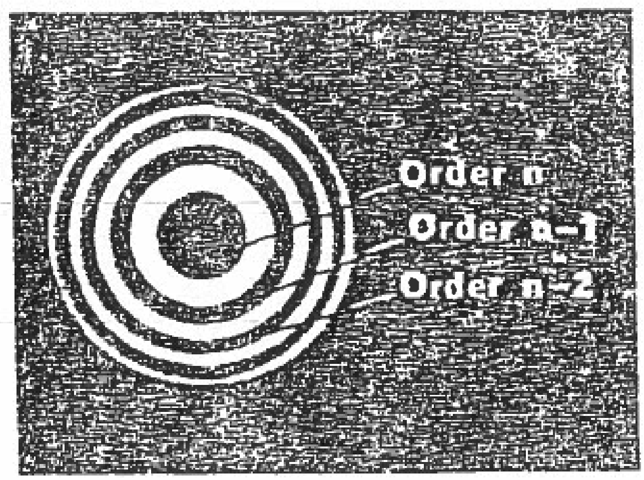


Figure 6. Pattern of Circular Fringes

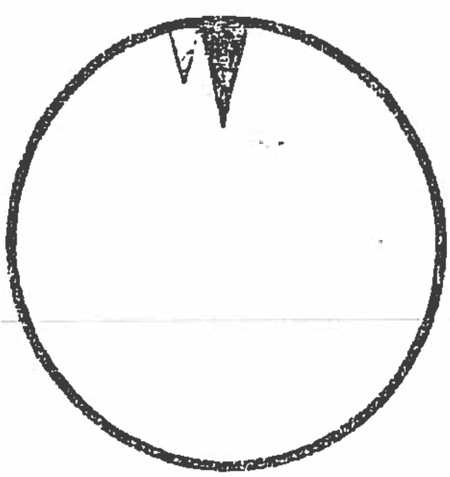


Figure 5. Superimposing the Images of the Pointer

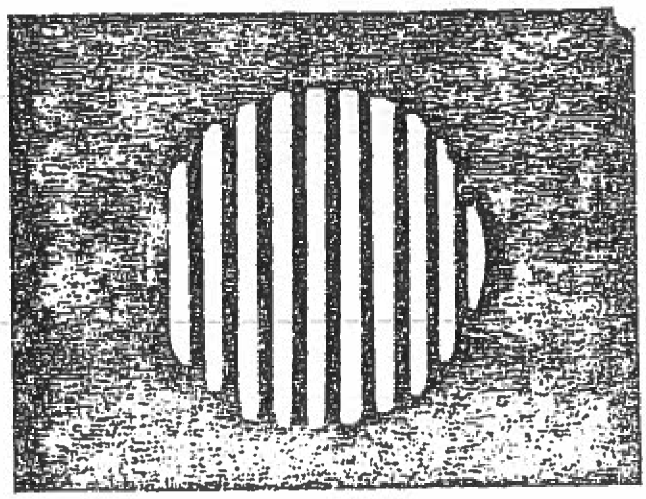


Figure 7. Pattern of Straight Fringes

When the tilt controls are adjusted using the mercury lamp a series of about ten vertical fringes will be produced. Turn the path length control until the fringes are straight (Fig. 7). Switch the tungsten lamp on and continue turning the path length control very slowly until a group of bright fringes with a distinctive dark band in the middle is seen. Bring the dark band to the centre and switch off the mercury lamp. The fringes will remain visible.

An extremely small adjustment to the tilt controls will cause a single constructive-interference fringe to completely fill the field. If the fringe tends to pass out of the field the path length control should be adjusted to effect its re-entry.

When a very thin parallel plate of thickness d and refractive index μ is placed in the beam near either mirror A or B, to cover half the field, the fringes through the plate will be displaced. The retardation is $2\,d\,(\mu - 1)$, the air having been replaced by the material. Since a displacement of one complete fringe is equivalent to a retardation of one wavelength, the thickness for a retardation of x fringes is:

$$d = \frac{x\lambda}{2\,(\mu - 1)}$$

the thickness being given in the same unit as that in which the main wavelength λ is measured.

10. DIFFRACTION OF LIGHT

OBJECT: (i) To study the diffraction patterns formed by single and double slits under laser illumination.

(ii) To measure the wavelength of a helium-neon laser source from diffraction patterns.

(iii) To measure the center to center distance between the slits using diffraction patterns.

APPARATUS: Helium-neon laser, three single slits of different widths, three double slits of different slit separation, Optical bench, mounts and holders, travelling microscope, diffuse screen and tracing paper sheets.

INTRODUCTION: Diffraction is the name given to the deviation from a straight line path when light passes through an aperture or around an obstacle. It is due to this "bending" property that light penetrates into the regions of geometrical shadow creating "fringes" i.e. alternate bright and dark regions especially on the periphery of the shadow. Diffraction phenomena are conveniently divided into two general classes outlined below:

a) Fraunhofer or Far-Field Diffraction-Here the source of light and the image screen are effectively at inifinite distance from the aperture causing the diffraction;

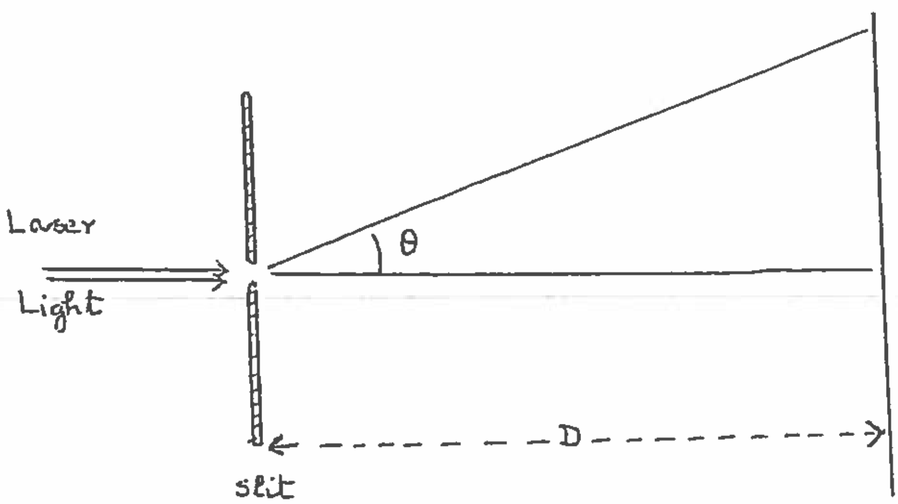
b) Frenel or Near-Field Diffraction where either the source or the screen or both are at finite distances from the aperture. Generally speaking, the near-field extends from the aperture to a distance of $\frac{a^2}{\lambda}$ where "a" is the size of the diffracting object and "λ" the source of light. Beyond $\frac{a^2}{\lambda}$, it is the far field.

In the near-field pattern, the image is clearly recognizable despite some "fringing" around its periphery. However, in the far-field pattern, the projected pattern bears little or no resemblance to the diffracting object.

We will study the Fraunhofer diffraction(far-field) from single and double slits.

SINGLE-SLIT DIFFRACTION

Fig.1 shows a section of a slit of width "a" illuminated by parallel monochromatic light of wavelength "λ". The diffraction pattern is observed on a screen which is at a distance "D" from the slit.



The intensity distribution "I_θ" for the diffraction pattern is given by (see Halliday and Resnick pages 1031-32).

$$I_\theta = I_m \left(\frac{\mathrm{Sin}\alpha}{\alpha}\right)^2 \quad ----- \quad (1)$$

where "I_m" is the maximum intensity, and

$$\alpha = \frac{\pi\ a\ \mathrm{Sin}\theta}{\lambda} \quad ----- \quad (2)$$

Here "θ" defines the angular position of the point of observation with respect to undeviated line of propagation (Fig.1). Fig. 2a shows a plot of the intensity ratios $\frac{I_\theta}{I_m}$ as a function of "α".

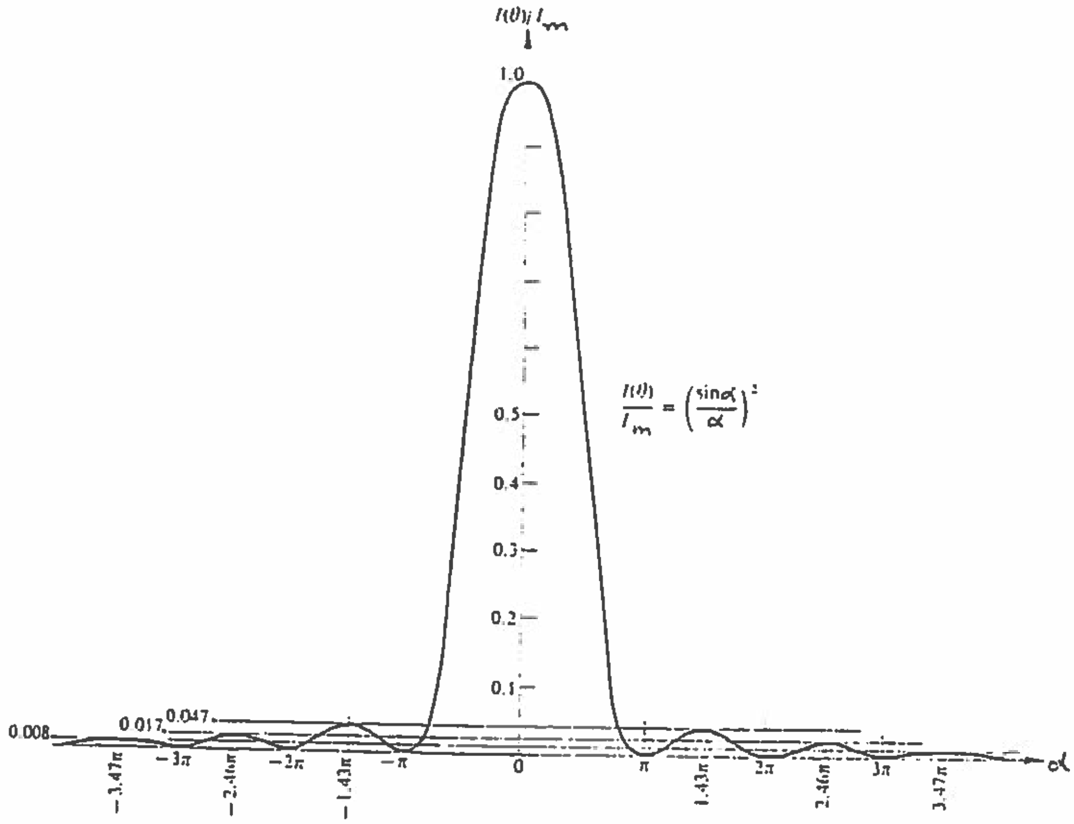


Fig 2a: The Fraunhofer diffraction pattern of a single slit.

Fig. 2b is a typical diffraction pattern from a single vertical slit under a point source illumination. The pattern consists of a series of bright (maximum intensity) and dark (minimum intensity) spots.

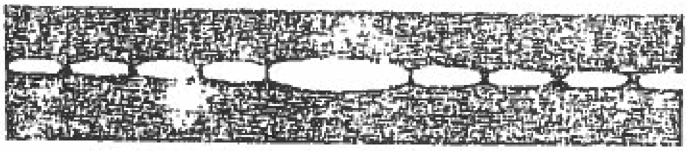


Fig. 2b

Central Intensity Maxima: The central or principal maxima corresponds to values of $\alpha = 0$ because $\lim_{\alpha \to 0} \left(\frac{\sin\alpha}{\alpha}\right) = 1$. The width of this central maxima is twice the width of secondary maxima lying on either side of this central maxima.

Intensity Minima: Minima in intensity distribution occur when $\alpha = m\pi$, $m = 1,2,3\ldots$ Using equation (2) this yields

$$a \sin\theta = m\lambda, \quad m = 1,2,3,\ldots \qquad (3)$$

as the condition of minima

The secondary maxima lie approximately half way between the minima and may be found for values of "α" given by

$\alpha \cong (m+\frac{1}{2})\pi \qquad m = 1,2,3, \ldots$

Usually, "θ" is very small and from small angle approximation

$\sin\theta \simeq \theta$, we get from equation (3)

$$\lambda = \frac{a\,\theta}{m} \qquad \text{-----} \qquad (4)$$

If "θ" cannot be measured directly, the distance "D" between the slit and the screen is measured. By measuring the center to center distance "p" between the minima of the same order on either side of the central maxima, we can approximate equation (3) using $\sin\theta \approx \tan\theta \approx \theta$ to

$$\lambda = \frac{a}{m}\frac{p}{D} \quad --- \qquad (5)$$

DOUBLE-SLIT DIFFRACTION

When a double slit is illuminated by monochromatic light of wavelength λ, a combined interference and diffraction pattern is observed. Suppose that the center to center distance between the slits is "d" and the width of each slit is "a". The intensity distribution is given by (see Halliday & Resnick Pages 1037-1040)

$$I_\theta = I_m \cos^2\beta \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad .... \qquad (6)$$

where

$$\alpha = \frac{\pi a}{\lambda} \sin\theta \quad .... \qquad (2)$$

and

$$\beta = \frac{\pi d \sin\theta}{\lambda} \quad .... \qquad (7)$$

The $\cos^2\beta$ in equation (6) is the "interference factor" (Young's double slit experiment) and the $\left(\frac{\sin\alpha}{\alpha}\right)^2$ is the "diffraction factor" representing the diffraction envelope. Fig 3a shows the plot of $\cos^2\beta$ (interference maxima) as a function of β and d Sin θ.

Fig. 3a

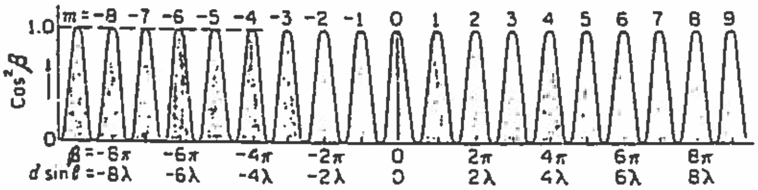
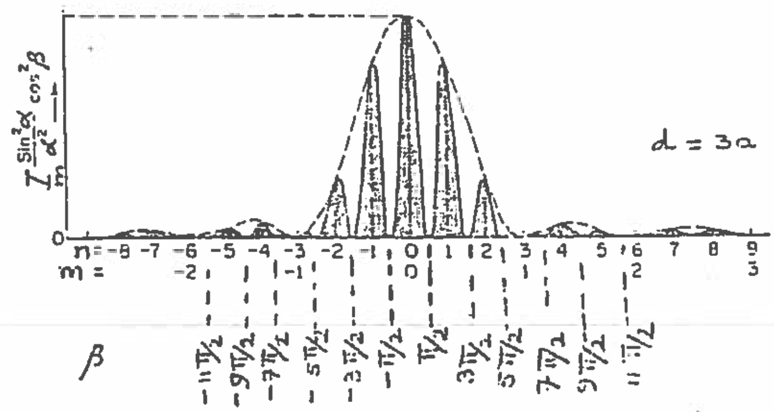


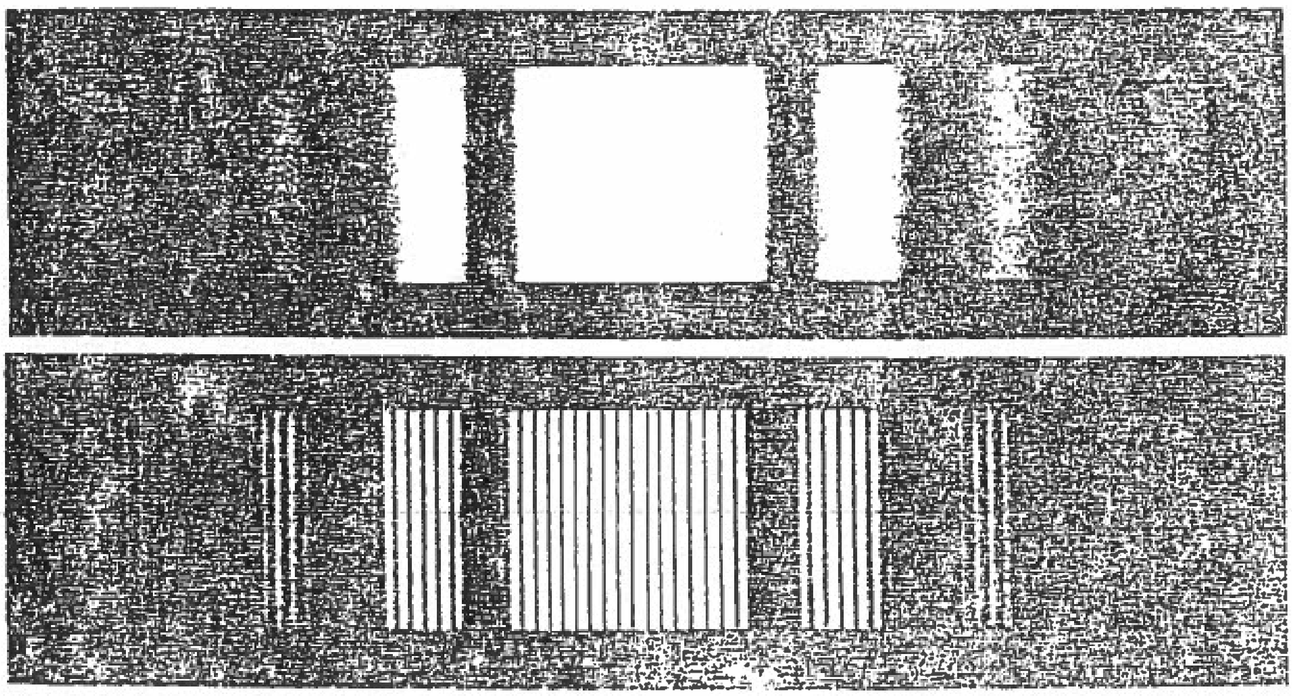
Fig 3b gives the variation of intensity in the double slit diffraction pattern (combined interference and diffraction) as a function of "β".

Fig. 3b



A comparison between single-slit and double-slit diffraction patterns is shown in Fig. 4.

Fig. 4



Position of Intensity Minima: The intensity will be zero whenever

$$\text{(i)}\ \beta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots = (n+\tfrac{1}{2})\pi \qquad n = 0,1,2,\ldots$$

$$\text{and/or(ii)}\ \alpha = \pi, 2\pi, \ldots = m\pi \qquad m = 1,2,3,\ldots$$

Using (6) and (2) above: these can be written as:

$$\left.\begin{array}{lll} \text{(i)} & d\ \mathrm{Sin}\theta = (n+\tfrac{1}{2})\lambda & n = 0,1,2,\ldots \\ \text{and/or(ii)} & a\ \mathrm{Sin}\theta = m\lambda & m = 1,2,3,\ldots \end{array}\right\} \quad \begin{array}{l} -8(a) \\ -8(b) \end{array}$$

Position of Intensity Maxima: The exact positions of maxima cannot be determined by any precise relationship, but their approximate positions may be found by neglecting the variations of the diffraction factor $(\frac{\mathrm{Sin}\,\alpha}{\alpha})^2$. This assumption will be justified if the maxima near the center of the pattern are considered. This corresponds to

$$\left.\begin{array}{ll} \beta = n\pi & n=0,1,2,\ldots \\ \text{or}\quad d\mathrm{Sin}\theta = n\lambda & n=0,1,2,\ldots \end{array}\right. \quad \text{—(9)}$$

In the center of the pattern these maxima are equally spaced (Fig. 3) and the distance between the adjacent maxima is given by (Halliday & Resnick Page 998)

$$\Delta y = \frac{\lambda\ D}{d} \qquad \text{—(10)}$$

Missing Orders: We have a situation where the conditions for maximum of interference term and minimum of diffraction term are both fulfilled for the same order. This order will be "missing".

$$\text{Thus} \qquad d\ \mathrm{Sin}\theta = n\lambda \qquad n = 1,2,3\ldots\ (9)$$

and

$$a\ \mathrm{Sin}\theta = m\lambda \qquad m = 1,2,3,\ldots (8b)$$

correspond to missing orders.

From (9) and (8b), we get $\frac{d}{a} = \frac{n}{m}$ —— (11)

Thus the orders which are missing correspond to ratios

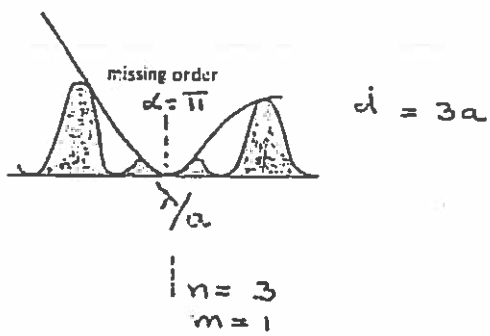
$$\frac{d}{a} = 1,2,3, \text{---} \qquad (11a)$$

In the case of the double-slit diffraction pattern shown in Fig. 3b the missing orders are:

$$n = 3,\ 6,\ 9,\ \ldots$$
$$m = 1,\ 2,\ 3,\ \ldots$$

Details of the pattern are shown in Fig. 5 for missing orders n=3, m=1.

Fig. 5



PROCEDURE:

a) Single Slit:

i) A helium-neon laser, a single slit and a diffuse screen are positioned on the optical bench such that the laser beam can pass through the slit and the pattern can be seen on the screen (same height approximately). The slit-screen distance should be at least 2 m.

ii) The laser is turned on and the slit is illuminated.

CAUTION: DO NOT LOOK DIRECTLY INTO THE LASER BEAM.

* SWITCH OFF THE LASER AS SOON AS YOU HAVE TAKEN MEASUREMENTS.

iii) The diffuse screen is adjusted so that the plane of the screen is exactly at right angles to the laser beam propagation direction.

iv) A sheet of tracing paper is fixed at the back of the diffuse screen.

v) Looking from behind the screen, the diffraction pattern is sketched on the tracing paper as accurately as possible.

vi) The laser is switched off and a slit of different width is placed in the path of the laser beam. The screen is lowered or raised to expose unused space on the tracing paper. The pattern is again obtained on the tracing paper.

vii) The procedure is repeated for the third slit.

viii) The slit orientation is changed from vertical to horizontal and at least one pattern is obtained in this position.

ix) The tracing paper is removed from the screen and using a travelling microscope, the widths of the central maxima and any secondary maxima are measured and compared for each slit.

x) The center to center distance "p" between the minima of the same order on either side of the central maxima is measured with the travelling microscope for each slit.

xi) If the width "a" of the slit is not given, it is measured with the travelling microscope.

xii) The distance "D" between the slit and the screen is measured.

xiii) The wavelength of the laser is calculated using equation (5). An average value is obtained and compared with the standard value of 6328A^{0}.

xiv) The results are discussed in the light of possible sources of error.

b) Double Slit

i) The single slit in part (a) above is replaced by the double slit.

ii) Proceeding as in part (a) above, the diffraction patterns are sketched on the tracing paper for each double slit.

iii) The center to center distance between interference maxima or minima are measured with the travelling microscope. The distance between several interference maxima (or minima) within the central diffraction maxima may be measured in order to minimize errors. The appropriate distance between adjacent maxima is then obtained.

iv) The wavelength of light is calculated using equation (10) for each pattern. The value of "d", if not given, may be determined as given below in steps v and vi: The value of "a", if not given, should be measured with travelling microscope.

v) The missing orders are determined from the diffraction patterns.

vi) Using equation (11), the values of "d"- the slit separation may be calculated.

vii) The results are compared with known values. Any deviations are accounted for by mentioning the possible sources of error in this experiment.

<u>note</u>: One rotation of the graduated knob on the travelling microscope moves the graticule seen through the eyepiece a distance of 0.08 mm.

11. GRATING AND SPECTROSCOPY

OBJECT:

-- Examine the difference between continuous and line spectra.

-- Meeting the diffraction grating, the basic instrument of spectroscopy and determine the grating constant d.

-- Meeting the structure of electronic energy levels of atoms, determine the separation between the Sodium D lines.

Spectroscopy is something that touches every branch of experimental science because the characteristic spectrum is the 'thumb-print' of an atom; it is a vital tool from the astrophysical laboratory to the crime laboratory!

WARNING: When you start using the discharge tube power supply please DO NOT touch the tube holder or replace the tube because of the high voltage which will cause a serious electrical shock. Ask someone in charge to replace the tube for you.

INTRODUCTION

An important property of an atom is its characteristic set of energy levels. Each energy level corresponds to one of the different configurations of orbital electrons allowed by the quantum theory of atomic structure. The energy levels may be shown on an energy level diagram, such as, Fig. (1) (drawn for a hypothetical atom.)

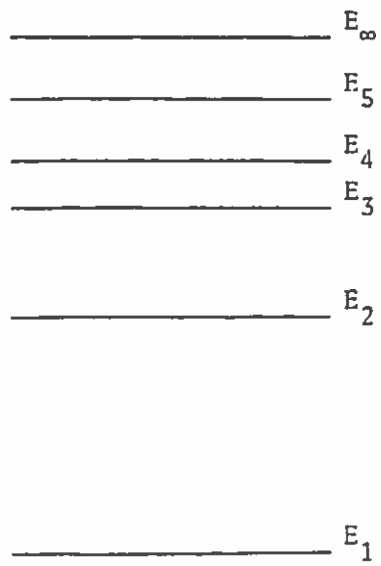


Figure (1)

In Chemistry, in connection with the Periodic Table, one is interested in the <u>occupied</u> energy levels. In spectroscopy the interest is in the higher energy levels that are not normally occupied but are available to the electron that is most weakly bound to the atom. The atom is normally in the state of lowest energy, the ground state E_1 in Fig. (1) which corresponds to the normal atom of chemistry with its full complement of electrons in their usual orbitals. If the valence electron gets the right amount of extra energy from incoming radiation or from a collision, it can jump to one of the higher energy orbitals: now the atom is in an 'excited' state E_2 or E_3 or E_4 etc. E_∞ represents the state where this electron is removed completely from the atom, so that the atom is ionized.

An atom in one of the excited levels can make a spontaneous transition to one of the lower levels with the emission of a light quantum. The fundamental law of quantum theory is that the frequency of the light is proportional to the energy <u>change</u>. Thus if an atom initially in the state E_2 makes a transition to the ground state E_1, the frequency of the light is given by

$$\Delta E = E_2 - E_1 = hf$$

where h is a universal constant (Planck's constant). This particular transition is indicated by the arrow in Figure (1).

In optical spectroscopy we do not measure the frequency f directly, but rather the wavelength λ. Since $f = c/\lambda$ where c is the velocity of the wave in the medium (air), then

$$\Delta E = hc/\lambda$$

In optics it is customary to measure the wavelength λ in nanometres (1 nm = 10^{-9}m) and the energy difference in electron-volts (1 eV = the energy gained by an electron accelerated through a potential difference of 1 volt). Because of this, it is convenient to express the constant hc in units of eV nm (although, of course, other units can be used.) In these units

$$hc = 1.240\ (10^3)\ (\text{eV nm})$$

The relationship between the energy difference and the wavelength can then be written as

$$\Delta E = 1.240\ (10^3)/\lambda\ (\text{eV})$$

*QUESTION: The visible spectrum extends from around 390 nm in the violet to 720 nm in the red. (The actual limits depends on the intensity and on the observer.) To what range of ΔE does this correspond?

The set of energy levels is characteristic of the type of atom, so that, for example, the spectrum emitted by hydrogen atoms will be different from the spectrum of helium or of any other type of atom. For a low pressure discharge tube the energy levels are sharp and the transitions between these levels produce a line spectrum - lines of definite wavelength. From these wavelengths we can find immediately the energy differences ΔE and (after a complicated analysis) also the energy levels. Of course the zero energy in the energy level diagram is arbitrary; usually the zero is chosen as the ground state energy. For an incandescent filament (ordinary light bulb) there are no sharp energy levels and we get a continuous spectrum.

So far we have only discussed transitions between the two lowest states of the atom, states E_1 and E_2 in Fig.(1). We now want to discuss the transitions between other states in order to point out the existence of the so-called selection rules. Suppose for instance that we have some excited atoms in the state E_3. From this state two transitions are energetically possible: some atoms may make a transition to E_2 while others may go directly to the ground state E_1. Under these circumstances we would therefore get two spectral lines originating from the level E_3. But because of the structure of a particular atom it may happen that a transition that is energetically possible nevertheless does not occur. For example, it may happen that the transition E_3 to E_1 does not occur, so that the corresponding spectral line is missing. In this case we say that the transition is forbidden by a selection rule. These selection rules eliminate many of the energetically possible transitions. For our hypothetical atom we have assumed that the transition E_2 to E_1 is in fact allowed but for some atoms the corresponding transition may be forbidden. For example for both He and Hg the transition from the first excited state to the ground state is forbidden. The He spectrum will be examined in Experiment 9.

THE SPECTROMETER

CAUTION:

(1) The surface of the grating is easily damaged so take care not to touch it.

(2) Some of the spectrometer knobs have been adjusted and should NOT be turned. Do not start adjustments until you have identified all the controls. (See STUDY 1).

1. Description of Instrument

The spectrometer is shown schematically in Fig.(2). It has a central table on which the grating is mounted, a collimator (the tube with the slit at the outer end) and a telescope (the tube with the eyepiece at the outer end). The light from the source under study passes through the slit of the collimator, and the rays are made parallel by the collimator lens. In this experiment the grating table is fixed in such a way that the light rays are incident perpendicularly on the grating. The diffracted rays are focussed by the telescope onto its cross wires, and the spectrum can be observed through the eyepiece.

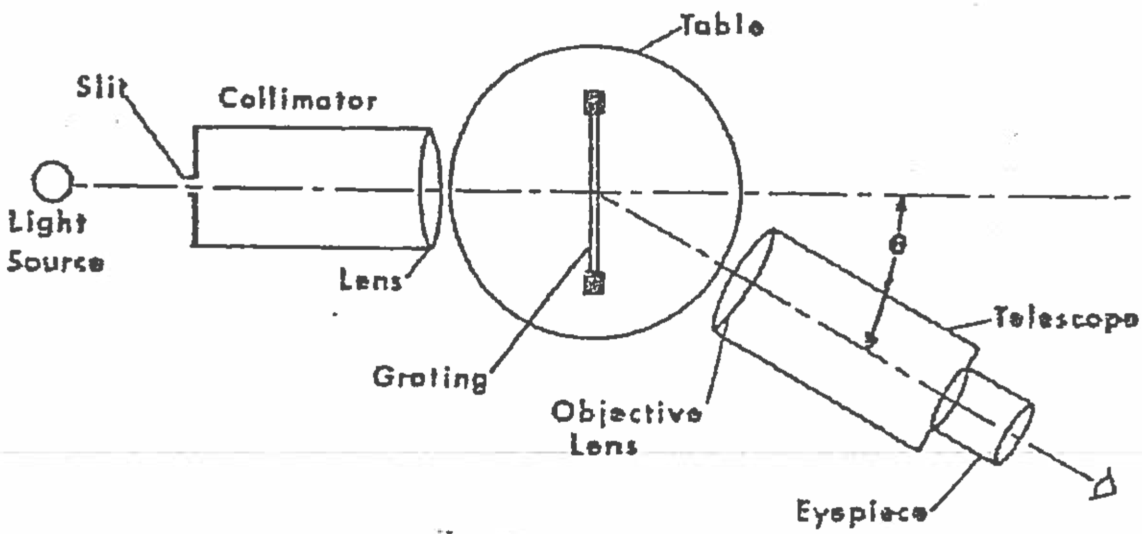


Figure (2)

2. Grating Equation

The diffraction grating is a piece of glass upon which a thin layer of plastic, with many parallel rulings, has been deposited. Such a grating has typically about 6,000 rulings per cm. For a given wavelength, λ, the grating produces bright maxima at angles given by:

$$d \sin \phi = m\lambda, \qquad m = 0, 1, 2 \ldots . \qquad (2)$$

where d is the grating spacing. The use of many 'slits' instead of two, sharpens the maxima considerably so that the broad maxima observed become sharply defined spectrum lines. When a mixture of frequencies are present these can then be viewed as distinct lines. The resolving power of a diffraction grating is a measure of its ability to resolve two spectrum lines that have almost the same wavelength. A two-slit device has low resolving power, but this increases in proportion to the number of slits used.

The integer m is called the Order of the spectrum. For the zero order ($m = 0$) we have $\sin \phi = 0$, so that $\phi = 0$ for all wavelengths, i.e. part of the light passes straight through the grating. For $m = 1$ we get the first order spectrum, for $m = 2$ the second order spectrum and so on. For the visible spectrum λ varies from about 400 nm (violet) to about 700 nm (red) Thus 2λ varies from 800 to 1,400 nm and 3λ from 1,200 to 2,100 nm. Hence the violet end of the third order spectrum overlaps the red end of the second order spectrum.

STUDY 1: OPERATION OF THE SPECTROMETER

1. Spectrometer Control Knobs

Examine the instrument to locate all the control knobs. DO NOT TURN AN of these until you have identified them all.

The following knobs or screws should NOT BE TURNED AT ALL by students. If they need adjustment, you should ask your instructor to do it. If you have by mistake turn one of these, ask someone in charge to readjust it immediately.

1. The three screws under the grating table that control the tilt of the table.
2. The knob that clamps the grating table. On some spectrometers it is located below the collimator, on others it is a horizontal screw below the grating table, close to the collimator.

2. Routine Adjustments

Having located all the controls, you should now carry out the following routine adjustments. The purpose of this procedure is to check that the grating table is clamped properly and that the instrument is in focus, to set the slit width and to focus the telescope eyepiece to suit your own vision. The procedure should be repeated at the beginning of each lab period.

For these adjustments use the sodium lamp for the light source. Sodium emits a pair of yellow lines, very close together in wavelength. This pair of lines is an example of a doublet. This particular doublet is often referred to as the sodium D lines, a term introduced by Fraunhofer.

1. Check by eye that the grating is perpendicular to the collimator. If it is not, ask someone in charge.
2. Move the instrument so that the slit of the collimator is next to the sodium lamp. Move the telescope out of the way and look through the grating into the collimator. You should see the illuminated slit; otherwise the slit is closed and should be opened by turning the screw clockwise at the end of the collimator. Now bring the telescope in line with the collimator and look through the telescope at the slit image. Narrow down the slit as far as possible with the image remaining at full brightness. (Turn the screw counterclockwise to narrow the slit).
3. Move the eyepiece housing back and forth until you see the cross wires clearly with no eyestrain; the eye should be relaxed and focussed for ∞. Note that to avoid eyestrain while looking through the telescope you should keep your other eye open but covered with your hand. Check that the cross wires are vertical and horizontal. If not you will also need to rotate the tube holding the eyepiece.

4. Now check that the slit is in focus on the cross wires. This means that there should be no parallax, i.e. if you move your eye slightly in front of the eyepiece, the image of the slit should not move relative to the cross wires. If there is some parallax, do Step 5 and 6, then carry out the steps 'Focussing the Spectrometer' described in experiment (7).

5. As a further check on the focus locate the first order sodium doublet on both sides. You should clearly see two lines. The room lights can be blocked off with a black cloth but make sure that the cloth does not obstruct the light path.

6. Check also that the two first order spectra on either side are in the middle of the field of view. If one is too high and the other too low, the grating table needs adjustment. But DO NOT attempt to do this yourself; ask someone in charge to help.

NOTE: Some spectral lines you will be asked to locate are very dim. To help locate them, first check that the slit is directly in line with the light source, and as close as possible to the source. When trying to locate the line through the telescope, open the slit very wide, to permit a great deal of light through. All spectral lines will become very broad, and the dimmer lines will become visible. Once you have located the required line, narrow the slit down again, for a precise measurement of the angle.

STUDY 2 : CONTINUOUS SPECTRUM

This is a brief qualitative study of the continuous spectrum emitted by an ordinary light bulb.

Look at the straight image of the slit illuminated by the light bulb and check that the slit is wide enough to give a bright image. Then find the first order spectrum on one side. Turn the telescope through the spectrum and continue to the second order spectrum, etc. How many orders can you see? Which orders overlap? If the spectra are too dim to see clearly in the higher orders, open the slit wider until you can see the colours more clearly. Repeat your observations on the other side of the zero order.

STUDY 3 : THE SODIUM SPECTRUM

In this study we use the known mean wavelength of the sodium D lines (589.3 nm) to determine the grating constant d (the distance between rulings). You will need this constant for the subsequent studies.

1. Use the sodium vapour lamp as source and check that the spectrometer is adjusted properly. Locate the first order image of the sodium D lines on one side. Clamp the telescope and move it to bring the cross wires in between the two sodium D lines. Record the angular position of the telescope. Repeat for the first order image on the other side. Find the difference 2ϕ in the angles. Note that if you have moved the telescope through zero you will have to add 360^o to one angle. Knowing that $\lambda = 589.3$ nm and that $m = 1$, you can find d from equation (2). Since λ is measured in nm, it is convenient to express d also in nm.

2. Repeat your readings using the second order spectra ($m = 2$) and find d. How much more accurate do you think this value of d is? Use this value for d in all subsequent calculations.

3. Measure the diffraction angles ϕ_1 and ϕ_2 for each of the two "D" lines in the second order; i.e. two lines on each side of straight through, hence FOUR angles are to be measured, all for the second order. Calculate λ_1 and λ_2 corresponding to ϕ_1 and ϕ_2, recalling m=2 therefore

$$m\ \lambda_1 = d \sin \phi_1$$

$$m\ \lambda_2 = d \sin \phi_2$$

and $m(\lambda_2 - \lambda_1) = d(\sin \phi_2 - \sin \phi_1)$

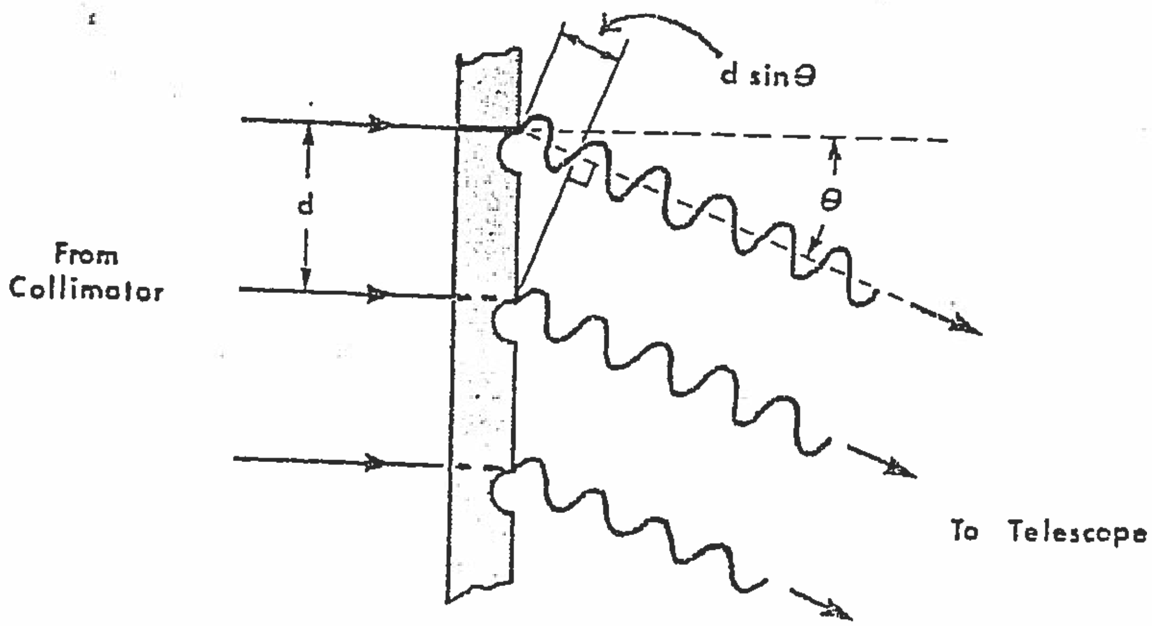
The last equation will give the difference in wavelength $\lambda_2 - \lambda_1$ directly.

4. Measure the angle between the two D lines in the second order as accurately as you can; to do this it is advisable to calibrate the fine control knob on the telescope motion, i.e. obtain the angular movement per turn. Knowing the average ϕ from step 2,

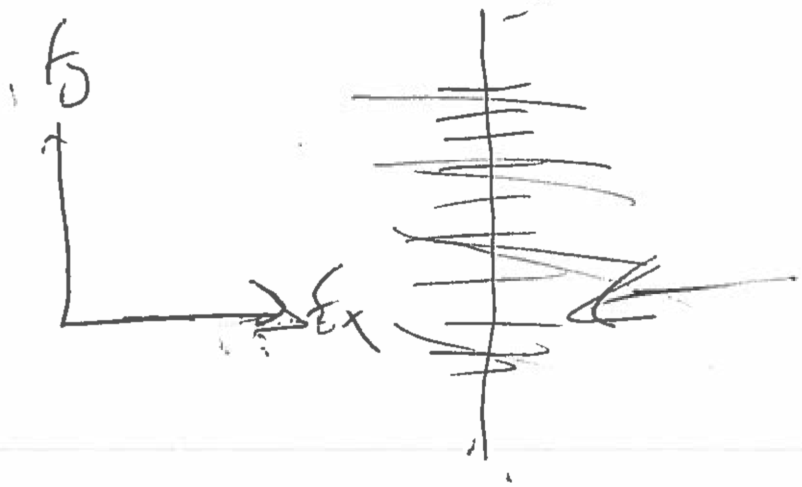
find the difference in wavelength between the two lines. To do this, differentiate the grating equation to find the small shift in wavelength $d\lambda$ corresponding to a small change of angle $d\phi$. Don't forget that m=2.

Do you get the same difference in wavelength as in step 3?
Which of the two methods is more accurate and why?

5. Locate the weaker green and red doublets in the first order. To see these clearly, the sodium lamp must be hot. Center the crosswires on the dark gap between the two green or the two red lines and measure the "average" diffraction angle; then calculate the average wavelength for each doublet.



$\vec{A} = A_x \hat{i} + A_y \hat{j}$



12. POLARIZATION OF LIGHT

OBJECT

-- encountering polarization phenomena, the terminology used, and an application.

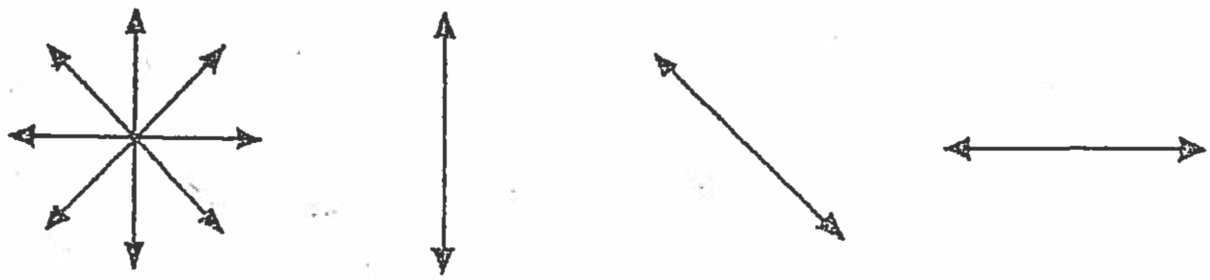
-- understanding the meaning and use of the polarization vector, including its mathematical manipulation by taking vector components.

-- errors and differentiation.

INTRODUCTION

Polarization is a phenomenon associated with transverse waves, that is, waves for which the vibration of the 'medium' is perpendicular to the direction of wave propagation.

For example, for a stretched string carrying a wave, a given point of the string vibrates in the plane perpendicular to the equilibrium string: moreover all points of the string vibrate in the same parallel direction and the wave is said to be plane polarized or linearly polarized. The vibration direction (for a wave travelling in a direction perpendicular to the paper) is shown by arrors in Figure 1 b, c and d.



(a) (b) (c) (d)

Figure (1)

In the case of light the 'vibration' is the fluctuation of the electric and magnetic fields in the plane perpendicular to the direction of wave travel. The electric and magnetic fields are perpendicular to each other, and customarily the direction of vibration that is selected to represent the polarization is that of the E-field.

Unpolarized light (e.g. from a light bult) consists of pulses (photons) each of which is a wave-train roughly a centimetre long and of a certain polarization: but the polarization of the different wave-trains is random so that unpolarized light is a mixture of all possible polarizations, as indicated in Figure 1.a.

When unpolarized light is incident on a polarizing agent and plane polarized light is produced, the agent is termed a "polarizer". All vibrations of the light waves incident on the polarizer can be resolved into 2 vector components, one perpendicular to, and one parallel to the select polarizing direction of the polarizer called its transmission axis. The polarizer transmits the component which is parallel to its transmission axis, and absorbs the perpendicular component.

To determine whether a beam of light is polarized or not, we pass the light through a second polarizing agent. When the latter is rotated, there is no change in the transmitted intensity if the light is unpolarized, while the intensity goes from the maximum to zero if the beam is plane polarized. A polarizing agent used in this way is called an "analyzer" (see Figure 2). The polarizer and analyzer in Fig. 2 are said to be crossed.

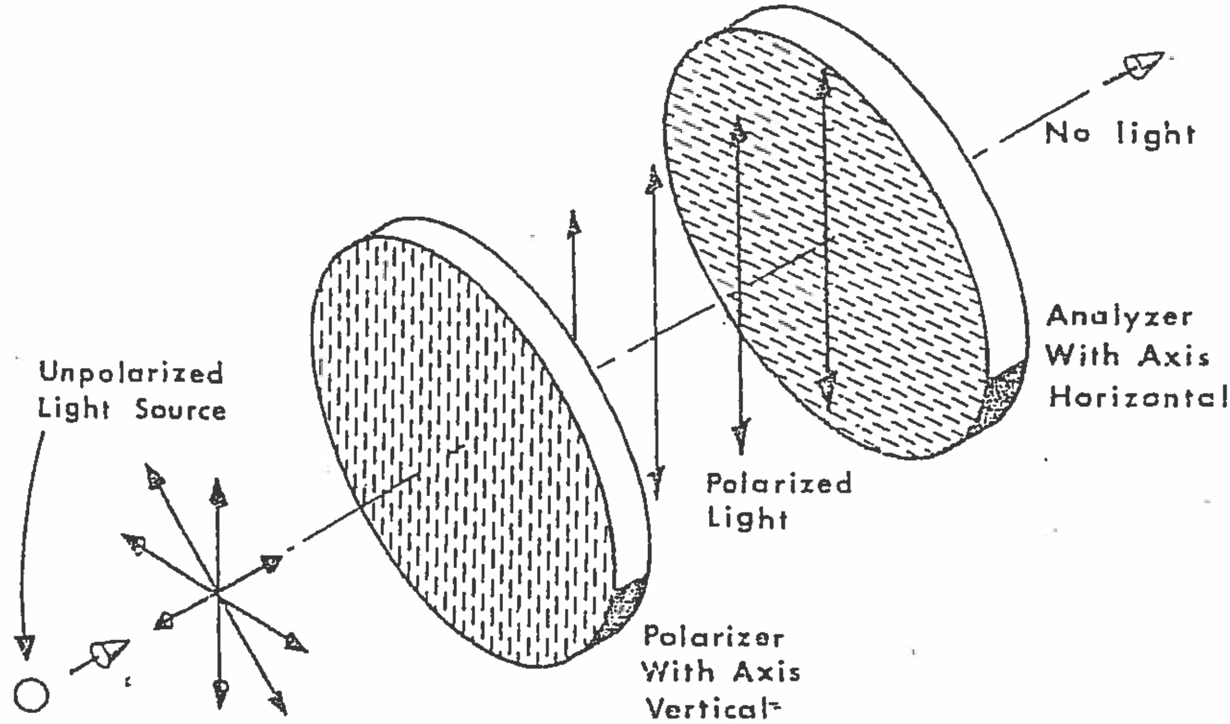


Figure 2

If a beam of plane polarized light falls on an analyzer set to transmit vibrations making an angle θ with that of the incident beam, a portion of the incident beam is transmitted. To calculate the fraction transmitted, the vibration amplitude A_1 of the incident beam is resolved into two components (Fig.3), one parallel to the transmission axis of the analyzer and the other perpendicular. The analyzer passes the parallel component and absorbs the perpendicular one. The amplitude transmitted A_2 is given by

$$A_2 = A_1 \cos\theta \qquad (1)$$

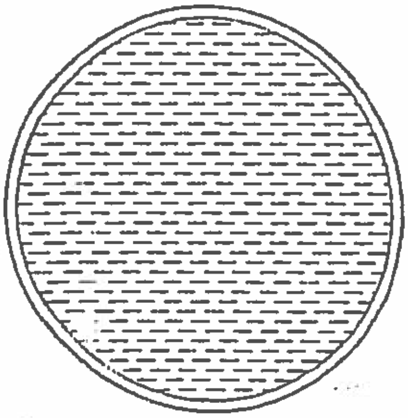
Wave theory shows that intensity (power per unit area) is proportional to the square of the amplitude; the fraction of the incident intensity I_1 transmitted is therefore given by

$$\frac{I_2}{I_1} = \frac{A_2^2}{A_1^2} = \cos^2\theta$$

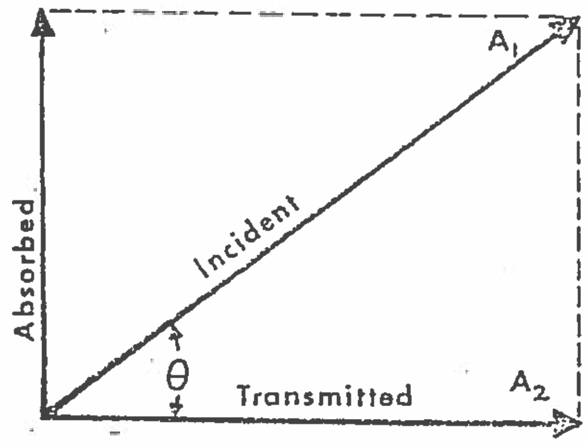
or

$$I_2 = I_1 \cos^2\theta \qquad (2)$$

(Malus' Law)



Polarizing Disk
With Axis
Horizontal



Resolution into Absorbed and
Transmitted Components

Figure 3

STUDY 1 : TRANSMISSION THROUGH ONE AND TWO POLAROIDS

In this experiment, the polarizing agents are polaroid sheets; the sheets contain long polymeric chains of polyvinyl alcohol molecules that have been given a preferred direction by stretching. Do not touch the polaroid material. The intensity of the light transmitted through the polaroid sheets is measured with a light meter calibrated in foot-candles (i.e. lumens per ft.^{2}); the S.I. unit is the lux (lumens per m^2). The light bulb and the fixed polaroid nearest the bulb) should not be removed or adjusted. The two polaroids with angle markings around them can be rotated, or removed altogether by lifting straight up.

1. Cover the light meter with a piece of paper to protect it from bright light. Remove one of the polaroids and look through it at the light bulb. N.B. do not let the polaroid touch the bulb. Rotate the polaroid through 360°. Similarly examine light from the sky, preferably in a direction perpendicular to the sun. (Do not look directly at the sun!); also examine a patch of reflected light, e.g. from your bench surface.

QUESTION (to be answered in your notebook)

- explain qualitatively the decrease in intensity when you look at the light bulb through the polaroid
- which of the 3 sources (bulb, sky, reflection) are at least partly polarized?
- why are all polaroid sunglasses made with the same transmission axis relative to the frame?

Now observe the light bulb through the two movable polaroid sheets held in your hands, rotating the second sheet with respect to the first. Notice that when the polarizer and analyser are crossed, you can still see a little deep purple light. Unfortunately, absorption by the polaroid material is a function of wavelength, and a pair of crossed polaroids is not perfectly effective for eliminating the far red and far violet radiat[illegible]

2. Insert one of the movable polaroids into the hole closest to the light meter; rotate the movable polaroid until the light meter gives a maximum reading; then the transmission axes of the two polaroids are the same ($\theta = 0$), (though we haven't found the actual plane of polarization of either polaroid.).

Measure the transmitted light intensity through the two polaroids as a function of the angle measured from $\theta = 0$. Take measurements I_+ and I_- at the same angle on either side of $\theta = 0$ and average of the two intensities; to a first approximation this eliminates any systematic zero error in the angle measurement (i.e. error due to the fact that $\theta = 0$ is not precisely located). If measurements I_+ and I_- differ substantially, then you should re-set your $\theta = 0$ reading and start again.

θ degrees	$\cos\theta$	$\cos^2\theta$	I_+ ft. cndle.	I_- ft. cndl.	$\bar{I} = \frac{1}{2}(I_+ + I_-)$ ft. cndl.

While taking measurements, be sure that there are no light-coloured objects behind the light bulb - like heads, hands, shirts etc.; spurious reflections will perturb the meter readings.

3. Plot a graph of $\bar{I}$ against $\cos^2\theta$ to verify the theoretical law expressed by equation (2). Does the 'background intensity' have any effect on your graph?

4. Errors. Suppose that the uncertainty in a given setting of θ is $\pm\Delta\theta$. Find an expression for the corresponding error in $\cos^2\theta$ by using the differentiation technique, i.e. find $\Delta(\cos^2\theta)$.

   -- Estimate the error $\Delta\theta$ in your angle settings (assumed to be the same uncertainty for all readings).
   -- At what angle is the error $\Delta(\cos^2\theta)$ a maximum?
   -- Find the maximum error in $\cos^2\theta$.

STUDY 2: TRANSMISSION THROUGH THREE POLAROIDS

Insert an additional polaroid between two crossed polaroids. Rotate the middle polaroid until a maximum amount of light is transmitted; at what orientation θ (see Fig.4) is the maximum amount of light transmitted?

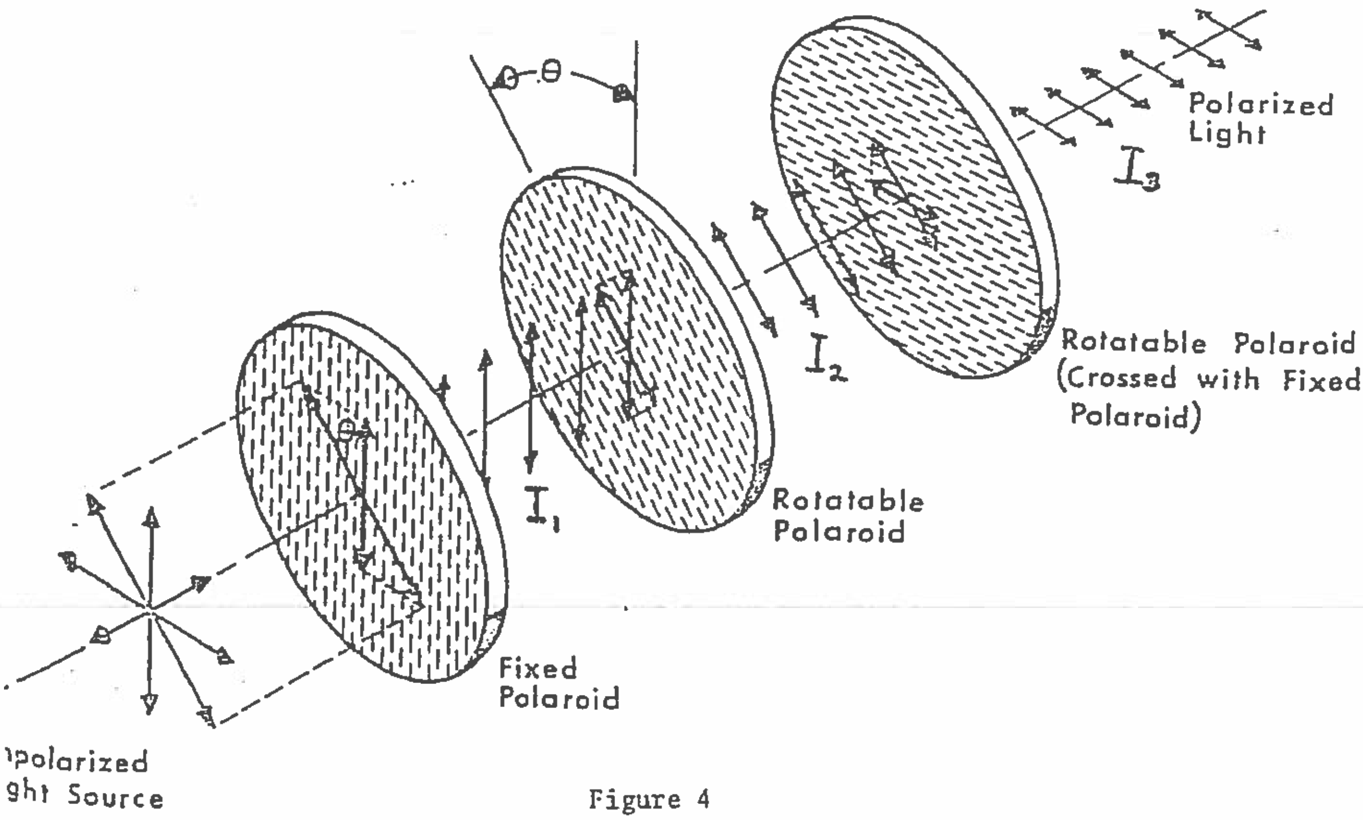


Figure 4

Explain why some light can get through the third polaroid (in spite of the fact that it is crossed with respect to the first polaroid) by extending t 'components of a vector' argument used to derive equation (2).

Find the general relationship between I_3 and I_1 as follows:

-- write down the relation between I_1 and I_2 (as in equation (2))

-- write down the relation between I_2 and I_3

-- eliminate I_2 to find the relation between I_1 and I_3.

Show that the relation you have derived has a maximum for I_3 at the orientation angle measured experimentally (I_1 being a constant).

STUDY 3: POLARIZATION COLOURS

This is a qualitative study only; there is no need to record anything in your notebook.

1. When a birefringent material is placed between crossed polaroids, colours can be seen that vary with the orientation of the material; try this with the mica sheet and the 'Scotch Tape'.
2. Some materials become birefringent under stress. Place the lucite 'nut-crackers' between crossed polaroids; is there any stress present? Now squeeze and watch what happens. Notice where the stress is a maximum (why do airplanes have round windows?). These stress patterns are the basis of 'photoelastic stress analysis', - see Scientific American, November 197? the STructural Analysis of Gothic Cathedrals.

APPENDIX: Trignometry

Frequently in physics (and engineering), as in this experiment, it is necessary to carry out mathematical manipulations with trignometric functions. Finger-tip knowledge of the elementary trignometry relationships is useful to any scientist and absolutely essential to any one pursuing physics beyond the 1st Year level.

13. ATOMIC CONSTANTS

OBJECT

-- The aim of this experiment is to examine the Balmer series of spectral lines emitted by hydrogen and from the measured values of the wavelengths to arrive at the value of the Rydberg constant and Planck's constant.

INTRODUCTION

In the Bohr or planetary model of the hydrogen atom, the electron is considered to move in a circular orbit about the nucleus. At any instant, the electron can be in one of a select number of orbits characterized by quantum numbers n = 1, 2, 3, 4, . . . In the innermost orbit where n = 1 the angular momentum of the electron would be equal to $\frac{h}{2\pi}$ where h is Planck's constant; in the next orbit, out, the angular momentum would be $2(\frac{h}{2\pi})$ and so on.......

Whenever the electron jumps from a higher energy outer orbit to an inner orbit of lower energy, a well defined amount of energy ΔE is emitted in the form of electromagnetic radiation; i.e. a photon is emitted having a frequency

$$f = \frac{\Delta E}{h}$$

and a wavelength:

$$\lambda = \frac{c}{f} = \frac{ch}{\Delta E} \qquad (1)$$

where c is the speed of light in vacuum.

The visible or Balmer series of spectral lines emitted by a hydrogen source corresponds to electrons in the atoms jumping from various outer orbits to the orbit of quantum number n = 2: the red line associated with electrons in a number of the atoms jumping from n = 3 to n = 2, the bluish-green line associated with the n = 4 to n = 2 transition . . . The relative number of atoms of the source in which electrons go from a particular initial energy state to the final (n = 2) state determines the relative intensity of the particular spectral line corresponding to this transition.

The energy of an electron in the n^{th} orbit is given by $E_n = \frac{E_1}{n^2}$ where E_1, the energy in the innermost orbit (equal numerically to that energy needed to ionize a hydrogen atom), has a value of -13.58 ev. (Note that 1 electron-volt = 1.60×10^{-19} joules.)

In 1885, Balmer obtained a simple relationship between the wavelengths of the visible lines emitted by hydrogen and the quantum numbers associated with the transitions giving rise to these lines, i.e.

$$\frac{1}{\lambda_n} = R_H \left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad (2)$$

where n = 3, 4, 5, . . . and R_H is Rydberg's constant.

In this Study, the wavelengths of the first four Balmer lines will be measured using a spectrometer and diffraction grating. Rydberg's constant will then be determined graphically making use of the relationship indicated as equation (4) in experiment 8. Finally Planck's constant will be be determined from equation (3) in experiment 8 rewritten in the form:

$$h = \frac{(\Delta E)\lambda n}{c} = \frac{(E_n - E_2)}{c}\lambda_n \qquad n = 3, 4, 5, \ldots \qquad (3)$$

1. Adjustment of Spectrometer (Note: do this part very carefully)

   (a) Focus the telescope for parallel light by sighting on a distant object through a window.

   (b) Eliminate parallax between the image of the distant object and the cross hairs of the telescope by adjusting the eyepiece until there is no relative motion between the cross hairs and the image of the distant object when you move your eye from side to side slightly while viewing through the telescope.

(c) Align the spectrometer so that the collimator and the clamped telescope are in line. Insert the hydrogen spectrum tube into the holder on the power supply and then turn on the power supply.

WARNING:

After the power supply is turned on DO NOT touch the tube holder or replace the tube because of the high voltage which will cause a serious electrical shock.

2. Balmer Series Wavelengths

(a) Fix the diffraction grating on the prism table by means of the holder provided. Orient the table so that the plane of the grating is approximately perpendicular to the path of the light arriving at it from the collimator slit and note the angular reading on the scale.

(b) Move the telescope slowly to the right and locate the four lines of the Balmer series to be examined (in viewing order: two violet lines, a bluish green line, and a red line). Note the angular position of each.

Repeat this step to the left of the central image.

(c) Calculate the wavelength of each of the four lines using the grating relation:

$$m \lambda = d \sin \theta \qquad m = 0, 1, 2, \ldots$$

where d is the grating constant (separation between the lines) scratched on the grating material) determined in Experiment 7 and θ is the average angular separation between a spectral line and the central image.

3. Atomic Constants

(a) Plot a graph of $\frac{1}{\lambda}$ versus $\frac{1}{2^2} - \frac{1}{n^2}$ using the measured wavelength values. From the slope of the resulting line, compute R_H and then compare this result to the accepted value.

(b) Calculate the energy values for orbits of quantum numbers 2, 3, 4, 5, & (Note that they will all be negative).

Calculate $E_n - E_2$ for each of the spectral lines and, then, h using equation (3).

Compare the average experimental value of h to the accepted value.

Accepted Values:

(1) Rydberg Constant $R_H = 1.097 \times 10^7\ m^{-1}$

(2) Planck's Constant $h = 6.626 \times 10^{-34}$ J-sec.

14. TWO-ELECTRON SPECTROSCOPY

OBJECT

-- To study transitions in the Helium atom and to introduce the student to basic atomic structure.

INTRODUCTION

A neutral helium atom has two orbital electrons, and in the theory the interaction between the electrons has to be taken into account, as well as the separate interactions with the nucleus. The energy levels that result, depend on the principal quantum number $n(=1, 2, 3...)$ as for hydrogen, but for each n there may be more than one energy level, depending on the angular moment quantum number, ℓ. For historical reasons, the states are labelled in terms of ℓ as follows:

ℓ:	0	1	2	3
Label:	s	p	d	f

The electron orbitals (or corresponding energy levels) in <u>any</u> atom are designated by the n-number followed by the ℓ-label:

1s 2s 2p 3s 3p 3d.......

N.B. Orbitals do not exist for $\ell > n$.

The helium spectrum is further complicated by the fact that the electrons have 'spin', that is, they have some angular momentum (and magnetic moment) of their own, quite apart from their orbital motion. The spins of the two electrons are very strongly coupled and can exist in only two states - parallel or antiparallel. In effect, there are two sorts of atom corresponding to these two states; there is the <u>singlet</u> helium atom with the spins opposed (and this includes the lowest energy state - the ground state), and there is the triplet helium atom with the spins lined up. The terminology comes from magnetic effects; in a magnetic field the triplet level splits into three different energy levels while the singlet is not affected (because the magnetic effect of the two opposed spins is cancelled out).

The energy levels for the helium atom are shown in Fig.(1); the singlet and triplet levels have been separated, so that we don't need an extra superscript.

From the energy level diagram there would appear to be a very large number of possible transitions, and hence spectrum lines. Fortunately some selection rules operate:

(a) $\Delta \ell = \pm 1$

i.e. transitions 3s→2p, 4d→3p, 3p→3s etc. are allowed

transitions 3s→2s, 5d→4d, 3d→3s etc. are forbidden

(this rule is a very general one)

(b) Transitions between singlet states and triplet states (or vice versa) are forbidden. It is because of this rule that we can separate the states completely as in Fig.(1), and talk about 'two sorts of helium atom'.

PROCEDURE

1. Construct a matrix showing the energy transitions for the singlet states, and another matrix for the triplet states. List the states in order of increasing energy, and fill out the top-right corner only, e.g. for singlet series -

TRANSITION FROM		1s	2s	2p	3s	3d	3p	etc.
TO	1s	0	X	UV	X	X	UV	
	2s		0		etc.			
	2p			0				
	etc.				0			

As shown here, label forbidden transitions X

ultra-violet transitions UV

infra-red transitions IR

and only include numerical values for allowed transitions in the visible region. It may help to draw in the transitions in Fig. (1) in the manual as you proceed.

2. Examine the spectrum of the He Geissler tube. Measure all 12 lines of the $m = 1$ spectrum. Enter your results in a Table.
   Identify the observed lines and enter this in your Table, e.g. $5d \rightarrow 3p$ (T) (for triplet). Indicate if there is some doubt on the identification.

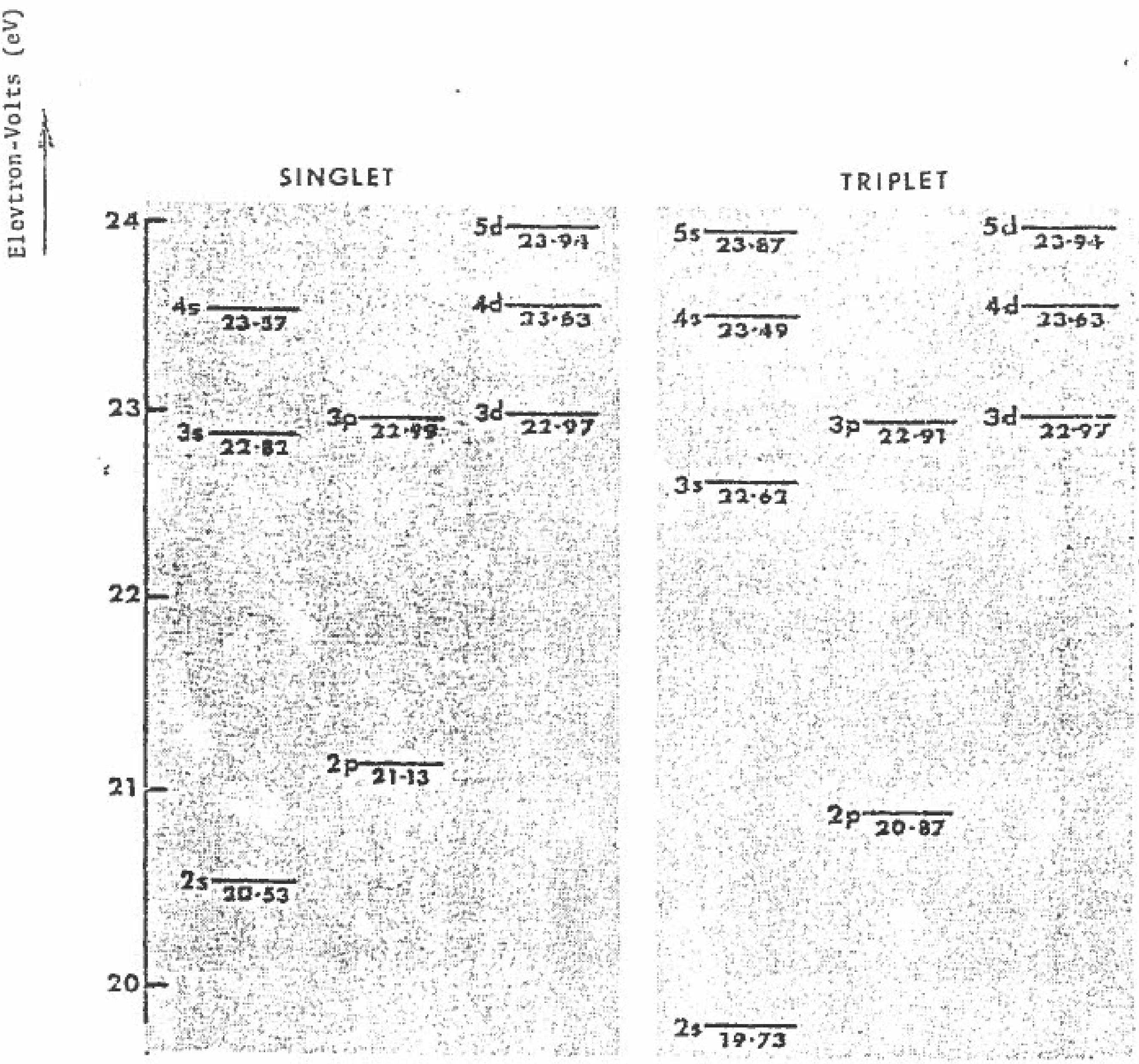


Figure (1)

NOTE: (a) the ground state is the 1s state of the singlet series with energy 0 on this scale; there is no 1s state in the triplet series.

(b) there is a small error - in the singlet series, 3d should be slightly below 3p.

15. RADIATION DETECTION

1. The Inverse Square Law

Nuclear radiations, like light rays, are emitted from a source isotropically, i.e. equally in all directions. If a radioactive source is placed at the center of a spherical shell, the number of radiations crossing per unit area per unit time can be written as

$$I = I(o) / A = I(o)/4\pi R^2$$

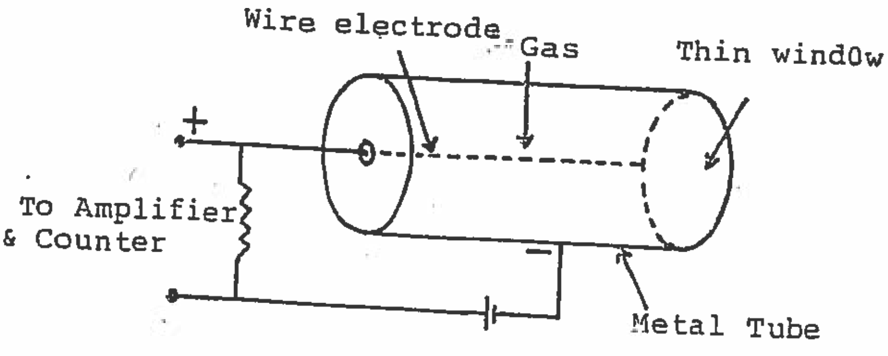
where

$I(o)$ = total number of radiations from the source per unit time.

R = radius of the shell.

This can be written as $I = f(1/R^2)$. i.e. the intensity varies as $1/R^2$ This is the so-called inverse square law.

1. Set the G.M. counter at the proper operating voltage.
2. Place a Cs - 137 source 1 cm away from the face of the window. One should try to remove any object nearly which could cause scattering.
3. Determine the count rate as a function of distance for d = 1, 2, 3, 5,.... 10 cm. Increase the distance until the background count rate is obtained.
4. Plot the net count rate versus the distance on log-log graph paper and draw the best straight line through the points. If the Square Law is applicable then the slope of the straight line should be - 2. A smaller slope would suggest the presence of significant amounts of scattered radiation and/or the size of the source is large in comparison with the detector-source distance.



A. Absorption Law

The decrease of intensity of radiation as it passes through a relatively thin absorber is exponential:

$$= I(o)e^{-\mu_m t_d}$$

where

I = intensity after the absorber
I_o = intensity before the absorber
μ_m = total mass absorption coefficient in $cm^2.g^{-1}$
t_d = density thickness in $g.cm^{-2}$

This property will be investigated with gamma rays from a Cs-137 source which emits gamma photons of energy 0.662 MeV.

1. Set the G.M. counter at the proper operating voltage.

2. Determine the count rate of the source without adding any absorber between source and counter.

3. Determine the count rate after placing one layer of lead absorber between source and counter.

4. Repeat the measurement, adding increasing thickness of absorber layers in place.

5. Determine the half value thickness [$t_{1/2}$] of Pb for the given gamma source. Compare with the given value 6.5 mm. The density of Pb= 13.50 g/cm^3. Explain any discrepancy betweens the results and the simple exponential law (i.e. not a straight line when plotted on log-linear graph paper).

6. Determine the linear attenuation coefficient μ_l in cm $^{-1}$) of Lead from the slope. The actual linear attenuation coefficient of Lead is 1.07 cm^{-1}. Compare the two values.