$$\psi = \left(\frac{2}{L}\right)^{3/2} \sin\frac{n_1 \pi x}{L} \sin\frac{n_2 \pi y}{L} \sin\frac{n_3 \pi z}{L} \; e^{-iE_{n_1 n_2 n_3} t/\hbar}$$

$$E_{n_1 n_2 n_3} = \frac{\pi^2 \hbar^2}{2m}\left[\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right]$$

Table 8.3 The Spherical Harmonics $Y_\ell^{m_\ell}(\theta, \phi)$

$$Y_0^0 = \frac{1}{2\sqrt{\pi}}$$

$$Y_1^0 = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cdot \cos\theta$$

$$Y_1^{\pm 1} = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}} \cdot \sin\theta \cdot e^{\pm i\phi}$$

$$Y_2^0 = \frac{1}{4}\sqrt{\frac{5}{\pi}} \cdot (3\cos^2\theta - 1)$$

$$Y_2^{\pm 1} = \mp\frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot \sin\theta \cdot \cos\theta \cdot e^{\pm i\phi}$$

$$Y_2^{\pm 2} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot \sin^2\theta \cdot e^{\pm 2i\phi}$$

$$Y_3^0 = \frac{1}{4}\sqrt{\frac{7}{\pi}} \cdot (5\cos^3\theta - 3\cos\theta)$$

$$Y_3^{\pm 1} = \mp\frac{1}{8}\sqrt{\frac{21}{\pi}} \cdot \sin\theta \cdot (5\cos^2\theta - 1) \cdot e^{\pm i\phi}$$

$$Y_3^{\pm 2} = \frac{1}{4}\sqrt{\frac{105}{2\pi}} \cdot \sin^2\theta \cdot \cos\theta \cdot e^{\pm 2i\phi}$$

$$Y_3^{\pm 3} = \mp\frac{1}{8}\sqrt{\frac{35}{\pi}} \cdot \sin^3\theta \cdot e^{\pm 3i\phi}$$

Table 8.2 Some Associated Legendre Polynomials $P_\ell^{m_\ell}(\cos\theta)$

$$P_0^0 = 1$$
$$P_1^0 = 2\cos\theta$$
$$P_1^1 = \sin\theta$$
$$P_2^0 = 4(3\cos^2\theta - 1)$$
$$P_2^1 = 4\sin\theta\cos\theta$$
$$P_2^2 = \sin^2\theta$$
$$P_3^0 = 24(5\cos^3\theta - 3\cos\theta)$$
$$P_3^1 = 6\sin\theta(5\cos^2\theta - 1)$$
$$P_3^2 = 6\sin^2\theta\cos\theta$$
$$P_3^3 = \sin^3\theta$$

$$L = \sqrt{\ell(\ell+1)}\,\hbar$$

Table 8.5 Spectroscopic Notation for Atomic Shells and Subhells

n	Shell Symbol	ℓ	Shell Symbol
1	K	0	s
2	L	1	p
3	M	2	d
4	N	3	f
5	O	4	g
6	P	5	h
. . .		. . .	

$$L_z = m_\ell \hbar$$

$$\cos\theta = \frac{L_z}{L} = \frac{m_\ell}{\sqrt{\ell(1+\ell)}}$$

$$E_n = \frac{ke^2}{2a_0}\left(\frac{Z^2}{n^2}\right) \qquad n = 1, 2, 3 \ldots$$

n = Principal .... 1, 2 ...

L = angler momentum ...

ℓ = orbital 1. 2 ... n-1

m_ℓ = magnatic qauntum ... 0, ±1 ... ±ℓ

Table 8.4 The Radial Wavefunctions $R_{n\ell}(r)$ of Hydrogen-like Atoms for $n = 1$, 2, and 3

n	ℓ	$R_{n\ell}(r)$
1	0	$\left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$
2	0	$\left(\frac{Z}{2a_0}\right)^{3/2}\left(2-\frac{Zr}{a_0}\right)e^{-Zr/2a_0}$
2	1	$\left(\frac{Z}{2a_0}\right)^{3/2}\frac{Zr}{\sqrt{3}\,a_0}e^{-Zr/2a_0}$
3	0	$\left(\frac{Z}{3a_0}\right)^{3/2} 2\left[1-\frac{2Zr}{3a_0}+\frac{2}{27}\left(\frac{Zr}{a_0}\right)^2\right]e^{-Zr/3a_0}$
3	1	$\left(\frac{Z}{3a_0}\right)^{3/2}\frac{4\sqrt{2}}{3}\frac{Zr}{a_0}\left(1-\frac{Zr}{6a_0}\right)e^{-Zr/3a_0}$
3	2	$\left(\frac{Z}{3a_0}\right)^{3/2}\frac{2\sqrt{2}}{27\sqrt{5}}\left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}$

Selection Rules:

$$|\Delta \ell| = 1$$

$$\Delta n > 1$$

$$P_{ns} = 4\pi r^2 |\psi_{ns}(r)|^2$$

$$P_{ns} = 4\pi r^2 |R(r)|^2$$

radial part

$$P(r) = r^2 |R_{n,\ell}(r)|^2$$

$$P(\theta, \phi) = |Y_\ell^{m_\ell}(\theta, \phi)|^2$$

average value of r

$$\int_0^\infty P(r) = 1$$

$$\langle r \rangle = \int_0^\infty r\, P(r)\, dr$$

integer

$$\int_0^\infty x^2 e^{-x}\, dx = 2!$$