* the quantum Oscillator matches the Correspondence Principle

---

Expectation Values

$$\bar{x} = \sum x P_x$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x \, |\psi(x,t)|^2 \, dx$$

Expectation Value.

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \, |\psi(x,t)|^2 \, dx$$

Uncertainity

$$SDE = \sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

$$\sum_i \frac{x_i^2 - 2\bar{x}x_i + \bar{x}^2}{N}$$

$$\sum_i^N \frac{x^2}{N} - 2\bar{x} \sum_i^N \frac{x_i}{N} + \bar{x}^2 \sum_i^N \frac{1}{N}$$

$$\overline{x^2} - 2\bar{x}\,\bar{x} + \bar{x}^2$$

$$\overline{x^2} - 2\bar{x}^2 + \bar{x}^2$$

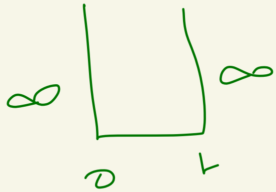
$$\overline{x^2} - \bar{x}^2$$

the uncertainity in Position

$$\sigma = \sqrt{\overline{x^2} - \bar{x}^2} \quad \overset{\text{Q.M.}}{=\!=} \quad \boxed{\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

Ex. Particle in a Box



$$\psi_n(x) = \sqrt{\frac{2}{L}}\, \sin \frac{n\pi x}{L}$$

$n=1$ $$\psi(x) = \sqrt{\frac{2}{L}}\, \sin \frac{\pi x}{L}$$

$$\frac{2}{L}\int_0^L x\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2\pi x}{L}\right)dx$$

$$\frac{2}{L}\left[\frac{L^2}{4} - \frac{1}{2}\int_0^L x\cos\frac{2\pi x}{L}\,dx\right]$$

$$-\frac{1}{2}\left[x\,\frac{\sin 2\pi x/L}{2\pi/L} + \frac{\cos\frac{2\pi x}{L}}{2^2\pi^2/L^2}\right]_0^L$$

$$\begin{array}{cc} x & \cos\frac{2\pi x}{L} \\ 1 & \dfrac{\sin\frac{2\pi x}{L}}{2\pi/L} \\ 0 & \dfrac{-\cos\frac{2\pi x}{L}}{2^2\pi^2/L^2} \end{array}$$

$$\frac{2}{L}\left[\frac{L^2}{4} - \cancel{\frac{1}{2}\left[0 + \frac{1}{2^2\pi^2/L^2} - \frac{1}{2^2\pi^2/L^2}\right]}\right]$$

$$\boxed{= \frac{L}{2} = \langle x \rangle}$$

What is Δx ?

$$\langle x^2 \rangle = \int_0^L x^2 |\psi|^2 \, dx = \frac{2}{L}\int_0^L x^2 \sin^2\frac{\pi x}{L} \, dx$$

$$= \frac{2}{L}\int_0^2 x^2\left[\frac{1}{2} - \cos\frac{2\pi x}{L}\right]$$

$$= \frac{1}{L}\int_0^L x^2 - x^2\cos\frac{2\pi x}{L}$$

$$\frac{1}{L}\left[\frac{x^3}{3} - \left[\cancelto{0}{\frac{x^2\sin 2\pi x/L}{2\pi/L}} + 2x\,\frac{\cos 2\pi x/L}{4\pi^2/L^2} - \cancelto{0}{2\,\frac{\sin 2\pi x/L}{8\pi^3/L^3}}\right]\right]_0^L$$

$$\langle x^2 \rangle = \frac{x^3}{3} - 2x \frac{\cos 2\pi x/L}{4\pi^2/L^2} \Big|_0^L$$

$x^2 \quad \cos \frac{2\pi x}{L}$

$2x \quad + \quad \frac{\sin 2\pi x/L}{2\pi x/L}$

$2 \quad - \quad -\frac{\cos 2\pi x/L}{4\pi^2 x^2/L^2}$

$0 \quad + \quad -\frac{\sin 2\pi x/L}{8\pi^3 x^3/L^3}$

$$\langle x^2 \rangle = \frac{L^3}{3} - \frac{2L}{4\pi^2/L^2}$$

$$\langle x^2 \rangle = \frac{1}{L}\left\{ \frac{L^3}{3} - \frac{L^3}{2\pi^2} \right]$$

$$\langle x^2 \rangle = \frac{L^2}{3} - \frac{L^2}{2\pi^2}$$

$$\langle x \rangle^2 = \frac{L^2}{4}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{L^2}{3} - \frac{L^2}{2\pi^2} - \frac{L^2}{4}}$$

$$\boxed{\Delta x = 0.181 L}$$

$$\frac{L}{2} \pm \frac{L}{5}$$

$$\langle p \rangle = \int_{-\infty}^{\infty} p\,|\psi|^2\,dx = \int_{-\infty}^{\infty} \psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\psi\,dx$$

x-space

$$p = \left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)$$

$$\langle p \rangle = \frac{2}{L}\frac{\hbar}{i}\int_0^L \sin\frac{\pi x}{L}\left(\frac{d}{dx}\sin\frac{\pi x}{L}\right)dx$$

$$= \frac{2h}{iL}\int_0^L \sin\frac{\pi x}{L}\cos\frac{\pi x}{L}\,dx$$

$$= \frac{2h}{iL}\int_0^L \frac{\sin\frac{2\pi x}{L}}{2} = 0$$

$$\langle p^2 \rangle = \frac{2}{L} \int_0^L \sin\frac{\pi x}{L} \left(\frac{\hbar}{i} \frac{\partial}{\partial x} \right)^2 \sin\frac{\pi x}{L} \, dx$$

$$\langle p^2 \rangle = -\frac{2}{L} (-\hbar^2) \frac{\pi}{L} \int_0^L \sin\frac{\pi x}{L} \quad \sin\frac{\pi x}{L} \, dx$$

$$= \frac{2\hbar^2}{L} \frac{\pi^2}{L^2} \left(\frac{L}{2} \right) = \frac{\hbar^2 \pi^2}{L^2}$$

$$\Delta p = \frac{\pi \hbar}{L}$$

$$\Delta x \, \Delta p = (0.1811 L) \left(\frac{\pi \hbar}{2} \right) \gtrsim \frac{\hbar}{2} \quad !$$

you can proove it in any case !

$\langle L \rangle, \langle K \rangle, \langle U \rangle, \langle E \rangle$

obsererable values

$\hat{p} = \frac{\hbar}{i} \frac{\partial}{\partial x}$ you need them to find

the values

Table 6.2 Common Observables and Associated Operators

Observable	Symbol	Associated Operator
Position	x	x
Momentum	p	$\frac{\hbar}{i}\frac{\partial}{\partial x}$
Potential energy	U	$U(x)$
Kinetic energy	K	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
Hamiltonian	H	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + U(x)$
Total energy	E	$i\hbar\frac{\partial}{\partial t}$

$$\langle \hat{\ } \rangle = \int \psi^* \hat{\ } \psi \, dx$$

$$\psi_1 = Ae^{ik_1x} + Be^{-ik_1x}$$

$$E < U$$

$$\psi_2 = Ce^{-\alpha x}$$

$$\alpha = \sqrt{\frac{2m}{\hbar^2}(U-E)}$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}$$

ψ and ψ' are cont

$A \rightarrow$ incidAt

Reflectd $\leftarrow B$

at $x=0$, $\psi_1 = A + B = C$

$$\psi_1' = Aike^{ikx} - ikBe^{-ikx}$$

$$\psi_2' = -C\alpha e^{\alpha x}$$

$x=0$, $\psi_1' = \psi_2'$

$$ik(A-B) = -\alpha C$$

$$A + B = C$$

$$ik(A-B) = -\alpha(A+B)$$

$$ikA - Bik = -\alpha A - \alpha B$$

$$(ik+\alpha)A = (ik-\alpha)B$$

$$\frac{B}{A} = \frac{ik+\alpha}{ik-\alpha}$$

$$R = \frac{B}{A} * \left(\frac{B}{A}\right)^* = \frac{ik+\alpha}{ik-\alpha} \cdot \frac{-ik+\alpha}{-ik-\alpha}$$

Reflection coff

$$\frac{-ik+\alpha}{-ik+\alpha} = 1$$

classical

100% will be Reflected

C, A

$$ik(A-B) = -\alpha C$$

$$ik \cdot A+B = C \cdot ik$$

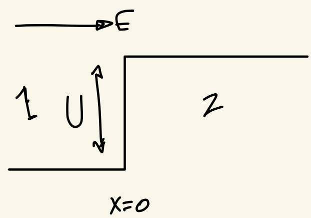
$$2ikA = C(ik-\alpha)$$

$$\frac{C}{A} = \frac{2ik}{ik-\alpha}$$

$$T = \frac{C}{A}\cdot\frac{C^{*}}{A} = \frac{2ik}{ik-\alpha}\cdot\frac{-2ik}{-ik-\alpha} = \frac{4k^2}{\alpha^2+k^2} > 0$$

Transmition value

What if $E > U$?



C.M : $T = 100\%$, $R = 0\%$

$$\frac{d^2\psi}{dx^2} + \underbrace{\frac{2mE}{\hbar^2}}_{k^2}\psi = 0 \qquad 0 < x$$

$$\psi_I = Ae^{ikx} + Be^{-ikx}$$

$x > 0$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

$$\psi = Ce^{ik'x} + \cancel{De^{-ik'x}} \quad 0$$

$$k' = \frac{2m}{\hbar^2}(E-U)$$

$$\psi(0) \rightarrow A+B = C$$

$$\psi_1'(0) = \psi_2' = ik(A-B) = ik'C$$

$$ik'C = ik(A+B)$$

$$(k-k')A = (k+k')B$$

$$\frac{B}{A} = \frac{k-k'}{k+k'} =$$

$$R = \frac{(k-k')^2}{(k+k')^2} > 0$$

there is a value

$$T = \frac{4kk'}{(k+k')^2}$$

$$k' = \sqrt{\frac{2m}{\hbar}(E-U)}$$

if $E >>> U$

$$E - U = E$$

$$k \sim k'$$

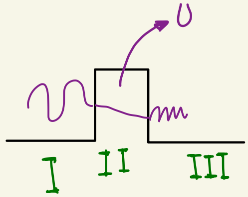
$R = 0$

$T = 1$

$E << U$

$R = 1, \ T = 0$

E high → T higher



$$U(x) \begin{cases} 0 & x<0 & \psi_{I} \\ U & 0<x<L & \psi_{II} \\ 0 & x>L & \psi_{III} \end{cases}$$

$$\psi_{I} = \underbrace{A e^{ikx}}_{\text{incident}} + \underbrace{B e^{-ikx}}_{\text{Ref}}$$

$$\psi_{III} = F e^{ikx} + \cancel{G e^{-kx}} \quad 0$$

II

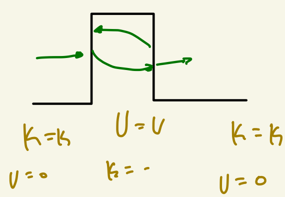
$$\frac{d^2\psi}{dx^2} - \underbrace{\frac{2m}{\hbar^2}(U-E)}_{\alpha^2}\psi = 0$$

$$\psi_{II}(x) = Ce^{-\alpha x} + De^{+\alpha x}$$

I = III

has the

Same E



because of the conservation of E but A is diff

Bondry conditions

$U < E$

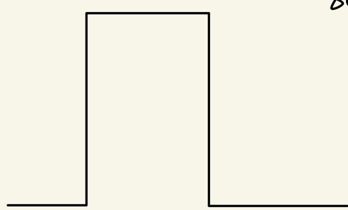
$$T = \left| \frac{F}{A} \right|^2$$

$$T = \left[1 + \frac{1}{4} \left[\frac{U^2}{E(U-E)} \right] \sinh^2 \alpha L \right]^{-1}$$

$$\alpha = \sqrt{\frac{2m}{\hbar}(U-E)}$$

$E >> U$

Only solution



$E > U$

$$\psi_{II} = C e^{ik'x} + D e^{-ik'y}$$

ψ_I the same

ψ_{III} the same

Bondry conditions

$$T = \left[1 + \frac{1}{4} \left[\frac{U^2}{E(E-U)} \right] \sin^2 k' L \right]^{-1}$$

$$E_n = U + \frac{n^2 \pi^2 \hbar^2}{2m L^2} \longrightarrow T = 100\%$$

How at $n = 1, 2, \ldots$ is R = zero?

Constructive and distructive

U has two

central U

Coulmb U $\longrightarrow$ $U = \frac{2kZe^2}{r}$ daughter

$T = \exp\left[-4\pi Z\sqrt{\frac{E_0}{E}} + 8\sqrt{\frac{ZR}{r_0}}\right]$ dauter

radius of

$R = 1.3A^{1/3}$

mass

Energy of α Particle

half life time

$N_0 \longrightarrow \frac{N_0}{2}$

r_0 = Bohr radius $\alpha = \frac{\hbar^2}{m_\alpha k e^2} = \frac{1}{7295}$ 0.0524 nm

7.25×10^{-15} m

$E_0 = R$

$$\frac{ke^2}{2r_0} = \left(\frac{a_0}{r_0}\right)\left(\frac{ke^2}{2a_0}\right) = 7245\,(13.6\,eV)$$

$$= 0.0993\ MeV$$

$\lambda =$ decay rate

$\lambda = f\,T = 10^{21}\,T$

$\downarrow$

frequency =

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{10^{21}\,T}$$

$$^{232}_{90}Th \xrightarrow{\alpha} {}^{228}_{88}Ra \xrightarrow{\alpha} {}^{224}_{86}Rn \xrightarrow{\alpha} {}^{220}_{84}Po \longrightarrow {}^{116}_{82}Pb$$

$$\downarrow \alpha$$

$$\downarrow \beta$$

Stable

Pb

CH 8

A Paraticle in a Box

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + U(x)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

$U(x) \longrightarrow U(x,y,z)$; $\psi(x,t) \longrightarrow \psi(x,y,z,t)$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2$$

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + U(\vec{r})\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

if U does not depend on t $\Longrightarrow$ $\psi(\vec{r},t) = \psi(\vec{r})\,\phi(t)$

time-independent

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r}) + U(r)\psi(\vec{r}) = E\psi(\vec{r})$$

$$P(\vec{r},t) = |\psi(\vec{r},t)|^2 = |\psi(r)|^2$$

$$\phi(t)^2 = 1$$

Stationary states E

$$U = \begin{cases} 0 & 0 < x,y,z < L \\ \infty & \text{otherwise} \end{cases}$$

x, y, z are independent coordinates

$$\psi(x,y,z) = \psi_1(x)\,\psi_2(y)\,\psi_3(z)$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y,z) = E\psi(x,y,z)$$

$$-\frac{\hbar^2}{2m}\psi(x,y,z) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\psi(x,y,z) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\psi(x,y,z) = E\psi(x,y,z)$$

$$-\frac{\hbar^2}{2m}\left[\psi_2\psi_3\frac{d^2\psi_1}{dx^2} + \psi_1\psi_3\frac{d^2\psi_2}{dy^2} + \psi_1\psi_2\frac{d^2\psi_3}{dz^2}\right] = E\psi_1\psi_2\psi_3$$

$$\psi_1\psi_2\psi_3$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{1}{\psi_1}\frac{d^2\psi_1}{dx^2}} - \boxed{\frac{\hbar^2}{2m}\frac{1}{\psi_2}\frac{d\psi_2}{dy^2}} - \boxed{\frac{\hbar^2}{2m}\frac{1}{\psi_3}\frac{d^2\psi_3}{dz^2}} = E$$

$$E_1 + E_2 + E_3 = E$$

$$\frac{-\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} = E_1\psi_1 \Longrightarrow \frac{d^2\psi}{dx^2} + \frac{2mE_1}{\hbar^2}\psi_1 = 0$$

$$\psi_1 = A\sin k_1 x + \cancelto{0}{B\cos k_1 x}$$

$$\psi_1 = A\sin k_1 x = A\sin\frac{n_1\pi}{L}x$$

$$\psi_2 = A'\sin\frac{n_2\pi}{L}y$$

$$\psi_3 = A''\sin\frac{n_3\pi}{L}z$$

$n_1, n_2, n_3 = 1, 2, 3 \ldots$

$\psi = \psi_1 \psi_2 \psi_3$

$$\psi(x,y,z) = A \sin \frac{n_1 \pi x}{L} \sin \frac{n_2 \pi}{L} y \sin \frac{n_3 \pi}{L} z$$

$$1 = \iiint |\psi|^2 \, dx\, dy\, dz =$$

$$A^2 \int_0^L \sin \frac{n_1 \pi}{L} x \, dx \int_0^L \sin \frac{n_1 \pi y}{L} \, dy \int_0^L \sin^2 \frac{n_3 \pi}{L} z \, dz = 1$$

$\cancel{A}$ $\cancel{A}$

0

$$A^2 \int_0^L \frac{1}{2} - \frac{1}{2} \cos \frac{2 n_1 \pi x}{L} \, dx \; \ldots\ldots$$

$$A^2 \quad \frac{L}{2} \cdot \frac{L}{2} \cdot \frac{L}{2} = 1$$

$$A^2 \left(\frac{L}{2}\right)^3 = 1$$

$$\boxed{A = \left(\frac{2}{L}\right)^{3/2}}$$

$$E = E_1 + E_2 + E_3$$

$$\downarrow$$

$$\psi_1(L) = 0$$

$$p = \frac{h}{\lambda}$$

$$\psi_1 = A \sin k_1 L = 0$$

$$k_1 L = n\pi \implies k_1 = \frac{n\pi}{L}, \quad k_1 = \frac{2\pi}{\lambda} \rightarrow \frac{p}{\hbar}$$

$$\rightarrow k_1 = \frac{p}{\hbar} \longrightarrow p = k_1 \hbar \longrightarrow p = \frac{n_1 \pi \hbar}{L}$$

$$E_1 = \frac{p^2}{2m} = \frac{n_1^2 \pi^2 \hbar^2}{2L^2 m}$$

$$E = E_1 + E_2 + E_3 = \frac{\hbar^2 \pi^2}{2mL^2} (\overbrace{n_1^2 + n_2^2 + n_3^2}^{n^2})$$

$$E_2 = \frac{n_2^2 \pi^2 \hbar^2}{2L^2 m}$$

$$\psi = \psi(\vec{r})\, e^{-i\omega t}$$

$$\omega = \frac{E}{\hbar}$$

$$E_3 = \frac{n_3 \pi^2 \hbar^2}{2L^2 m}$$

total

$$\psi_{n_1 n_2 n_3} = \left(\frac{2}{L}\right)^{3/2} \sin \frac{n_1 \pi x}{L} \sin\left(\frac{n_2 \pi y}{L}\right) \sin\left(\frac{n_3 \pi z}{L}\right) e^{-iE_{n_1 n_2 n_3} t/\hbar}$$

$$E_{1,2,3} = \frac{\pi \hbar^2}{2mL^2}(n_1^2 + n_2^2 + n_3^2)$$

* Ψ_{21} and Ψ_{12} have the same Energy but different wave function "called degenerate, degeneracy"

$$\Psi_{221} = A \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\sin\left(\frac{\pi z}{L}\right)e^{-iE_{221}t/\hbar}$$

$$\Psi_{212} = A \sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{\pi y}{L}\right)\sin\left(\frac{2\pi z}{L}\right)e^{-iE_{212}t/\hbar}$$

$$\Psi_{122} = A \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)\sin\left(\frac{2\pi z}{L}\right)e^{-iE_{122}t/\hbar}$$

$$E_{221} = E_{212} = E_{122} = \frac{9\pi^2\hbar^2}{2mL^2}$$

Table 8.1 Quantum Numbers and Degeneracies of the Energy Levels for a Particle Confined to a Cubic Box*

n_1	n_2	n_3	n^2	Degeneracy
1	1	1	3	None
1	1	2	6	Threefold
1	2	1	6	
2	1	1	6	
1	2	2	9	Threefold
2	1	2	9	
2	2	1	9	
1	1	3	11	Threefold
1	3	1	11	
3	1	1	11	
2	2	2	12	None

*Note: $n^2 = n_1^2 + n_2^2 + n_3^2$.

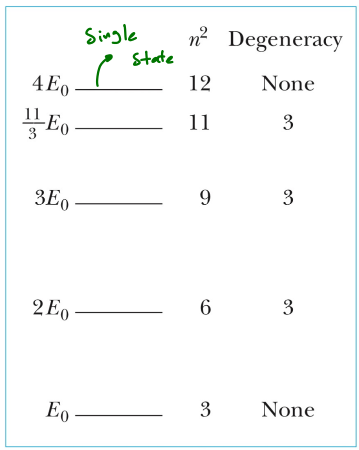


Figure 8.3 An energy-level diagram for a particle confined to a cubic box. The ground-state energy is $E_0 = 3\pi^2\hbar^2/2mL^2$, and $n^2 = n_1^2 + n_2^2 + n_3^2$. Note that most of the levels are degenerate.

I will see the differance if I apply M or E field force if it is not cube:

$$E_{1,2,3} = \frac{\pi^2 \hbar^2}{2m}\left[\frac{n_1^2}{L_1^2} + \frac{n_2^2}{L_2^2} + \frac{n_3^2}{L_3^2}\right]$$

Real case H atom

$$\boxed{U(r) = -\frac{Z k e^2}{r}}$$

r : the radius

θ = the angle between z and r

ϕ = is the Projection for x,y plane

$$\frac{-\hbar^2}{2m_e}\nabla^2\psi(\vec{r}) + U(r)\,\psi(\vec{r}) = E\,\psi(\vec{r})$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \left(\frac{2}{r}\right)\frac{\partial}{\partial r} + \frac{1}{r^2}\left[\frac{\partial}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \csc^2\theta\,\frac{\partial^2}{\partial\phi^2}\right]$$

$$\psi(r,\theta,\phi) = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$\frac{d^2 \Phi}{d\phi^2} + m_l^2 \Phi(\theta) = 0$$

m_l: magnetic quantum number = integer

$$\Phi(\phi) = \exp(i m_l \phi)$$

$$e^{i m_l \phi} = e^{i m_l (\phi + 2\pi)} = e^{i m_l \phi}\, e^{i m_l 2\pi}$$

1

integer

$P_l^{m_l}$

$m_l = 0, \pm 1, \pm 2 \dots \pm l$

orbital quantum number

$l = 0, 1, 2, 3 \dots max$

+ only

$$Y_l^{m_l} = \Theta \Phi = P_l^{m_l}(\theta)\, e^{i m_l \phi}$$

$$P_0^0 = 1$$
$$P_1^0 = 2 \cos \theta$$
$$P_1^1 = \sin \theta$$
$$P_2^0 = 4(3 \cos^2 \theta - 1)$$
$$P_2^1 = 4 \sin \theta \cos \theta$$
$$P_2^2 = \sin^2 \theta$$
$$P_3^0 = 24(5 \cos^3 \theta - 3 \cos \theta)$$
$$P_3^1 = 6 \sin \theta (5 \cos^2 \theta - 1)$$
$$P_3^2 = 6 \sin^2 \theta \cos \theta$$
$$P_3^3 = \sin^3 \theta$$

Table 8.3 The Spherical Harmonics $Y_\ell^{m_\ell}(\theta, \phi)$

$$Y_0^0 = \frac{1}{2\sqrt{\pi}}$$
$$Y_1^0 = \frac{1}{2}\sqrt{\frac{3}{\pi}} \cdot \cos\theta$$
$$Y_1^{\pm 1} = \mp\frac{1}{2}\sqrt{\frac{3}{2\pi}} \cdot \sin\theta \cdot e^{\pm i\phi}$$
$$Y_2^0 = \frac{1}{4}\sqrt{\frac{5}{\pi}} \cdot (3\cos^2\theta - 1)$$
$$Y_2^{\pm 1} = \mp\frac{1}{2}\sqrt{\frac{15}{2\pi}} \cdot \sin\theta \cdot \cos\theta \cdot e^{\pm i\phi}$$
$$Y_2^{\pm 2} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \cdot \sin^2\theta \cdot e^{\pm 2i\phi}$$
$$Y_3^0 = \frac{1}{4}\sqrt{\frac{7}{\pi}} \cdot (5\cos^3\theta - 3\cos\theta)$$
$$Y_3^{\pm 1} = \mp\frac{1}{8}\sqrt{\frac{21}{\pi}} \cdot \sin\theta \cdot (5\cos^2\theta - 1) \cdot e^{\pm i\phi}$$
$$Y_3^{\pm 2} = \frac{1}{4}\sqrt{\frac{105}{2\pi}} \cdot \sin^2\theta \cdot \cos\theta \cdot e^{\pm 2i\phi}$$
$$Y_3^{\pm 3} = \mp\frac{1}{8}\sqrt{\frac{35}{\pi}} \cdot \sin^3\theta \cdot e^{\pm 3i\phi}$$

it lefts R!

$\Phi = Y_\ell^{m_\ell}(\theta, \phi)$ → Spherical harmonics

the same for all the atoms

$E_{n,\ell}$

quanized!

$$\boxed{L = \sqrt{\ell(\ell+1)}\,\hbar \quad \text{for } \ell = 0, 1, 2 \ldots}$$

→ total Angular momentum orbital

for the same r → different ℓ!

$\vec{L} = L_x, L_y, L_z$

L_x, L_y changing

L_z → constant

quanized

$L_z = m_\ell \hbar$ for $m_\ell = 0, \pm 1, \pm 2 \ldots\ldots \pm \ell$

quanized!

$$\cos\theta = \frac{L_z}{|L|} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}}$$

* E does not depend on m_ℓ

* ψ depends on m_ℓ

different solutions with the same E. ("degenrcy")

$2\ell + 1 \Longrightarrow$ degree of degenrcy

How many states with the same E?

L_x, L_y = can not be calculated

Table 8.5 Spectroscopic Notation for Atomic Shells and Subshells

n	Shell Symbol	ℓ	Shell Symbol
1	K	0	s
2	L	1	p
3	M	2	d
4	N	3	f
5	O	4	g
6	P	5	h
...		...	

it lefts R!

only for H and H like atom (one $\bar{e}$ only)

$$E_n = \left(\frac{ke^2}{2a_0}\right)\left(\frac{Z^2}{n^2}\right) \quad n = 1, 2, 3 \ldots \qquad E_n = -\frac{13.6\ eV}{n^2}$$

$Z = 1$ for H

n = Principal quantum number 1, 2, 3, ...

L = angular momentum quantum number 0, 1, 2, 3 ... n-1

m_l = magnetic quantum number 0, ±1, ±2 ... ±l

l = orbital quantum number

The degree of degeneracy is = n^2

Table 8.4 The Radial Wavefunctions $R_{n\ell}(r)$ of Hydrogen-like Atoms for $n = 1, 2,$ and 3

n	ℓ	$R_{n\ell}(r)$
1	0	$\left(\frac{Z}{a_0}\right)^{3/2} 2e^{-Zr/a_0}$
2	0	$\left(\frac{Z}{2a_0}\right)^{3/2} \left(2 - \frac{Zr}{a_0}\right) e^{-Zr/2a_0}$
2	1	$\left(\frac{Z}{2a_0}\right)^{3/2} \frac{Zr}{\sqrt{3}\,a_0} e^{-Zr/2a_0}$
3	0	$\left(\frac{Z}{3a_0}\right)^{3/2} 2\left[1 - \frac{2Zr}{3a_0} + \frac{2}{27}\left(\frac{Zr}{a_0}\right)^2\right] e^{-Zr/3a_0}$
3	1	$\left(\frac{Z}{3a_0}\right)^{3/2} \frac{4\sqrt{2}}{3} \frac{Zr}{a_0}\left(1 - \frac{Zr}{6a_0}\right) e^{-Zr/3a_0}$
3	2	$\left(\frac{Z}{3a_0}\right)^{3/2} \frac{2\sqrt{2}}{27\sqrt{5}} \left(\frac{Zr}{a_0}\right)^2 e^{-Zr/3a_0}$

Selection Rules

forbidden = impossible

$\Delta \ell = \pm 1 \qquad |\Delta \ell| = 1$

$\Delta n > 1$

Probablity density for r

$$P(r) = r^2 |R_{n,l}(r)|^2$$

$$P(\theta, \phi) = |\Theta_{l,m_l}(\theta)\,\Phi_{m_l}(\phi)|^2$$