

# PHYS#215 - CH#18 - Q&A

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## Concept Review

**Cosmology** is the study of the Universe on its grandest scale – its birth, evolution, and ultimate fate (introductory paragraph). **Olbers's paradox** – the darkness of the night sky – has profound implications and is resolved primarily by the finite age of the Universe ([Section 18.1](#)).

Historically, the first piece of evidence for this definite beginning, the **big bang**, was Edwin Hubble's and Georges Lemaître's discovery of the expansion of the Universe ([Section 18.2a](#)): most galaxies are moving away from us, and the speed of any galaxy, determined from the **redshift** of its spectrum, is proportional to its distance. The plot of redshift (or recession speed) vs. distance is the **Hubble diagram**, the proportionality is known as **Hubble's law** (or the **Hubble–Lemaître law**), and the constant of proportionality is called **Hubble's constant**,  $H_0$ . The expansion of the Universe has no definite center within any physically accessible dimensions ([Section 18.2b](#)). Note that *space itself* expands; galaxies are *not* flying apart from each other through a preexisting space ([Section 18.2c](#)).

There are at least two ways in which to determine the age of the Universe ([Section 18.3a](#)). First, the Universe must be at least as old as the oldest objects within it. Second, the age of the Universe is the elapsed time since the big bang, when all the material that formed our Galaxy and any other

was essentially in the same place. The [Hubble time](#) is the expansion age of the Universe under the assumption that gravity has not been slowing down the expansion; its value depends on Hubble's constant. If the true expansion age of the Universe is only two-thirds of the Hubble time, as had been expected by some theorists, then its value is smaller than the ages of the oldest globular star clusters, leading to an age crisis.

The quest for an accurate measurement of Hubble's constant has been long and arduous, with many conflicting results ([Section 18.3b](#)). Generally, astronomers have used the inverse-square law of light to determine the distances of galaxies from the luminosities and measured brightnesses of certain kinds of stars within them. Cepheid variables have played a very important role in this regard. A major project of the Hubble Space Telescope was to determine the Hubble constant by measuring the distances of galaxies with Cepheid variables, which can be identified by looking at their light curves to see that they (1) have a sharp rise and slower fall, and (2) have periods of days or weeks rather than shorter than a day ([Section 18.3c](#)). One difficulty is that the recession speeds of relatively nearby galaxies can deviate from the [Hubble flow](#), the assumed smooth expansion of the Universe ([Section 18.3d](#)). Such [peculiar motions](#) are produced by the gravitational tug of the Local Group and other mass concentrations. Nevertheless, several techniques were combined in the late 1990s and in the subsequent two

decades to achieve a convincing value for Hubble's constant, with the 2018 measured values being  $H_0 = 67$  km/s/Mpc from the cosmic microwave background radiation and 73 km/s/Mpc from Type Ia supernovae ([Section 18.3e](#)); here we adopt a value of 71 km/s/Mpc.

Mathematically, studies of the expansion history and overall geometry of the Universe are conducted with Einstein's general theory of relativity ([Section 18.4a](#)). To simplify the equations and achieve reasonable progress, we assume the [cosmological principle](#): on the largest size scales, the Universe is [homogeneous](#) (has the same average density everywhere) and [isotropic](#) (looks the same in all directions). Observationally, this assumption appears to be valid. In addition, we initially assume that Einstein's [cosmological constant](#) ( $\Lambda$ , Lambda), or any other source of long-range repulsion, is zero ([Section 18.4b](#)). However, as we discuss later in the chapter, this constant (or something that behaves like it) actually appears to have a nonzero value.

The result (ignoring  $\Lambda$ ) is that the ultimate fate of the Universe depends only on the ratio of the average matter density of the Universe to a specific [critical density](#) ([Section 18.4c](#)). If the average density exceeds the critical density, then this ratio, known as  $\Omega_M$ , would be greater than 1 and gravity will eventually cause the Universe to collapse to a fiery death informally called the "big crunch." On the other hand, if the average density is low, such that  $\Omega_M$  is less than or equal to 1, the Universe will expand forever, though more

and more slowly with time; indeed, in the case that  $\Omega_M$  is equal to 1, the expansion would halt as time approaches infinity.

In the absence of  $\Lambda$ , the value of  $\Omega_M$  also determines the overall geometry of the Universe ([Section 18.4c](#)). If the average density equals the critical density, we live in a [flat \(critical\) universe](#) whose volume is formally infinite, and whose age is two-thirds ( $\frac{2}{3}$ ) of the Hubble time. A [closed universe](#) has  $\Omega_M > 1$ ; it has positive curvature and finite volume, resembling a sphere (technically, a “hypersphere”). Its age is less than  $\frac{2}{3}$  of the Hubble time. The last alternative, an [open universe](#), occurs if  $\Omega_M < 1$ ; it has negative curvature (“hyperbolic”), and its volume is formally infinite. Its age is  $\frac{2}{3}$  to 1 times the Hubble time. Two-dimensional analogues for such flat, positively curved, and negatively curved universes are an infinite sheet of paper, a sphere, and an infinite horse’s saddle or potato chip ([Section 18.4d](#)). We can attempt to predict the ultimate fate of the Universe from a number of methods ([Section 18.4e](#)), all of which are difficult and have substantial uncertainties ([Section 18.4f](#)).

One can determine the expansion history of the Universe by measuring the distance and redshift of objects at a very wide range of distances ([Section 18.5a](#)). The most successful recent technique for determining accurate distances at high redshifts has been to measure the apparent brightness of Type Ia (incinerating-white-dwarf) supernovae ([Section 18.5b](#)). Since the very distant ones are fainter than expected

given our past understanding of an expanding Universe, the result is that the Universe appears to be expanding faster now than in the past: we currently live in an [accelerating universe](#) ([Section 18.5c](#))! This observation resolves the age crisis of the Universe and shows that the Universe will expand forever if the acceleration continues.

Apparently, a long-range “antigravity” effect exists in the Universe, like the cosmological constant that Einstein had previously postulated with a specific value to obtain a static universe ([Section 18.5d](#)). Physically, the cosmological constant may correspond to a vacuum energy consisting of [quantum fluctuations](#), the spontaneous formation (followed by rapid destruction) of virtual pairs of particles and antiparticles. However, there are many alternatives to the cosmological constant; we don’t know the nature of this [dark energy](#) that fills the Universe ([Section 18.5e](#)). Dark energy is ubiquitous, and it constitutes about 70 per cent of the Universe. All together, there is enough luminous matter, dark matter, and dark energy to make the Universe spatially flat (Euclidean) over the largest scales.

The above conclusions are so astonishing that alternative explanations have been sought, but they have not been compelling ([Section 18.5f](#)). One triumph is that measurements of supernovae from a time when the Universe was half its present age show that, as expected, the expansion rate was initially decelerating. The change from deceleration to acceleration, a “cosmic jerk,” occurred about 5

billion years ago. Moreover, there are now additional observations, independent of supernovae, which imply the presence of dark energy and cosmic acceleration (or, alternatively, to a failure of the general theory of relativity over very large distances).

If the Universe lasts forever, many interesting physical processes will occur ([Section 18.6](#)). We now live in the [stelliferous era](#), filled with stars, but eventually all normal stars will burn out, and the Universe will enter the [degenerate era](#), dominated by objects like white dwarfs and neutron stars. After a very long time, these degenerate objects will disintegrate, leaving the Universe in the [black-hole era](#). But even black holes decay owing to the Hawking evaporation process after exceedingly long times; the Universe will enter its last stage, the [dark era](#), as a cold, nearly empty space consisting of extremely low-energy photons and a few elementary particles.

## Questions

1. Describe Olbers's paradox – the darkness of the night sky – and several possible resolutions.
2. Summarize the first observational evidence found for the expansion of the Universe and a possible beginning of time.
3. Why does the recession of galaxies not necessarily imply that the Milky Way Galaxy is at the center of the Universe?
4. Explain how the effective center of expansion can be in an unobservable spatial dimension.
5. **(a)** What do we mean when we say that the Universe is expanding? **(b)** Are galaxies moving through some preexisting space? **(c)** Do humans, planets, stars, and galaxies themselves expand?
6. What are two ways of estimating the age of the Universe?
7. Describe how the current value of Hubble's constant is measured.
8. Why can relatively nearby galaxies give erroneous values for Hubble's constant?
- †9. Calculate the Hubble time (that is, the expansion age of the Universe assuming no deceleration) if Hubble's constant is  $71 \text{ km/s/Mpc}$ .

- 10.** Suppose we assume that the recession speed of a given galaxy never changes with time. Why is the derived age of the Universe likely to be an overestimate of its true age?
- 11.** If the expansion rate of the Universe were increasing (rather than decreasing) with time, would the true expansion age be less than or greater than that derived by assuming a constant expansion rate?
- 12.** What is the “age crisis” that rocked cosmology until the late 1990s? Discuss two developments that helped alleviate this crisis.
- 13.** Why are exploding white dwarf stars so useful for measuring the distances of galaxies?
- 14.** State the cosmological principle. Why is it a reasonable assumption?
- 15.** Define what we mean by the cosmological constant,  $\Lambda$ .
- 16.** Summarize the possible types of overall geometry for a homogeneous, isotropic universe with a cosmological constant (or dark energy) equal to zero.
- 17.** Describe how the geometry of the Universe is intimately connected to the ultimate fate of the Universe, assuming the cosmological constant (or dark energy) is zero.
- 18.** Explain what we mean by the critical density of the Universe.

**19.** Discuss why  $\Omega_M$ , the ratio of the average matter density to the critical density of the Universe, is such an important parameter.

**20. (a)** Summarize different methods for determining the value of  $\Omega_M$ . **(b)** What are the difficulties in actually implementing these methods?

**21.** Does the average matter density appear to be large enough to close the Universe?

**22.** Discuss the main conclusion of the distant supernova studies, and its implications.

**23.** Explain how Einstein's cosmological constant is relevant to the conclusion in question 22.

**24.** What do we mean by "dark energy"? How might it affect the expansion of the Universe?

**25.** If the expansion of the Universe is currently accelerating, can we conclude that the Universe will necessarily expand forever?

**26.** What is the overall shape of the Universe, as implied by the total amount of luminous matter, dark matter, and dark energy?

**27.** Why might high-redshift supernovae appear fainter than expected, other than an acceleration of the expansion rate of the Universe?

- 28. (a)** Has the expansion rate of the Universe always been accelerating? **(b)** What could cause a change from deceleration to acceleration?
- 29.** What are some of the processes that will eventually occur if the Universe expands forever?
- 30. True or false?** The curvature of three-dimensional space cannot be detected because we live within the three spatial dimensions.
- 31. True or false?** Current observational evidence suggests that the Universe will eventually collapse; the expansion rate is decelerating rapidly.
- 32. True or false?** Most astronomers believe that “Olbers’s paradox,” the darkness of the night sky, is resolved by the fact that the Universe has a finite age.
- 33. True or false?** According to the “cosmological principle,” the Universe is homogeneous and isotropic on the largest size scales, but its properties can change with time.
- 34. True or false?** A given straight line in a positively curved Universe has zero straight lines parallel to it.
- 35. True or false?** Current observations suggest that the average density of matter (not including energy) in the Universe is consistent with being equal to the critical density; that is,  $\Omega_M = 1$ .

**36. True or false?** The “cosmological constant” was a concept introduced by Albert Einstein to support a static universe, one that is neither expanding nor contracting.

**37. True or false?** A value of  $\Omega_{\text{total}}$  that is greater than 1 indicates that the Universe has positive curvature, while a value less than 1 indicates that the Universe has negative curvature.

**38. True or false?** According to Einstein’s equation,  $E = mc^2$ , dark energy and dark matter are essentially the same thing.

**39. True or false?** The reason galaxies are not being ripped apart by dark energy is that the gravitational attraction of matter over the distances between stars is much greater than the repulsive effect of dark energy.

**40. Multiple choice:** Which one of the following is an assumption made as part of the “cosmological principle”? **(a)** The Universe is homogeneous on scales smaller than galaxies. **(b)** The Universe has had a constant average density over time. **(c)** The Universe is isotropic on scales of billions of light-years. **(d)** The age of the Universe is finite. **(e)** The Universe is expanding at an accelerating rate.

**†41. Multiple choice:** Using incomplete data, suppose you find that the value of Hubble’s constant in our Universe is about 670 km/s/Mpc instead of about 67 km/s/Mpc (as found from data obtained by the Planck satellite). Roughly how old would you calculate the Universe to be? **(a)** 138 billion years.



(b) 1/670 billion years. (c) 1.38 billion years. (d) 670 billion years. (e) 13.8 billion years.

†42. **Multiple choice:** Suppose from the top of your palace you notice a circular patch of land of radius 10 meters that needs landscaping. You hire Edwin, who charges \$5 per square meter. Edwin completes the job and comes back with a bill of \$3000. What type of homogeneous, isotropic space can you deduce that you live on? (a) Spherical. (b) Hyperbolic. (c) Euclidean. (d) Shaped like a doughnut. (e) A space devoid of logic.

43. **Multiple choice:** Which one of the following statements about the expansion of the Universe is *false*? (a) Dark energy acts as a long-range repulsive effect, causing the expansion of the Universe to currently accelerate. (b) If the expansion of the Universe were to always accelerate, the Universe would expand forever, even if it is closed (finite in volume). (c) During most of the first 10 billion years of its existence, the expansion of the Universe decelerated with time. (d) If all the matter in the Universe were suddenly changed to antimatter, the Universe would expand even faster. (e) If the expansion of the Universe continues to accelerate, there will eventually be no other galaxies visible in the night sky.

44. **Multiple choice:** Which one of the following statements about Hubble's constant,  $H_0$ , is *false*? (a) Hubble's constant changes with time. (b) The value of Hubble's constant is best determined with the very nearest galaxies, whose distances

can be measured accurately. **(c)** Hubble's constant is believed to be constant throughout the Universe at a given time. **(d)** If the expansion of the Universe has always been constant (neither speeding up nor slowing down), then the age of the Universe is  $1/H_0$ . **(e)**  $H_0$  is the constant of proportionality between the observed recession speeds ( $v$ ) and distances ( $d$ ) of galaxies.

**†45. Multiple choice:** If we measure the recession speed of isolated galaxy Lemaître to be 35,000 km/s, what is Lemaître's approximate distance? **(a)** 50 Mpc. **(b)** 50 million light-years. **(c)** 500 Mpc. **(d)** 500 million light-years. **(e)** 350 Mpc.

**46. Multiple choice:** Which one of the following is *not* a logically possible solution to Olbers's paradox, regardless of whether it is correct or consistent with current observations? **(a)** The Universe has a finite age. **(b)** The Universe has a finite size. **(c)** The Universe is infinite in size, but there are few stars beyond a certain distance from us. **(d)** The Universe is infinite in size, but according to the inverse-square law we receive progressively less light from progressively more-distant stars. **(e)** The Universe is infinite in size, but expanding so fast that the light from stars in extremely distant galaxies is redshifted beyond the visible wavelengths.

**47. Multiple choice:** Which one of the following statements about the big-bang theory is *false*? **(a)** The redshifts of distant galaxies are a consequence of the wavelength of light

stretching during its journey to us. **(b)** All, or almost all, clusters of galaxies move away from each other. **(c)** Everything used to be much closer together and hotter than at the present time. **(d)** The matter density of the Universe is decreasing with time. **(e)** There is a unique center within the Universe that coincides with where the big bang happened.

**48. Multiple choice:** Observations of what kind of objects were the first to strongly imply that the expansion of the Universe is currently accelerating? **(a)** The cosmic background radiation left over from the big bang. **(b)** Type Ia supernovae. **(c)** Gamma-ray bursts. **(d)** Clusters of galaxies. **(e)** Gravitational lenses.

**49. Multiple choice:** Which one of the following distributions is isotropic but is *not* homogeneous? **(a)** The distribution of stars in a globular cluster, as measured from its center. **(b)** The distribution of globular clusters in our Galaxy, as measured from Earth. **(c)** The distribution of stars in our Galaxy, as measured from Earth. **(d)** The distribution of galaxy clusters in the Universe averaged over large scales, as measured from Earth. **(e)** The distribution of galaxy clusters in the Universe averaged over large scales, as measured from anywhere.

**50. Multiple choice:** Which one of the following is *not* a possible homogeneous, isotropic, expanding universe according to Einstein's general theory of relativity? (Assume there is no dark energy.) **(a)** A "flat" universe in which the laws

of Euclidean geometry are satisfied. **(b)** A “closed” universe in which there is no such thing as parallel lines. **(c)** A “closed” universe that will expand forever. **(d)** An “open” universe whose volume is infinite, and has been infinite since the beginning of time. **(e)** An “open” universe in which the volume of a large sphere of radius  $R$  is greater than  $(\pi R^3)$ .

**51. Fill in the blank:** Two types of stars that have played central roles as cosmological yardsticks are \_\_\_\_\_ variables and \_\_\_\_\_ supernovae.

Cepheid variables type Ia supernova

**52. Fill in the blank:** A time far in the future of the Universe, when the density of photons and elementary particles will be very low, is known as the \_\_\_\_\_.

Dark era

**53. Fill in the blank:** The expansion of the Universe appears to be accelerating, driven by the repulsive effect of the cosmological constant or, more generally, \_\_\_\_\_.

faster

**54. Fill in the blank:** A universe with \_\_\_\_\_ curvature is said to be “hyperbolic,” shaped somewhat like a potato chip.

negative

**†55. Fill in the blank:** In a universe where the Hubble constant is  $50 \text{ Gyr}^{-1}$  and in which  $\Omega_M = 0$  and there is no dark energy, you can determine that the universe has an age of \_\_\_\_\_.

$2 \times 10^{10} \text{ yr}$

†This question requires a numerical solution.



### Figure It Out 18.2 The Critical Density and $\Omega_M$

The “critical density,”  $\rho_{\text{crit}}$ , is defined to be that density of matter that would allow the Universe to expand forever, but only just barely. Its formula is

$$\rho_{\text{crit}} = 3H_0^2/(8\pi G),$$

where  $H_0$  is Hubble’s constant and  $G$  is Newton’s universal constant of gravitation. If the average matter density of the Universe ( $\rho_{\text{ave}}$ ) is greater than, equal to, or less than  $\rho_{\text{crit}}$ , then  $\Omega_M$  (Omega of the matter) is greater than, equal to, or less than 1, respectively, since we have defined  $\Omega_M = \rho_{\text{ave}}/\rho_{\text{crit}}$ .

For  $H_0 = 71$  km/s/Mpc, we find that  $\rho_{\text{crit}} = 9.4 \times 10^{-30}$  g/cm<sup>3</sup>, which is the equivalent of only about 5.6 hydrogen atoms per cubic meter of space! Clearly, our local surroundings are much denser, but this isn’t relevant. Only the average density of the *Universe as a whole* should be compared with  $\rho_{\text{crit}}$  to determine the value of  $\Omega_M$ . In fact, we find that  $\Omega_M$  is almost certainly less than 1. This means that on large scales the Universe is remarkably empty.

Note that, as the Universe ages, the average density of matter in the Universe decreases. However, if the value of  $\Omega_M$  initially exceeds 1, it does not later drop to 1 or less. This is because the value of  $H_0$  decreases with time, and hence the

critical density also decreases. The ratio of the average density to the critical density,  $\Omega_M$ , remains either above 1 or below 1 (or is exactly equal to 1) forever.

As we will discuss in [Section 18.5](#), we now know that in addition to matter, there is mysterious “dark energy” that contributes to the total density of the Universe.