Swinging Atwood Machine

A swinging Atwood machine consists of two non-colliding masses connected by an inextensible string over two frictionless support points (see figure below). The mass **M** moves **only vertically**, that is up and down, while the **mass m oscillate in the vertical plane**. Since the length of the string is **fixed** then the system has **two degrees of freedom**, the swinging angle **θ** of mass m and length of the swinging wire ***r*** **(**the distance of the swinging mass m from its point of support). The motion of the swinging mass can be regular or chaotic depending on **initial conditions** and the ration of the two masses **μ=M/m**. **This numerical assignment allows investigating the motion as a function of μ and the initial conditions**. In practice, the horizontal part of the string should be long enough to prevent collisions of the two masses.

The Lagrangian for the swinging mass m is

$$L_m = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + mgrcos\theta$$

Since the string is of constant length b, the distance of the counterweight mass **M** from its point of support is ***(b- r).*** The Lagrangian of **M** is

$$L_M = \frac{1}{2}M\left[\frac{d}{dt}(b-r)\right]^2 - Mgr = \frac{1}{2}M\dot{r}^2 - Mgr$$

where the constant term in the potential can be ignored. Adding together the two contributions gives the system full Lagrangian,

$$L_s = \frac{1}{2}(m+M)\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + gr(mcos\theta - M)$$

Check that Lagrange equations are given by

$$(m+M)\ddot{r} - mr\dot{\theta}^2 - mgcos\theta + Mg = 0$$

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta}^2 + mgrsin\theta = 0$$

Introducing the mass ratio μ and cancelling common factors in the second equation, the system to be written as

$$\ddot{r} = \frac{r\dot{\theta}^2 + g\cos\theta - \mu g}{1+\mu}; \ \ddot{\theta} = -\frac{2\dot{r}\dot{\theta} + g\sin\theta}{r}$$

a) **After checking all above derivations**, we need to proceed with the numerical integration of the above **second order differential equations** in time using the **following initial conditions**:

$$r(0) = a\ ; \dot{r}(0) = b\ ;\ \theta(0) = c\ ;\ \dot{\theta}(0) = d$$

Where a, b, c and d are to be chosen in the following **domains**:

$$a \in [1-2];\ b \in [0-1]\ ;\ c \in [0-2\pi]\ ;\ d \in [0-2]$$

These parameters are to be indicated under each graph along with the value of μ.

b) **Plot r(t), θ(t)** for μ = 0.2, 0.5, 1, 2, 5 for **two sets of initial conditions** that you select.

c) **Select one value of** μ and **one set of initial conditions to plot a phase diagram** $\boldsymbol{\dot{r}}$ **versus** ***r.***

d) You can also **design an interactive graphic** that allows displaying the trajectory of the swinging mass m when μ and the above initial conditions are varied.