PHYS.300

QUIZ # 1 Fall 211

Q1. Consider the following **rotation matrix**

$$\lambda = \begin{pmatrix} \sqrt{3}/2 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \sqrt{3}/2 \end{pmatrix}$$

a) Is this a **rotation matrix?** About which **axis** and by which **angle?** **(4pts)**

$\lambda = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}$ is a rotation matrix about $\hat{y}$- axis with angle $\theta = \frac{\pi}{3}$ or 30°

$|\lambda| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$ as required for proper rotations.

$-1 \leq \lambda_{ij} \leq 1$ as required for a rotation matrix since $\lambda_{ij} = \cos(\hat{x}_i', \hat{x}_j)$

b) Check that $\lambda\lambda^t = 1$ and deduce λ^{-1}. **(4pts)**

$$\lambda\lambda^t = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ \frac{1}{2} & 0 & \sqrt{3}/2 \end{pmatrix}\begin{pmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\lambda^{-1} = \lambda^t = \begin{pmatrix} \sqrt{3}/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ -1/2 & 0 & \sqrt{3}/2 \end{pmatrix}$$

c) Find (x′,y′,z′) the transformed of the **point** $(\sqrt{3}, 0, -1)$ under λ **(2pts)**

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix}\begin{pmatrix} \sqrt{3} \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

Q2. Consider a **cube of size L** and mass **m** that floats in a liquid of density ρ. When this cube is slightly pushed down from its equilibrium position then it will oscillate with a frequency ω. Assume that $\omega = f(L, m, \rho, g)$

a) Use dimensional analysis to deduce the **explicit dependence of ω** on the variables (L, m, ρ, g). **(4pts)**

$\omega = C L^{\alpha} m^{\beta} \rho^{\gamma} g^{\delta}$; C = dimensionless Constant.

$[\omega] = [L]^{\alpha} [M]^{\beta} [ML^{-3}]^{\gamma} [LT^{-2}]^{\delta}$

$T^{-1} = L^{\alpha - 3\gamma + \delta} M^{\beta + \gamma} T^{-2\delta}$

$\Rightarrow -2\delta = -1 \Rightarrow \delta = 1/2$

$\alpha - 3\gamma + \delta = 0$; $\beta + \gamma = 0$

We have 3 equation and 4 unknown, so one remains undefined. In this case we express all variable in terms of one, say γ

then $\beta = -\gamma$ and $\alpha = 3\gamma - \frac{1}{2}$

So that

$\omega = C L^{3\gamma - 1/2} m^{-\gamma} \rho^{\gamma} g^{1/2} = C\sqrt{\frac{g}{L}} \left(\frac{\rho L^3}{m}\right)^{\gamma}$

$\omega = C\sqrt{\frac{g}{L}}\, \Pi^{\gamma}$

$\Pi = \frac{\rho L^3}{m}$ is the dimensionless parameter

b) Can you **identify** the **dimensionless variable** that occurs? **(1pt)**

$\Pi = \frac{\rho L^3}{m}$ the dimensionless Variable from Π-Theorem.