KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DEPARTMENT OF PHYSICS

PHYS.300- Classical Mechanics I (TERM 211)

Solution.

Quiz #2

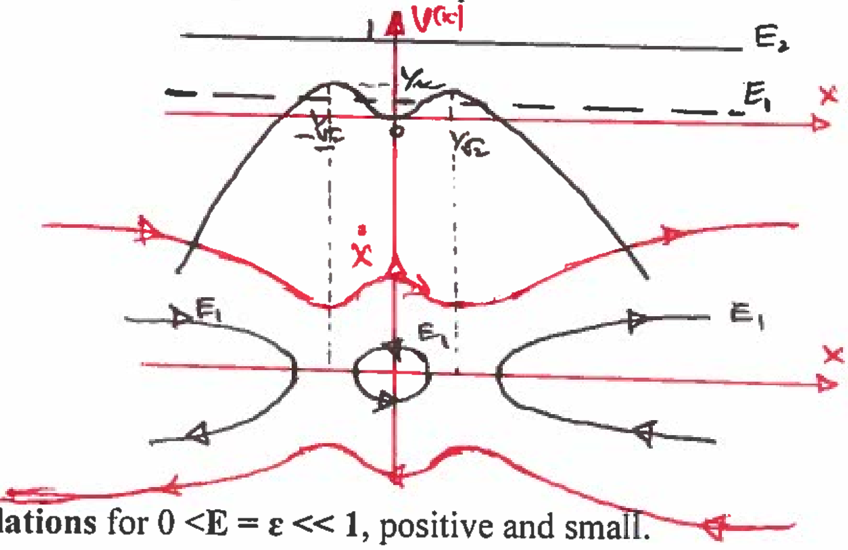
1. Consider the following paraboloid surface **$\Phi(x,y,z) = z - x^2 - y^2 = 1$**, with $z > 0$ find the **unit vector normal to this surface** at **(1/2, 1/2, 3/2)**.

$$\vec{\nabla}\Phi = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)\phi = (-2x, -2y, 1)$$

$$\hat{n} = \frac{\vec{\nabla}\Phi}{|\vec{\nabla}\Phi|} = \frac{-2x\vec{i} - 2y\vec{j} + \vec{k}}{\sqrt{(2x)^2 + (2y)^2 + 1^2}}\Bigg|_{(1/2, 1/2, 3/2)} = \frac{-\vec{i} - \vec{j} + \vec{k}}{\sqrt{3}}$$

2. Consider the following potential energy **$V(x) = x^2 - x^4$**, x in]- ∞, + ∞ [.

a. Sketch V(x) and the corresponding **phase diagram** for small positive **$0 < E_1 = \varepsilon \ll 1$** and for **$E_2 = 1$**.



b. Find the **period of small oscillations** for $0 < E = \varepsilon \ll 1$, positive and small.

Since for $x \ll 1$ we have $V(x) \simeq x^2 = \frac{1}{2} k x^2 \Rightarrow \frac{1}{2}k = 1 \Rightarrow k = 2$

then $\omega = \sqrt{\frac{k}{m}}$ and $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{2}} = \pi\sqrt{2m}$

Of course you get the same result through energy conservation

$$\mathcal{E} = \frac{1}{2}m\dot{x}^2 + V(x) \Rightarrow \dot{x}^2 = \frac{2}{m}(\varepsilon - V(x))$$

$$\Rightarrow \dot{x} = \frac{dx}{dt} = \sqrt{\frac{2}{m}}(\varepsilon - V(x)) \Rightarrow dt = \sqrt{\frac{m}{2}}\frac{dx}{\sqrt{\varepsilon - V(x)}}$$

$$T = \sqrt{2m}\int_{-x_0}^{x_0}\frac{dx}{\sqrt{\varepsilon - x^2}} = \sqrt{2m}\int_{-\sqrt{\varepsilon}}^{+\sqrt{\varepsilon}}\frac{dx}{\sqrt{\varepsilon - x^2}} = \sqrt{2m}\,\sin^{-1}\frac{x}{\sqrt{\varepsilon}}\Bigg|_{-\sqrt{\varepsilon}}^{+\sqrt{\varepsilon}} = \pi\sqrt{2m}$$