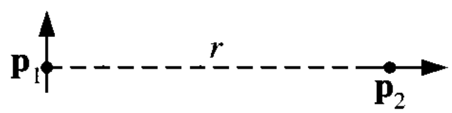
PHYS305 Homework# 5

Due on 25Nov2021

Q#1:

In the figure below $\vec{\boldsymbol{p}}_1$ and $\vec{\boldsymbol{p}}_2$ are perfect dipoles a distance r apart. What is the torque on p1 due to p2? What is the torque on p2 due to p1?



Q#2: Show that the interaction energy of two dipoles separated by a displacement r is:

$$U = \frac{1}{4\pi\varepsilon_o}\frac{1}{r^3}[\vec{p}_1.\vec{p}_2 - 3(\vec{p}_1.\hat{r})(\vec{p}_2.\hat{r})]$$

Q#3: A sphere of radius R carries a polarization $\vec{P}(r) = k\vec{r}$, where k is a constant and $\vec{r}$ is the vector from the center.

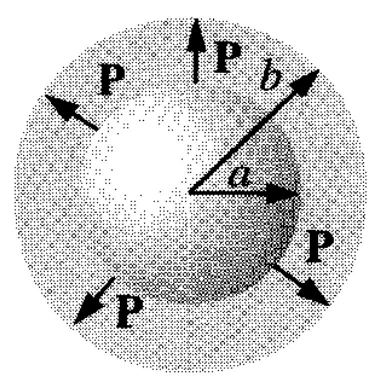
(a) Calculate the bound charges σ_b and ρ_b.
(b) Find the electric field inside and outside the sphere.

Q#4: A very long cylinder, of radius a, carries a uniform polarization P perpendicular to its axis. Find the electric field inside the cylinder. Show that the field outside the cylinder can be expressed in the form:

$$\vec{E}(r) = \frac{a^2}{2\varepsilon_o s^2}\left[2(\vec{P}.\hat{s})\hat{s} - \vec{P}\right]$$

Q#5: A thick spherical shell of inner radius a and outer radius b, is made of a dielectric material with a “frozen-in” polarization:

$$\vec{P} = \frac{k}{r}\hat{r}$$

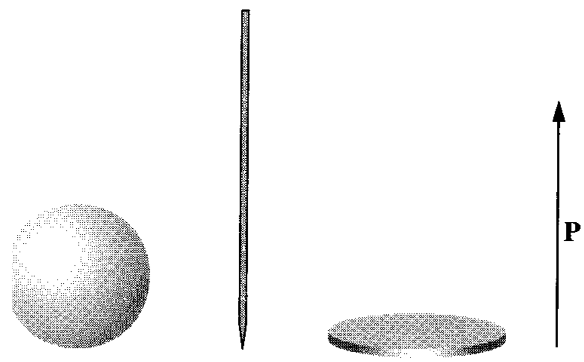


Where k is a constant and r is the distance from the center. Find the electric field in all three regions by:

a) Locate all the bound charge and use Gauss’s law to calculate the field it produces.
b) Find electric displacement D and then find E from using equation $[\vec{D} = \varepsilon_o\vec{E} + \vec{P}]$

Q#6: Suppose that the field inside a large piece of dielectric is $\vec{E}_o$, so that the electric displacement is $\vec{D}_o = \varepsilon_o \vec{E}_o + \vec{P}$

a) Now a small spherical cavity is hollowed out of the material. Find the field at the center of the cavity in terms of $\vec{E}_o$ and $\vec{P}$. Also find the electric displacement at the center of the cavity interms of $\vec{D}_o$ and $\vec{P}$. [Assume the polarization is frozen-in]
b) Do the same for a long needle-shaped cavity running parallel to $\vec{P}$.
c) Do the same for a thin wafer-shaped cavity perpendicular to $\vec{P}$.

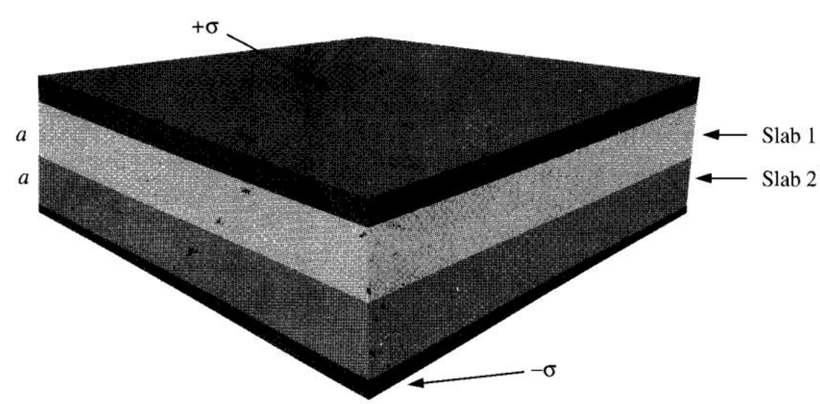


(a) Sphere (b) Needle (c) Wafer

Q#7: A metal sphere of radius a carries a charge Q. It is surrounded out to raidus b, by linear dielectric material of permittivity ε. Find the potential at the center of the sphere relative to infinity.

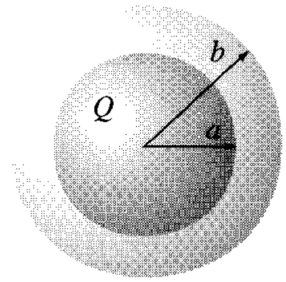
Q#8: The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a so that the total distance between the plates is 2a. Slab 1 has a dielectric constant of 2 and slab 2 has dielectric constant of 1.5. The free charge density on the top plate is σ and on the bottom plate is $-\sigma$.

a) Find the electric displacement in each slab.
b) Find the electric field in each slab.
c) Find the polarization in each slab.
d) Find the potential difference between the plates.
e) Find the location and amount of all bound charges.



Q#9: An uncharged conducting sphere of radius a is coated with a thick insulating shell (dielectric constant ϵ_r) out to radius b. This object is now placed in an otherwise uniform electric field Eo. Find the electric field in the insulator.

Q#10: A spherical conductor of raidus a, carries a charge Q as shown in the figure, It is surrounded by a linear dielectric material of susceptibility χ_e out to radius b. Find the energy of this configuration.



Q#11:

An electric dipole $\vec{p}$, pointing in the y-direction is placed midway between two large conducting plates, as shown in the figure below, each plate makes a small angle θ with respect to the x-axis and they are maintained at potentials $\pm V$.
What is the direction of net force on $\vec{p}$?