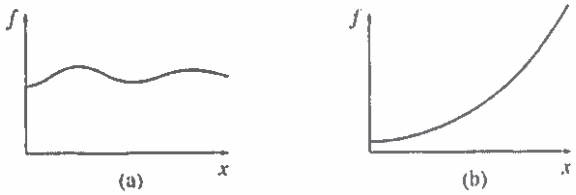
1.2 Differential Calculus

The derivative df/dx is the slope of the graph of f vs x.



$$df = \left(\frac{df}{dx}\right) dx$$

Gradient

If we have a function of three variables, lets say temperature T is a function of x, y, and z. then

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

We can write the above expression in terms of a dot product as:

$$dT = \left[\left(\frac{\partial T}{\partial x}\right)\hat{\imath} + \left(\frac{\partial T}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial T}{\partial z}\right)\hat{k}\right].\left[dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}\right] = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) T \cdot d\bar{l}$$

$$dT = (\overrightarrow{\nabla T}).\overrightarrow{dl}$$

$$dT = (\bar{\nabla}\bar{T} \cdot d\bar{l})$$

Where $\overrightarrow{\nabla T} = \left(\frac{\partial T}{\partial x}\right)\hat{\imath} + \left(\frac{\partial T}{\partial y}\right)\hat{\jmath} + \left(\frac{\partial T}{\partial z}\right)\hat{k}$ is the gradient of T, which is a vector quantity.

In abstract form: $dT = (\overrightarrow{\nabla T}).\overrightarrow{dl} = \nabla T\, dl \cos\theta$

Here θ is the angle between ∇T and dl, If we fix dl and start moving in every direction then the value of dT will be greatest when $\theta = 0$.

- **So gradient of a function ∇T is the direction of the maximum change in that function.**

The magnitude $|\nabla T| = dT/dl$ gives the slope of maximum change direction.

If we are standing on a hill and look all around then the steepest direction is the direction of gradient and the slope of this ascent is the magnitude of the gradient.

If $\vec{\nabla T} = 0$ then $dT = 0$, which means for small displacements there is no change in temperature T, this is a point of a maxima, a minima or a saddle point or a shoulder.

If you want to locate the extrema of a function of three variables then set its gradient equal to zero.

- **Example 4:** Find the gradient of a function $r = \sqrt{x^2 + y^2 + z^2}$

Ans: $\hat{r}$