Example 5: The height of certain hill given in feet is given by:

$h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where y is the distance (in miles) in north and x is the distance east of south Hadley.

(a) Where is the top of the hill located?
(b) How high is the hill? 720 ft
(c) How steep is the slope (in feet per mile) at a point 1 mile north and 1 mile east of South Hadley? In what direction the slope is steepest at that point.

Ans: (a) 3 miles north, 2 miles west of South Hadley. (b) 720 ft. (c) 311 ft/mile in north west direction.

a) $\bar{\nabla} h = 0$ $\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$

$= (20y - 60x - 0 - 18 + 0 + 0)\hat{i} + (20x - 0 - 80y - 0 + 28 + 0)\hat{j}$

$(20y - 60x - 180)\hat{i} + (20x - 80y + 280)\hat{j} = 0$

$20y - 60x - 180 = 0$

$-80y + 20x + 280 = 0 \quad \times 3 =$

$20y - 60x - 180 = 0$

$-240y + 60x + 840 = 0$

$-220y + 660 = 0$

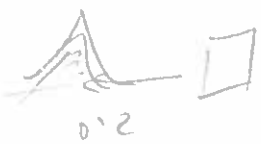
$y = \frac{660}{220} = 3$

$20 \times 3 - 60x - 180 = 0$

$60 - 60x - 180 = 0$

$-60x = 120$

$x = \frac{-120}{60} = -2$



location of hill $(-2, 3)$

(b) $h = 10 \times (2 \times (-2) \times 3 - 3 \times (-2)^2 - 4(3)^2 - 18 \times (-2) + 28 \times (3) + 12)$