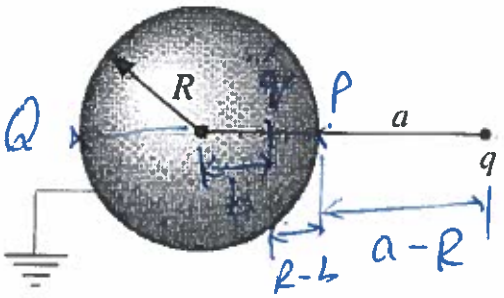
PHYS305 – Quiz#5 Date: 31Oct2021

Name: _____ Key ID: _____

Q#1: (1+5+2+1+2 points)

A point charge q is placed at a distance a from the center of a grounded conducting sphere of radius R, as shown in the figure.

(a) What are the boundary conditions of this problem?
(b) Determine the location and amount of image charge for this problem.
(c) Find the potential outside the sphere at a distance r from the center of the sphere. [You can take center of the sphere as origin].
(d) Find the Potential inside the sphere.
(e) Find the force between the charge and the sphere.



a) $V(R) = 0$ $V(\infty) = 0$

b) Let's take image charge

a) q' at distance b from the Center.

$V(R)$

at P $V(r=R) = 0$

$$\frac{1}{4\pi\epsilon_0}\frac{q}{a-R} + \frac{1}{4\pi\epsilon_0}\frac{q'}{R-b} = 0 \Rightarrow$$

$$q' = -\left(\frac{R-b}{a-R}\right)q \quad -(1)$$

at Q $V(r=R) = 0$

$$\frac{1}{4\pi\epsilon_0}\frac{q}{a+R} + \frac{1}{4\pi\epsilon_0}\frac{q'}{R+b} = 0 \Rightarrow \frac{q'}{R+b} = -\frac{q}{a+R} \quad -(2)$$

$$+\left(\frac{R-b}{a-R}\right)\frac{q}{R+b} = +\frac{q}{a+R}$$

$$(R-b)(a+R) = (R+b)(a-R) = 0$$

$$aR + R^2 - ab - bR - aR + R^2 - ab + bR = 0$$

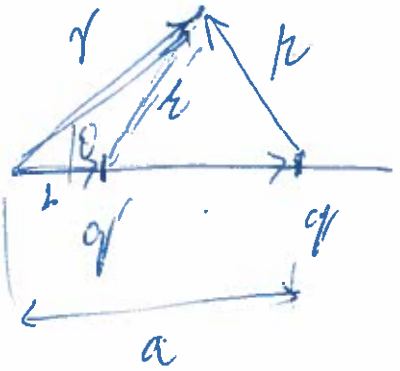
$$-2ab + 2R^2 = 0 \Rightarrow \boxed{b = \frac{R^2}{a}}$$

(b) $\Rightarrow \quad q' = -\left(\frac{R-b}{a-R}\right) q = -\left(\frac{R-\frac{R^2}{a}}{a-R}\right) q = -\left(\frac{qR-R^2}{a-R}\right)\frac{q}{a}$

$$= -\frac{R\,(a-R)}{(a-R)} \cdot \frac{q}{a} = -\frac{R}{a} q$$

$$\boxed{q' = -\frac{R}{a} q}$$

(c) $V(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{\xi} + \frac{1}{4\pi\varepsilon_0}\frac{q'}{\xi'}$



$\bar{r} = \bar{\xi} + \bar{a}$

$\bar{\xi} = \bar{r} - \bar{a}$

$\bar{\xi}\cdot\bar{\xi} = \xi^2 = r^2 + a^2 - 2ra\cos\theta$

$\bar{r} = \bar{b} + \bar{\xi}' \Rightarrow \bar{\xi}' = \bar{r} - \bar{b} \Rightarrow \bar{\xi}'\cdot\bar{\xi}' = \xi'^2 = r^2 + b^2 - 2rb\cos\theta$

$$V(r) = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{r^2+a^2-2ar\cos\theta}} - \frac{R}{a\sqrt{r^2+b^2-2rb\cos\theta}}\right]$$

(d) $V = 0$

(e) $F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{(a-b)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Rq^2}{a(a-b)^2}$