Key

Name: ______________________ ID: ______________

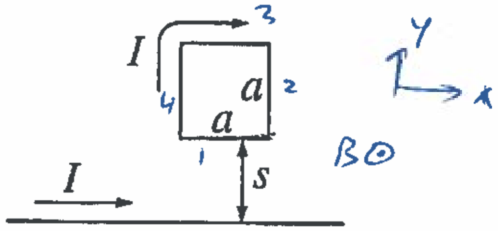
Q#1:

A square loop of side length a is placed near an infinite long wire. Both the wire and square loop carry a steady current I as shown in the figure.

(a) Find the magnitude and direction of magnetic force on each side of the loop due to the infinite wire.

(b) Find the magnitude and direction of net magnetic force on the loop.

$$B = \frac{\mu_0 i}{2\pi r}$$

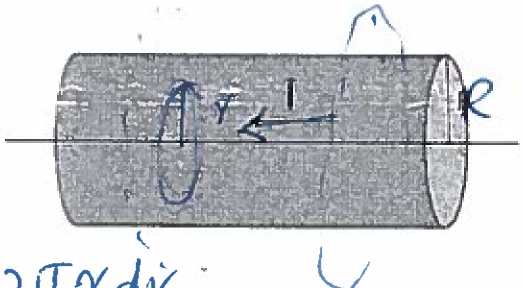


$\bar{F} = i\oint d\bar{l} \times \bar{B}$ for the loop

Side 1 $F_1 = i\int d\bar{l} \times \bar{B} = i\int d\bar{l} \times \frac{\mu_0 i}{2\pi s} = \frac{\mu_0 i^2}{2\pi s} a\,(+\hat{j})$

$$F_2 = i\int d\bar{l} \times \bar{B} = i\int_{s+a}^{s} dl\,\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i^2}{2\pi}\left(-\ln(a+s) + \ln(s)\right)\hat{i}$$

Q#2: A steady current I flows down a long cylindrical wire of radius R. Use Ampere's law to find the magnetic field both inside and outside of the wire if the current is distributed in such a way that $\vec{J} = k\vec{r}$ (Where k is a constant and $\vec{r}$ is a distance from the axis of the cylinder).



$$\oint \bar{B}\cdot d\bar{l} = \mu_0 i_{enc}$$

$$B \times 2\pi r = \mu_0 \int J\,da = \mu_0 \int_0^r kr \times 2\pi r\,dr$$

$$B \times \cancel{2\pi} r = \mu_0 k \times \cancel{2\pi}\,\frac{r^3}{3} \Rightarrow \boxed{B = \frac{\mu_0 k}{3} r^2 \hat{\phi}} \quad r<R$$

outside

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 i_{enc} = \mu_0 \int_0^R J\,da = \mu_0 k \times 2\pi \frac{R^3}{3}$$

$$B \times \cancel{2\pi} r = \frac{\mu_0 k \times \cancel{2\pi} R^3}{3} \Rightarrow \boxed{B = \frac{\mu_0 k R^3}{3r}\hat{\phi}}$$

Q#1 – Continued

(a)

$$\bar{F}_3 = i\int d\bar{l} \times \bar{B} = i\int d\bar{l} \times \frac{\mu_0 i}{2\pi(s+a)}(-\hat{j})$$

$$\bar{F}_3 = \frac{\mu_0 i^2 a}{2\pi(s+a)}(-\hat{j})$$

$$\bar{F}_4 = i\int d\bar{l} \times \bar{B} = i\int_{s}^{s+a} d\bar{l} \times \frac{\mu_0 i}{2\pi r}(\hat{i})$$

$$\bar{F}_4 = \frac{\mu_0 i^2}{2\pi}\left(\ln(s+a) - \ln(s)\right)\hat{i} = \frac{\mu_0 i^2}{2\pi}\left(\ln(s+a) - \ln(s)\right)\hat{i}$$

(b)

$$\bar{F}_{net} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4 = \left(\frac{\mu_0 i^2 a}{2\pi s} - \frac{\mu_0 i^2 a}{2\pi(s+a)}\right)\hat{j}$$

$$\boxed{\bar{F}_{net}} = \frac{\mu_0 i^2 a}{2\pi}\left(\frac{\cancel{s}+a-\cancel{s}}{s(s+a)}\right)\hat{j} = \boxed{\frac{\mu_0 i^2 a^2}{2\pi s(s+a)}\hat{j}}$$