Key

PHYS305 – Quiz#2

Date: 14Sep2021

Name: ______________________ ID: ______________________

Q#1:

3 (a) Calculate the divergence of the following function:

$$\vec{v} = x^2\hat{\imath} + +xz^2\hat{\jmath} - 2xz\hat{k}$$

3 (b) Also calculate the Laplacian of the function given above.

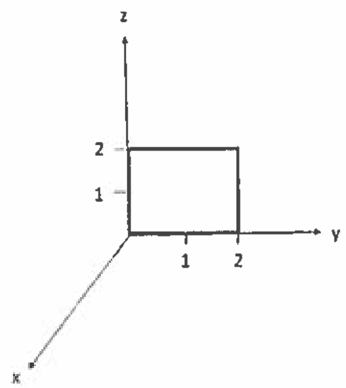
a) $\bar{\nabla}\cdot\bar{V} = \frac{\partial}{\partial x}V_x + \frac{\partial}{\partial y}V_y + \frac{\partial}{\partial z}V_z = 2x + 0 - 2x = 0$

b) $\nabla^2\bar{V} = \frac{\partial^2}{\partial x^2}\bar{V} + \frac{\partial^2}{\partial y^2}\bar{V} + \frac{\partial^2}{\partial z^2}\bar{V} = \frac{\partial^2}{\partial x^2}(x^2\hat{i} + xz^2\hat{j} - 2xz\hat{k}) + \frac{\partial^2}{\partial y^2}(x^2\hat{i} + xz^2\hat{j} - 2xz\hat{k})$

$+ \frac{\partial^2}{\partial z^2}(x^2\hat{i} + xz^2\hat{j} - 2xz\hat{k})$

$= (2\hat{i} + 0 - 0) + 0 + (0\hat{i} + 2x\hat{j} - 0) = \boxed{2\hat{i} + 2x\hat{j}}$

Q#2: For the vector function given in Q#1, test the Stoke's theorem for the surface shown in the figure below.



$3\int_S (\bar{\nabla}\times\bar{V})\cdot d\bar{a} =_1 \oint \bar{V}\cdot d\bar{l}\ _3$

$$\bar{\nabla}\times\bar{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xz^2 & -2xz \end{vmatrix} = \hat{i}(0 - 2xz) - \hat{j}(-2z - 0) + \hat{k}(z^2 - 0)$$

$\bar{\nabla}\times\bar{V} = -2xz\hat{i} + 2z\hat{j} + z^2\hat{k}$

$(\bar{\nabla}\times\bar{V})\cdot d\bar{a} = (-2xz\hat{i} + 2z\hat{j} + z^2\hat{k})\cdot dy\,dz\,\hat{i} = -2xz\,dy\,dz$

$\int(\bar{\nabla}\times\bar{V})\cdot d\bar{a} = \int_0^2\int_0^2 -2xz\,dy\,dz = 0$

$\oint \bar{v}\cdot d\bar{l} = \int_0^2 (x^2\hat{i} + xz^2\hat{j} - 2xz\hat{k})\cdot dy\hat{j} + \int^2 (x^2\hat{i} + xz^2\hat{j} - 2xz\hat{k})\cdot dz\hat{k} +$ $x=0$

$$+\int_{2}^{0}(x^2\hat{i}+xz^2\hat{j}-2xz\hat{k})\cdot dy\hat{j}\Big|_{z=2}+\int_{2}^{0}(x^2\hat{i}+xz^2\hat{j}-2xz\hat{k})\cdot dz\hat{k}\Big|_{y=0}$$

$$\oint \bar{v}\cdot d\bar{l} = \int_0^2 xz^2\,dy\Big|_{z=0} - \int_0^2 2xz\,dz\Big|_{y=2} + \int_2^0 xz^2\,dy\Big|_{z=2} - \int_2^0 2xz\,dz\Big|_{y=0}$$

$$= xz^2y\Big|_0^2{}_{x=0,\,z=0} - 2x\frac{z^2}{2}\Big|_0^2{}_{x=0,\,y=2} + xz^2y\Big|_2^0{}_{x=0,\,z=2} - 2x\frac{z^2}{2}\Big|_2^0{}_{x=0}$$

$$\boxed{\oint_P \bar{v}\cdot d\bar{l} = 0} = \boxed{\int_S (\bar{\nabla}\times\bar{v})\,da}$$