Chapter 11 – Radiation

11.1 Dipole Radiation

A charge at rest does not generate electromagnetic waves, nor does a steady current. Only accelerating charges and changing currents produce radiation. Once created, EM waves in vacuum propagate out to infinity, carrying energy with them, signature of radiation is this irreversible flow of energy away from the source.

Assume the source is localized near the origin. A gigantic spherical shell, out at radius r, the total power passing out through this surface is the integral of the Poynting vector:

$$P(r) = \oint \vec{S}.d\vec{a} = \frac{1}{\mu_o}\oint (\vec{E} \times \vec{B}).d\vec{a}$$

The power radiated is the limit of this quantity as r goes to infinity:

$$P_{rad} = \lim_{r\to\infty} P(r)$$

This is the energy per unit time that is transported to infinity and never comes back. Now the area of the square is $4\pi r^2$, so for radiation to occur, the Poynting vector must decrease no faster than $1/r^2$ (if the Poynting vector were to decrease by $1/r^3$, then P(r) would go by $1/r$ and P_{rad} would go to zero).

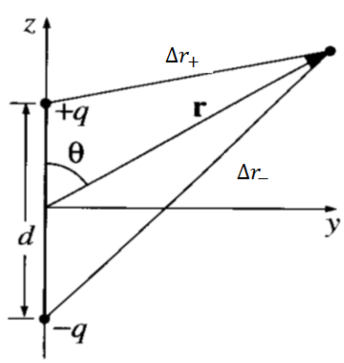
According to Coulomb's law, electrostatic fields fall off like $1/r^2$ (or even faster if the total charge is zero) and by Biot-Savart's law the magnetic field also goes like $1/r^2$, which will make Poynting vector $S{\sim}1/r^4$ for static configurations. Hence, static sources (and steady currents) do not radiate.

But according to Jefimenko's equations, the time dependent field include terms involving ($\dot{\rho}$ and $\dot{J}$) that go like $1/r$, it is these terms that are responsible for EM radiation.

11.1.2 Electric Dipole Radiation

Consider two tiny metal spheres separated by a distance d and connected by a fine wire. At time t the charge on the upper sphere is $q(t)$ and the charge on the lower sphere is $-q(t)$. Suppose we drive the charge back and forth through the wire, from one end to the other end, at an angular frequency ω:

$$q(t) = q_o \cos(\omega t)$$



The dipole moment of this dipole is:

$$p(t) = p_o \cos(\omega t)\,\hat{z}$$

Where $p_o = q_o d$ is the maximum dipole moment.

The retarded potential is given as:

$$V(\vec{\boldsymbol{r}},t) = \frac{1}{4\pi\epsilon_o}\left\{\frac{q(t_r)}{\Delta r_+} + \frac{-q(t_r)}{\Delta r_-}\right\}$$

$$V(\vec{\boldsymbol{r}},t) = \frac{1}{4\pi\epsilon_o}\left\{\frac{q_o\cos(\omega(t-\Delta r_+/c)}{\Delta r_+} - \frac{q_o\cos(\omega(t-\Delta r_-/c))}{\Delta r_-}\right\}$$

And by the law of cosines: $\Delta r_\pm = \sqrt{r^2 \mp rd\cos\theta + (d/2)^2}$

For a perfect dipole, the separation between the charges should be extremely small:

Approximation 1: $d \ll r$

$$\Delta r_\pm \cong r\left(1 \mp \frac{d}{r}\cos\theta\right)^{\frac{1}{2}} = r\left(1 \mp \frac{d}{2r}\cos\theta\right)$$

$$\frac{1}{\Delta r_\pm} = \frac{1}{r}\left(1 \pm \frac{d}{2r}\cos\theta\right)$$

$$\cos(\omega(t-\Delta r_+/c) \cong \cos\left[\omega(t-r/c) \pm \frac{\omega d}{2c}\cos\theta\right]$$

$$\cos\left[\omega(t-r/c) \pm \frac{\omega d}{2c}\cos\theta\right] \cong \cos[\omega(t-r/c)]\cos\left[\frac{\omega d}{2c}\cos\theta\right] \mp \sin[\omega(t-r/c)]\sin\left[\frac{\omega d}{2c}\cos\theta\right]$$

In the perfect dipole limit, we have the approximation:

Approximation 2: $d \ll \frac{c}{\omega} = \lambda/2\pi$

Under these conditions:

$$\cos(\omega(t-\Delta r_+/c) \cong \cos[\omega(t-r/c)] \mp \frac{\omega d}{2c}\cos\theta\sin[\omega(t-r/c)]$$

Hence the potential of an oscillating dipole is:

$$V(r,\theta,t) = \frac{p_o\cos\theta}{4\pi\epsilon_o r}\left\{-\frac{\omega}{c}\sin\big(\omega(t-r/c)\big) + \frac{1}{r}\cos\big(\omega(t-r/c)\big)\right\}$$

In the static limit $\omega = 0$, we get potential of a stationary dipole:

$$V = \frac{p_o\cos\theta}{4\pi\epsilon_o r^2}$$

We are interested in the fields that survive at large distances from the source, in the so-called **radiation zone**.

Approximation 3: $r \gg \frac{c}{\omega}$ or $r \gg \lambda$

In this region, potential reduces to:

$$V(r,\theta,t) = -\frac{p_o\omega}{4\pi\epsilon_o c}\left(\frac{\cos\theta}{r}\right)\sin\big(\omega(t-r/c)\big)$$

Meanwhile the vector potential is determined by the current flowing in the wire:

$$I(t) = \frac{dq}{dt}\hat{z} = -q_o\omega\sin(\omega t)\,\hat{z}$$

Retarded vector potential is defined as:

$$\vec{A}(\vec{\boldsymbol{r}},t) = \frac{\mu_o}{4\pi}\int\frac{\vec{J}(\vec{r}\,',t_r)}{\Delta r}d\tau' = \frac{\mu_o}{4\pi}\int_{-d/2}^{d/2}\frac{-q_o\omega\sin[\omega(t-\Delta r/c)]\,\hat{z}}{\Delta r}dz$$

Since the integration itself introduces a factor of d, we can replace the integrant by its value at the center.

$$\vec{A}(r,\theta,t) = -\frac{\mu_o p_o\omega}{4\pi r}\sin[\omega(t-r/c)]\,\hat{z}$$

Once we know the potentials, we can easily find the fields.

$$\vec{\nabla}V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat{\theta}$$

$$= -\frac{p_0\omega}{4\pi\epsilon_0 c}\left\{\cos\theta\left(-\frac{1}{r^2}\sin[\omega(t-r/c)] - \frac{\omega}{rc}\cos[\omega(t-r/c)]\right)\hat{\mathbf{r}} - \frac{\sin\theta}{r^2}\sin[\omega(t-r/c)]\,\hat{\boldsymbol{\theta}}\right\}$$

Using approximation 3, we can drop the first and last term:

$$\vec{\nabla}V \cong \frac{p_o\omega^2}{4\pi\epsilon_o c^2}\left(\frac{\cos\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{r}$$

Similarly,

$$\frac{\partial\vec{A}}{\partial t} = -\frac{\mu_o p_o\omega^2}{4\pi r}\cos[\omega(t-r/c)]\left(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\right)$$

Hence

$$\vec{E} = \vec{\nabla}V - \frac{\partial\vec{A}}{\partial t} = -\frac{\mu_o p_o\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\theta} \ldots (1)$$

Meanwhile

$$\vec{\nabla}\times\vec{\mathrm{A}} = \frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]\hat{\phi}$$

$$\vec{\nabla} \times \vec{A} = -\frac{\mu_o p_o \omega}{4\pi r}\left\{\frac{\omega}{c}\sin\theta\cos\left[\omega\left(t-\frac{r}{c}\right)\right]+\frac{\sin\theta}{r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\right\}\hat{\phi}$$

The second term can be eliminated by approximation 3, so

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\mu_o p_o \omega^2}{4\pi c}\frac{\sin\theta}{r}\cos[\omega(t-r/c)]\,\hat{\phi} \ldots (2)$$

Equations (1) and (2) represent monochromatic waves of frequency ω travelling in the radial direction at the speed of light. $\vec{E}$ and $\vec{B}$ are in phase and perpendicular to each other and perpendicular to the direction of the direction of propagation of the wave. And the ratio of their amplitudes is $\frac{E_o}{B_o} = c$.

These are actually spherical waves, not plane waves, and their amplitudes decrease like $1/r$ as they progress. But for larger r, they are approximately plane over small regions.

The energy radiated by an oscillating electric dipole is determined by the Poynting vector:

$$\vec{S} = \frac{1}{\mu_o}\left(\vec{E} \times \vec{B}\right) = \frac{\mu_o}{c}\left[\frac{p_o \omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\right]^2 \hat{r}$$

The intensity is obtained by taking the time average over a complete cycle:

$$\langle\vec{S}\rangle = \frac{\mu_o}{2c}\left(\frac{p_o \omega^2}{4\pi}\frac{\sin\theta}{r}\right)^2 \hat{r} = \left(\frac{\mu_o p_o^2 \omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{r}$$

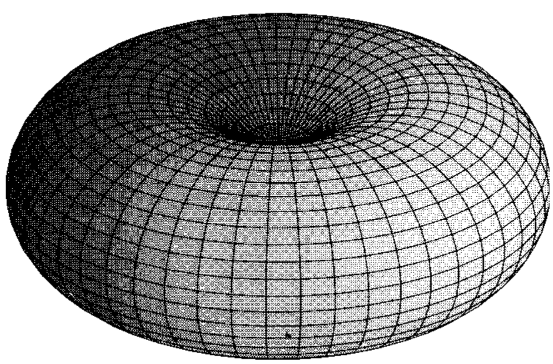
Note that there is no radiation in the direction of the dipole where $\sin\theta = 0$, the intensity profile takes the shape of the donut, with its maximum at the equatorial plane as shown in the figure below.

The total power radiated is found by integrating $\langle\vec{S}\rangle$ over a sphere of radius r:

$$\langle P\rangle = \int \langle\vec{S}\rangle . d\vec{a}$$

$$\langle P\rangle = \left(\frac{\mu_o p_o^2 \omega^4}{32\pi^2 c}\right)\int_0^{2\pi}\int_0^{\pi}\frac{\sin^2\theta}{r^2}\, r^2 \sin\theta \; d\theta \; d\phi$$

$$\langle P\rangle = \frac{\mu_o p_o^2 \omega^4}{12\pi c}$$



The power radiated is independent of the radius of the sphere, due to conservation of energy.

Example:

The sharp frequency dependence of the power formula is what accounts for the blueness of the sky. Sunlight passing through the atmosphere stimulates atoms to oscillate as tiny dipoles.

Sunlight covers a broad range of frequencies but the energy absorbed and radiated by the atmospheric dipoles is strongest at the higher frequencies because of the ω^4 factor in the radiated energy.

Since EM waves are transverse so dipoles oscillate in a plane orthogonal to the sun's rays. The redness of sunset is the other side of the same coin: sunlight coming in at a tangent to the earth's surface must pass through a much longer stretch of atmosphere than sunlight coming from overhead. Accordingly, much of the blue has been removed by scattering what's left is red.

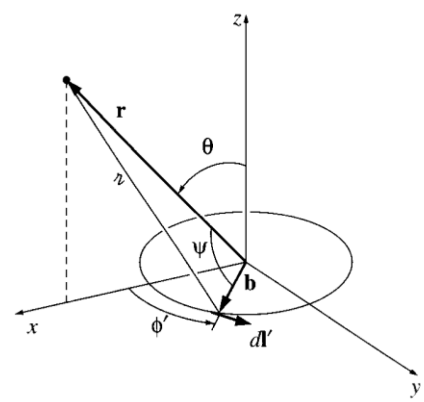
11.1.3 Magnetic Dipole Radiation

Suppose we have a wire loop of radius b, carrying an alternating current:

$$I(t) = I_o \cos(\omega t)$$

This is a model for an oscillating magnetic dipole, with magnetic moment:

$$\vec{m}(t) = \pi b^2 I(t)\hat{z} = \pi b^2 I_o \cos(\omega t)\,\hat{z}$$



Where $m_o = \pi b^2 I_o$ is the maximum value of the magnetic dipole moment. The loop is uncharged so the scalar potential is zero.

The retarded vector potential is:

$$\vec{A}(\vec{r},t) = \frac{\mu_o}{4\pi}\int \frac{I_o \cos[\omega(t-\Delta r/c)]}{\Delta r}\, d\vec{l}'$$

For a point $\vec{r}$ directly above the x-axis, $\vec{A}$ must aim in the y-direction, since the x-components from symmetrically placed points on either side of the x-axis will cancel.

$$\vec{A}(\vec{r},t) = \frac{\mu_o I_o b}{4\pi}\hat{y}\int \frac{\cos[\omega(t-\Delta r/c)]}{\Delta r}\cos\phi'\, d\phi'$$

Where

$$\Delta r = \sqrt{r^2 + b^2 - 2rb\cos\psi}$$

Where ψ is the angle between the vectors $\vec{r}$ and $\vec{b}$:

$$\vec{r} = r\sin\theta\,\hat{x} + r\cos\theta\,\hat{z} \text{ and } \vec{b} = b\cos\phi'\,\hat{x} + b\sin\phi'\,\hat{y}$$

$$rb\cos\psi = \vec{r}.\vec{b} = rb\sin\theta\cos\phi'$$

$$\Delta r = \sqrt{r^2 + b^2 - 2rb\sin\theta\cos\phi'}$$

For a perfect dipole we would like the loop to be small:

Approximation 1: $b \ll r$

$$\Delta r \cong r\left(1 - \frac{b}{r}\sin\theta\cos\phi'\right)$$

$$\frac{1}{\Delta r} \cong \frac{1}{r}\left(1 + \frac{b}{r}\sin\theta\cos\phi'\right)$$

$$\cos[\omega(t - \Delta r/c)] \cong \cos\left[\omega(t - r/c) + \frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

$$= \cos[\omega(t - r/c)]\cos\left[\frac{\omega b}{c}\sin\theta\cos\phi'\right] - \sin[\omega(t - r/c)]\sin\left[\frac{\omega b}{c}\sin\theta\cos\phi'\right]$$

We will also assume that the size of the dipole is small compared to the wavelength radiated:

Approxiomation 2: $b \ll \frac{c}{\omega}$

In that case,

$$\cos[\omega(t - \Delta r/c)] \cong \cos[\omega(t - r/c)] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin[\omega(t - r/c)]$$

So

$$\vec{A}(\vec{r}, t) = \frac{\mu_o I_o b}{4\pi}\hat{y}\int\frac{\cos[\omega(t - \Delta r/c)]}{\Delta r}\cos\phi'\,d\phi'$$

$$\vec{A}(\vec{r}, t) = \frac{\mu_o I_o b}{4\pi r}\hat{y}\int\left[\cos[\omega(t - r/c)] - \frac{\omega b}{c}\sin\theta\cos\phi'\sin[\omega(t - r/c)]\right]\left(1 + \frac{b}{r}\sin\theta\cos\phi'\right)\cos\phi'\,d\phi'$$

The first term integrates to zero because:

$$\int_0^{2\pi}\cos\phi'\,d\phi' = 0$$

The second term involves the integral of cosine squared:

$$\int_0^{2\pi}\cos^2\phi'\,d\phi' = \pi$$

Using the cosine integrals and noting that in general $\vec{A}$ points in the $\hat{\phi}$ direction, we get:

$$\vec{A}(r, \theta, t) = \frac{\mu_o m_o}{4\pi}\left(\frac{\sin\theta}{r}\right)\left\{\frac{1}{r}\cos[\omega(t - r/c)] - \frac{\omega}{c}\sin[\omega(t - r/c)]\right\}\hat{\phi}$$

In the static limit $\omega = 0$ and we recover the formula for potential of a magnetic dipole:

$$\vec{A}(r, \theta) = \frac{\mu_o}{4\pi}\frac{m_o\sin\theta}{r^2}\hat{\phi}$$

And in the radiation zone:

Approximation 3: $r \gg \frac{c}{\omega}$

The first term in $\vec{A}$ is negligible, so:

$$\vec{A}(r,\theta,t) = -\frac{\mu_o m_o \omega}{4\pi}\left(\frac{\sin\theta}{r}\right)\sin[\omega(t-r/c)]\,\hat{\phi}$$

We can get the electric field at large r:

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} = \frac{\mu_o m_o \omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\phi}$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{\mu_o m_o \omega^2}{4\pi c^2}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\,\hat{\theta}$$

The field are in phase, mutually perpendicular and transverse to the direction of propagation ($\hat{r}$), and the ratio of their amplitudes is $E_o/B_o = c$, all which is as expected for EM waves.

The energy flux for magnetic dipole radiation is:

$$\vec{S} = \frac{1}{\mu_o}\left(\vec{E}\times\vec{B}\right) = \frac{\mu_o}{c}\left\{\frac{m_o\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos[\omega(t-r/c)]\right\}^2\hat{r}$$

And the intensity is:

$$\langle\vec{S}\rangle = \frac{\mu_o m_o^2\omega^4}{32\pi^2 c^3}\frac{\sin^2\theta}{r^2}\hat{r}$$

And the total radiated power is:

$$\langle P\rangle = \frac{\mu_o m_o^2\omega^4}{12\pi c^3}$$

Magnetic dipole has the intensity profile as the shape of a donut and the power radiated goes like ω^4.

For configurations with comparable dimensions, the power radiated by electrical dipole is much greater than the power radiated by magnetic dipole:

$$\frac{P_{magnetic}}{P_{electric}} = \left(\frac{m_o}{p_o c}\right)^2$$

Where $m_o = \pi b^2 I_o$ and $p_o = q_o d$. The amplitude of current in the electrical case was $I_o = q_o\omega$

Now setting $d = \pi b$, for the sake of comparison:

$$\frac{P_{magnetic}}{P_{electric}} = \left(\frac{\omega b}{c}\right)^2$$

And $\frac{\omega b}{c}$ is exactly the quantity that we assumed was very small (approximation 2) and here it is squared. So we should expect electric dipole radiation to dominate.

11.1.4 Radiation from an arbitrary source

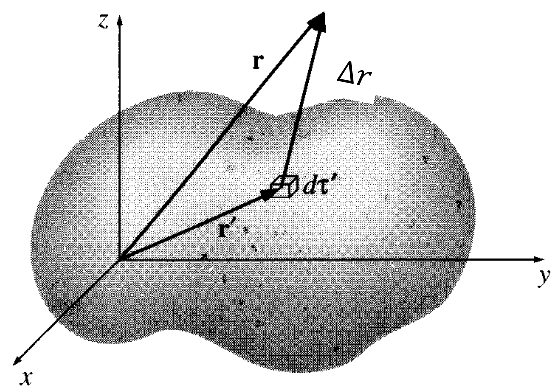
Assume there is a charge configuration localized within some finite volume near the origin.

The retarded scalar potential is:

$$V(r,t) = \frac{1}{4\pi\epsilon_o}\int \frac{\rho(\vec{r}',t-\Delta r/c\,)}{\Delta r}d\tau'$$

Where

$$\Delta r = \sqrt{r^2 + r'^2 - 2\vec{r}.\vec{r}'}$$



We assume that the field point r is far away compared to the dimension of the source:

Approximation 1: $r' \ll r$

Hence

$$\Delta r \cong r\left(1 - \frac{\vec{r}.\vec{r}'}{r^2}\right)$$

$$\frac{1}{\Delta r} \cong \frac{1}{r}\left(1 + \frac{\vec{r}.\vec{r}'}{r^2}\right)$$

$$\rho(\vec{r}',t-\Delta r/c) \cong \rho\left(\vec{r}',t-\frac{r}{c}+\frac{\hat{r}.\vec{r}'}{c}\right)$$

Expanding ρ as a Taylor series in t about the retarded time at the origin:

$$t_o = t - \frac{r}{c}$$

$$\rho(\vec{r}',t-\Delta r/c) \cong \rho(\vec{r}',t_o) + \dot{\rho}(\vec{r}',t_o)\left(\frac{\hat{r}.\vec{r}'}{c}\right) + \cdots$$

Expanding ρ as Taylor series in t about the retarded time at the origin:

$$t_o = t - \frac{r}{c} \;\ldots(1)$$

$$\rho\left(\vec{r}', t-\frac{r}{c}\right) \cong \rho(\vec{r}', t_o) + \dot{\rho}(\vec{r}', t_o)\left(\frac{\hat{r}.\vec{r}'}{c}\right) + \cdots$$

The next terms in the series can be dropped, provided:

Approximation 2: $r' \ll \frac{c}{|\ddot{\rho}-\dot{\rho}|}, \frac{c}{|\dddot{\rho}-\dot{\rho}|^{1/2}}, \frac{c}{|\ddddot{\rho}-\dot{\rho}|^{1/3}}, \dots$

For an oscillating system, each of these ratios is $\frac{c}{\omega}$, and we recover the old approximation 2.

The result of approximation 1 and 2 is to keep only the first-order terms in r'. Hence:

$$V(r,t) \cong \frac{1}{4\pi\epsilon_o r}\left[\int \rho(\vec{r}', t_o)d\tau' + \frac{\hat{r}}{r}.\int \vec{r}'\rho(\vec{r}', t_o)d\tau' + \frac{\hat{r}}{c}.\frac{d}{dt}\int \vec{r}'\rho(\vec{r}', t_o)d\tau'\right]$$

The first integral is simply the total charge Q at time t_o. Because, charge is conserved, however, Q is independent of time. The other two integrals represent the electric dipole moment at time t_o. Thus

$$V(r,t) \cong \frac{1}{4\pi\epsilon_o}\left[\frac{Q}{r} + \frac{\hat{r}.\vec{\boldsymbol{p}}(t_o)}{r^2} + \frac{\hat{r}.\dot{\boldsymbol{p}}(t_o)}{rc}\right] \dots (2)$$

In the static case the first two terms are the monopole and dipole contributions to the multipole expansion for V, the third term, however, would not be present.

The vector potential will be:

$$\vec{A}(\vec{r},t) = \frac{\mu_o}{4\pi}\int \frac{\vec{J}(\vec{r}', t-\Delta r/c\,)}{\Delta r}\, d\tau'$$

To first order in $\vec{r}'$ it suffices to replace Δr by r in the integrand:

$$\vec{A}(\vec{r},t) = \frac{\mu_o}{4\pi r}\int \vec{J}(\vec{r}', t_o\,)\, d\tau'$$

As we proved in chapter 5, the integral of $\vec{J}$ is the time derivative of the dipole moment.

$$\vec{A}(\vec{r},t) \cong \frac{\mu_o}{4\pi}\frac{\dot{\boldsymbol{p}}(t_o)}{r}$$

To find the electric and magnetic fields in the radiation zones, i.e. at large distances from the source, we keep terms only that go like $1/r$:

Approximation 3: discard $1/r^2$ terms in $\vec{E}$ and $\vec{B}$:

For instance, the Coulomb field, $\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$ coming from the first term in equation (1) does not contribute to the electromagnetic radiation. In fact the radiation comes entirely from the terms in which we differentiate the argument t_o.

$$\vec{\nabla} t_o = \vec{\nabla}(t - r/c) = -\frac{1}{c}\vec{\nabla} r = -\frac{1}{c}\hat{r}$$

Hence

$$\vec{\nabla} V \cong \vec{\nabla}\left[\frac{1}{4\pi\epsilon_o}\frac{\hat{r}.\dot{\boldsymbol{p}}(t_o)}{rc}\right] \cong \frac{1}{4\pi\epsilon_o}\left[\frac{\hat{r}.\ddot{\boldsymbol{p}}(t_o)}{rc}\right]\vec{\nabla} t_o = -\frac{1}{4\pi\epsilon_o}\frac{[\hat{r}.\ddot{\boldsymbol{p}}(t_o)]}{r}\hat{r}$$

Similarly,

$$\vec{\nabla} \times \vec{A} = \frac{\mu_o}{4\pi r}[\vec{\nabla} \times \dot{\boldsymbol{p}}(t_o)] = \frac{\mu_o}{4\pi r}[\vec{\nabla} t_o \times \ddot{\boldsymbol{p}}(t_o)] = -\frac{\mu_o}{4\pi r}[\hat{r} \times \ddot{\boldsymbol{p}}(t_o)]$$

$$\frac{\partial \vec{A}}{\partial t} \cong \frac{\mu_o}{4\pi}\frac{\ddot{\boldsymbol{p}}(t_o)}{r}$$

$$\vec{E}(\vec{r}, t) = \frac{\mu_o}{4\pi r}[(\hat{r}.\ddot{\boldsymbol{p}})\hat{r} - \ddot{\boldsymbol{p}}] = \frac{\mu_o}{4\pi r}[\hat{r} \times (\hat{r} \times \ddot{\boldsymbol{p}}(t_o))]$$

$$\vec{B}(\vec{r}, t) = -\frac{\mu_o}{4\pi r}[\hat{r} \times \ddot{\boldsymbol{p}}(t_o)]$$

If we use spherical coordinates, with the z-axis in the direction of $\ddot{\boldsymbol{p}}(t_o)$, then

$$\left.\begin{aligned} \vec{E}(r, \theta, t) &= \frac{\mu_o \ddot{\boldsymbol{p}}(t_o)}{4\pi}\left(\frac{\sin\theta}{r}\right)\hat{\theta} \\ \vec{B}(r, \theta, t) &= \frac{\mu_o \ddot{\boldsymbol{p}}(t_o)}{4\pi c}\left(\frac{\sin\theta}{r}\right)\hat{\phi} \end{aligned}\right\} \quad (4)$$

Note that $\vec{E}$ and $\vec{B}$ are mutually perpendicular to each other and perpendicular to the direction of propagation. Also the ratio $\frac{E_o}{B_o} = c$, as always for radiation fields.

The Poynting vector is:

$$\vec{S} = \frac{1}{\mu_o}(\vec{E} \times \vec{B}) = \frac{\mu_o}{16\pi^2 c}[\ddot{\boldsymbol{p}}(t_o)]^2\left(\frac{\sin^2\theta}{r^2}\right)\hat{r} \quad \ldots (5)$$

And the total radiated power is:

$$P \cong \int \vec{S}.d\vec{a} = \frac{\mu_o \ddot{\boldsymbol{p}}^2}{6\pi c} \quad \ldots (6)$$