KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF PHYSICS

PHYS.310 – Major Exam I (TERM 212)

Instructor: Dr. Hocine Bahlouli

Wednesday March 02, 8:00 pm, 6-105

Student Name: ______________________

ID. No. : _________________

- **Exam time : 120 Minutes**
- **Solve the following five problems and show all details of your work to earn a full mark.**
- **Since you are provided with a formula sheet, reference any equation you use from the formula sheet.**

Problem #	Grade
1	/20
2	/20
3	/25
4	/35
Total	/100
Normalized Final Grade	/20

Question 1:

Answer the following **independent** questions.

(a) Why **does** the **photoelectric current vanish below a certain threshold frequency** of the incident light on a metallic surface?
(5 pts)

(b) Why does **increasing the frequency** of incident light, keeping the **intensity constant, lead to a decrease** in the photoelectric current? (Note that intensity of light is energy per unit time per unit area)
(5pts)

(c) The photoelectron kinetic energy is given by

K= hν – W = hc/λ –W **(5 pts)**

If a metal has a work function of **1.5 eV** and is illuminated with light of wavelength **400 nm**, find required **stopping potential** that makes the photoelectric current vanish?

(d) **Which experiment** shows very prominently (easily) the **wave-like behavior** (for both light and matter waves)? Give an example. **(5 pts)**

Question 2:

Consider a particle of mass m moving in a one-dimensional **harmonic oscillator** potential $V(x)=\frac{1}{2}m\omega^2x^2$ with initial state at t = 0 given by

$$\Psi(x,0)=\frac{A}{\sqrt{12}}\Phi_1(x)+\frac{1}{\sqrt{6}}\Phi_2(x)+\frac{1}{\sqrt{3}}\Phi_3(x)+\frac{1}{2}\Phi_4(x) \quad ;H\Phi_n(x)=E_n\Phi_n(x)$$

$$E_n=\hbar\omega(n+\frac{1}{2}) \quad ;n=0,1,2,3... \quad ; \int_{-\infty}^{+\infty} dx\,\Phi_m(x)\Phi_n(x)=\delta_{m,n}$$

where A is a real **positive** constant, Φ_1 , Φ_2 , Φ_3 , Φ_4 are the harmonic oscillator eigenstates corresponding to n = 1, 2, 3, 4 excited states, respectively.

(a) Find **A** so that Ψ(x,0) is normalized to unity. **(7 pts)**

(b) Find the state of the system **Ψ(x,t)** as a function of **ωt** and **Φ_n**. **(6 pts)**

(c) If the associated **energy E** with state Ψ(x,t) was measured at a time t, find its expectation value in terms of **$\hbar\omega$**. **(7 pts)**

Question 3:

Answer the following **independent** questions.

(a) Show that in 1D: The allowed eigenergies for a stationary state are always **greater than the minimum** of the potential, V_{min}. **(6 pts)**

(b) Evaluate the commutator [x , p] by **operating** it on an arbitrary wave function **F(x)**. Then use this result to **prove that** $[x^2, p^2] = 2i\hbar(i\hbar + 2\,px)$. **(6 pts)**

Hints: [A, BC] = B[A,C] + [A, B]C = - [BC, A] and $p = -i\hbar d/dx$.

(c) Consider the **harmonic oscillator** creation and destruction operators:

$$N = a_+a_- \; ; [a_-, a_+] = 1 \, , H = \hbar\omega(N + \frac{1}{2}) \, , H\Phi_n(x) = E_n\Phi_n(x)$$

$$E_n = \hbar\omega(n + \frac{1}{2}) \quad ; n = 0,1,2,3... \quad ; \quad \int_{-\infty}^{+\infty} dx\, \Phi_m(x)\Phi_n(x) = \delta_{m,n}$$

Find $F_n = [N, (a_+)^n]$ **(7 pts)**

by evaluating this commutator for small values of **n =1,2.** Then generalize your result to **n.** This is done as follows: Assume that the result is valid for **n** and use this to prove that this implied that your result is also valid for **(n+1)**. This is called the **induction approach** in mathematics.

Deduce that $H\left(a_+^n\Phi_0(x)\right) = E_n\left(a_+^n\Phi_0(x)\right)$ **and hence** $a_+^n\Phi_0(x) = A_n\Phi_n(x)$ **(6 pts)**

Question 4:

Consider the **one dimensional scattering** of a particle of mass **m** through a single **δ-potential barrier** located at **x=a.** The Schrodinger equation for this case reduces to

$$\frac{d^2}{dx^2}\Psi(x) = -\frac{2m}{\hbar^2}\left(E - V_0\,\delta(x-a)\right)\Psi(x) \quad ; x \in]-\infty, +\infty[$$

where $V_0 > 0$ is the strength of the δ-potential and a is a positive number.

a) Show that scattering states are allowed, **E > 0**, but **NO** bound states. **(5 pts)**

b) Treat the scattering problem, **E > 0, show that** the wavefunction is given by

(5 pts)

$$\Psi(x)=\begin{cases} Ae^{ikx}+Be^{-ikx} & ; \; x<a \\ Ce^{ikx}+De^{-ikx} & ; \; x>a \end{cases} \quad ; \quad E=\frac{(\hbar k)^2}{2m} \quad ; \quad k=\frac{\sqrt{2mE}}{\hbar}$$

Use the above Schrodinger equation to show that the derivative of the wave function is **discontinuous** at **x = a** (show details) **(5 pts)**

$$\frac{d}{dx}\Psi(x)\bigg|_{a^+}-\frac{d}{dx}\Psi(x)\bigg|_{a^-}=\frac{2mV_0}{\hbar^2}\Psi(a) \quad ; \quad F(a^\pm)=_{\varepsilon}\underline{Lim_0}F(a\pm\varepsilon)$$

c) Using the **boundary conditions : continuity** of the wave function and **discontinuity** of its derivative at **x = a** as given in previous question, show that it give **(10 pts)**

$$\begin{pmatrix} C \\ D \end{pmatrix}=\begin{pmatrix} 1-i\alpha & -i\alpha e^{-i2ka} \\ +i\alpha e^{+i2ka} & 1+i\alpha \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}=M\begin{pmatrix} A \\ B \end{pmatrix} \quad ; \quad M=\begin{pmatrix} 1-i\alpha & -i\alpha e^{-i2ka} \\ +i\alpha e^{+i2ka} & 1+i\alpha \end{pmatrix}; \quad \alpha=\frac{mV_0}{\hbar^2 k}$$

d) Assume an **incident wave from the far left** towards the potential, then on the far right we should have only transmitted waves (i.e **D = 0**). **Show that** the transmission coefficient for this case is given by **(10 pts)**

$$T=\left|\frac{C}{A}\right|^2=\frac{1}{|M_{22}|^2} \quad ; \quad M=\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}=\begin{pmatrix} 1-i\alpha & -i\alpha e^{-i2ka} \\ +i\alpha e^{+i2ka} & 1+i\alpha \end{pmatrix}$$

Express T(E) in terms of **E** and **$E_0=mV_0^2/2\hbar$** and **plot T(E).**