Solution.

PHYS.310

QUIZ # 2 **Fall 212**

Q. Infinite Potential Well $V(x)=\begin{cases} 0 & 0<x<a \\ \infty & x<0\,;x>a \end{cases}$

We have shown in class that the eigenfunctions and associated eigenenergies are given b

$$\Psi_n(x)=\sqrt{\frac{2}{a}}\sin(\frac{n\pi}{a}x)\,;n=1,2,3,...;\,E_n=\frac{\hbar^2\pi^2}{2ma^2}n^2;H\Psi_n(x)=E_n\Psi_n(x);\int_0^a dx\Psi_n^*(x)\Psi_m(x)=\delta_{m,n}$$

Consider the wavefunction $\Psi(x)=\frac{1}{\sqrt{2}}(\Psi_1(x)+\Psi_4(x))$

(a) Show that the above function is **normalized** i.e. $\int_0^a dx\Psi^*(x)\Psi(x)=1$

$$\int_0^a \psi^*(x)\psi(x)\,dx = \frac{1}{2}\int_0^a(\psi_1^*(x)+\psi_4^*(x))(\psi_1(x)+\psi_4(x)) = \frac{1}{2}\Big[\underbrace{\int\psi_1^*\psi_1\,dx}_{1}+\overbrace{\int\psi_1^*\psi_4\,dx}^{0}+\overbrace{\int\psi_4^*\psi_1\,dx}^{0}+\underbrace{\int\psi_4^*\psi_4\,dx}_{1}\Big]$$

$$= \frac{1}{2}(1+1) = 1$$

(b) Calculate the **energy E** associated with this state, i.e. $E=\int_0^a dx\Psi^*(x)H\Psi(x)$

$$E=\int_0^a\frac{1}{2}(\psi_1^*(x)+\psi_4^*(x))\,H\,(\psi_1(x)+\psi_4(x)) = \frac{1}{2}\int_0^a(\psi_1^*(x)+\psi_4^*(x))(E_1\psi_1(x)+E_4\psi_4(x))$$

$$E=\frac{1}{2}E_1\underbrace{\int_0^a\psi_1^*(x)\psi_1(x)\,dx}_{1}+\frac{1}{2}E_4\underbrace{\int_0^a\psi_4^*(x)\psi_4(x)\,dx}_{1} = \frac{1}{2}(E_1+E_4)=\frac{\hbar^2\pi^2}{4ma^2}(1^2+4^2)=\frac{17\hbar^2\pi^2}{4ma^2}$$

(c) **Shift the origin** of coordinate to the center of the well i.e. $x'=x+\frac{a}{2}$ then show that

$\Psi_{2n}(x')\;\;;\Psi_{2n-1}(x')\quad n=1,2,3...$ have different parities.

(use Sin(A-B) = SinA CosB - CosA SinB)

$$\psi_n(x')=\sqrt{\frac{2}{a}}\sin\frac{n\pi}{a}(x'-\frac{a}{2})=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x'}{a}-\frac{n\pi}{2}\right)$$

$$=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x'}{a}\right)\cos\left(\frac{n\pi}{2}\right)-\sqrt{\frac{2}{a}}\cos\left(\frac{n\pi x'}{a}\right)\sin\left(\frac{n\pi}{2}\right)$$

Thus

$$\psi_{2n}(x')=\sqrt{\frac{2}{a}}\sin\left(\frac{2n\pi x'}{a}\right)\cos(n\pi)-\cancel{\sqrt{\frac{2}{a}}\cos\left(\frac{2n\pi x'}{a}\right)\underbrace{\sin n\pi}_{0}}$$

$$=\sqrt{\frac{2}{a}}(-1)^n\sin\left(\frac{2n\pi x'}{a}\right) \quad \text{odd function of } x'$$

$$\psi_{2n-1}(x')=\cancel{\sqrt{\frac{2}{a}}\sin\left(\frac{(2n-1)\pi x'}{a}\right)\cos(n-\tfrac{1}{2})\pi}-\sqrt{\frac{2}{a}}\cos\left(\frac{(2n-1)\pi x'}{a}\right)\sin(n-\tfrac{1}{2})\pi$$

$$\psi_{2n-1}(x')=\sqrt{\frac{2}{a}}(-1)^n\cos\left(\frac{(2n-1)\pi x'}{a}\right) \quad \text{even function of } x$$