PHYS.410

Solution QUIZ # 1 Fall 221

Q1. Infinite Potential Well $V(x)=\begin{cases} 0 & 0<x<a \\ \infty & x<0\,;x>a \end{cases}$

We have shown in class that the eigenfunctions and associated eigenenergies are

given by

$$\Psi_n^0(x)=\sqrt{\frac{2}{a}}\sin(\frac{n\pi}{a}x)=\langle x|n\rangle; \quad n=1,2,3,\ldots; \quad E_n^0=\frac{\hbar^2\pi^2}{2ma^2}n^2;$$

$$H^0\Psi_n^0(x)=E_n^0\Psi_n^0(x); \quad \int_0^a dx\Psi_n^{0*}(x)\Psi_m^0(x)=\delta_{m,n}$$

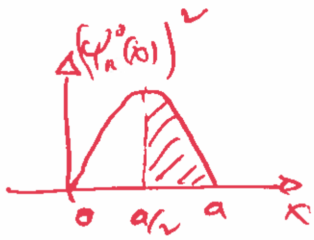
Consider a perturbation given by $H'(x)=\begin{cases} V_0 & a/2<x<a \\ 0 & elsewhere \end{cases}$; $H=H^0+H'$

Calculate the first order correction to the energy and show that $E_n^1=\frac{1}{2}V_0$

for the n-th bound state. (You may want to use $(\sin x)^2=\frac{1}{2}(1-\cos 2x)$)

$$E_n^1=\langle\Psi_n^0|H'|\Psi_n^0\rangle=V_0\frac{2}{a}\int_{a/2}^{a}dx\,\sin^2\left(\frac{n\pi x}{a}\right)=\frac{1}{2}V_0$$

since $\int_0^a(\Psi_n^0(x))^2dx=1=2\int_{a/2}^{a}(\Psi_n^0(x))^2dx \Rightarrow \int_{a/2}^{a}(\Psi_n^0(x))^2dx=\frac{1}{2}$



Q2. 1D Harmonic Oscillator

$$H^0=-\frac{d^2}{dx^2}+\frac{1}{2}kx^2 \quad ; \; k=m\omega^2\,; E_n^0=\hbar\omega(n+\frac{1}{2})\,; n=0,1,2,3,\ldots$$

Given that $a|n\rangle=\sqrt{n}\,|n-1\rangle,\; a^+|n\rangle=\sqrt{n+1}\,|n+1\rangle;\; x=\sqrt{\frac{\hbar}{2m\omega}}(a+a^+)$

$$H^0|n\rangle=E_n^0|n\rangle;\; \int_{-\infty}^{+\infty}dx\,\Phi_n^{0*}(x)\Phi_m^0(x)=\langle n|m\rangle=\delta_{m,n};\; \langle x|n\rangle=\Phi_n^0(x)$$

Consider a perturbation given by $H'(x)=\alpha x^2$, $H=H^0+H'$, α being a constant.

Calculate the first order correction to the energy and show that $E_n^1=\frac{\alpha}{k}E_n^0$

for the n-th bound state (this is an exactly solvable problem!).

$$E_n^1=\langle\Psi_n^0|H'|\Psi_n^0\rangle=\alpha\langle n|x^2|n\rangle=\alpha\frac{\hbar}{2m\omega}\langle n|(a+a^\dagger)^2|n\rangle$$

$$E_n^1=\alpha\frac{\hbar}{2m\omega}\left[\cancel{\langle n|a^2|n\rangle}+\cancel{\langle n|(a^\dagger)^2|n\rangle}+\langle n|aa^\dagger|n\rangle+\langle n|a^\dagger a|n\rangle\right]$$

$$E_n^1=\alpha\frac{\hbar}{2m\omega}[0+0+(n+1)+n]=\frac{\alpha}{m\omega^2}\hbar\omega(n+\tfrac{1}{2})=\frac{\alpha}{m\omega^2}E_n^0$$

$$E_n^1=\frac{\alpha}{k}E_n^0$$

using $k=m\omega^2$