Solution

PHYS.410

QUIZ # 2 **Fall 221**

Q1. Consider the following dimensionless **Hamiltonian in matrix form**

$$H = \begin{pmatrix} 2+\varepsilon & \varepsilon & 0 \\ \varepsilon & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \varepsilon \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = H^0 + H' \quad ; \quad |\varepsilon| \ll 1$$

a) **Check** that the **eigenvalues and normalized eigenvectors** of the $H^0 = H(\varepsilon = 0)$ are

$$|V_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; |V_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad for\ E_1^0 = E_2^0 = 2 \ \ and\ \ |V_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad for\ E_3^0 = 1$$

$$H^0|V_1\rangle = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2|V_1\rangle \ ; \quad H^0|V_2\rangle = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2|V_2\rangle$$

$$H^0|V_3\rangle = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = |V_3\rangle \quad \text{thus } E_1^0 = E_2^0 = 2, \ E_3^0 = 1.$$

b) Find the **first order correction** to the **non-degenerate** eigenenergy of $H^0(\varepsilon=0)$, $E_3^0 = 1$.

$$E_3^1 = \langle V_3|H'|V_3\rangle = \varepsilon(0\ 0\ 1)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \varepsilon(0\ 0\ 1)\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \boxed{0 = E_3^1}$$

not asked

$$E_3^2 = \sum_{n\neq 3} \frac{|\langle V_n|H'|V_3\rangle|^2}{E_3^0 - E_n^0} = \frac{|\langle V_1|H'|V_3\rangle|^2}{E_3^0 - E_1^0} + \frac{|\langle V_2|H'|V_3\rangle|^2}{E_3^0 - E_2^0} = 0 \text{ since } H'|V_3\rangle = 0$$

c) Find the **first order correction** to the **degenerate** eigenenergy of $H^0(\varepsilon=0)$, $E_1^0 = E_2^0 = 2$.

$$\begin{vmatrix} \langle V_1|H'|V_1\rangle - E^1 & \langle V_1|H'|V_2\rangle \\ \langle V_2|H'|V_1\rangle & \langle V_2|H'|V_2\rangle - E^1 \end{vmatrix} = 0 \text{ in degenerate space } \begin{pmatrix} |V_1\rangle \\ |V_2\rangle \end{pmatrix}$$

$$\langle V_1|H'|V_1\rangle = \varepsilon(1\ 0\ 0)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \varepsilon(1\ 0\ 0)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \varepsilon$$

$$\langle V_2|H'|V_2\rangle = \varepsilon(0\ 1\ 0)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \varepsilon(0\ 1\ 0)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$\langle V_1|H'|V_2\rangle = \varepsilon(1\ 0\ 0)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \varepsilon(1\ 0\ 0)\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \varepsilon$$

$$\begin{vmatrix} \varepsilon - E^1 & \varepsilon \\ \varepsilon & -E^1 \end{vmatrix} = E^1(E^1 - \varepsilon) - \varepsilon^2 \Rightarrow (E^1)^2 - \varepsilon E^1 - \varepsilon^2 = 0$$

$$\Rightarrow E^1 = \frac{+\varepsilon \pm \sqrt{5\varepsilon^2}}{2} = \boxed{\frac{1}{2}\varepsilon(1 \pm \sqrt{5})}$$