PHYS.410

Solution **QUIZ # 3** **Fall 221**

Q1. Consider a **spinless particle** of mass **m** and charge **q** constrained to move in the **xy plane** under the influence of a **two-dimensional harmonic oscillator** so that

$$H^0 = \frac{1}{2m}(p_x^2 + p_x^2) + V(x,y); \quad V(x,y) = \frac{1}{2}m\omega^2\left(x^2+y^2\right)$$

a) Construct the **ground state** and **first excited state** functions for this particle and write down **their energies**, E_0^0 and E_1^0. (use the convention $\left|n_x n_y\right\rangle$ to designate states) $H^0 = H_x^0 + H_y^0$; $H_x^0|n_x\rangle = E_{n_x}^0|n_x\rangle$; $H_y^0|n_y\rangle = E_{n_y}^0|n_y\rangle$

$E_0 = E_{n_x}^0 + E_{n_y}^0 = \hbar\omega\,(n_x + n_y + 1)$; $(n_x, n_y) = 0, 1, 2, 3, \ldots$

$|00\rangle = |\psi_0\rangle$; $E_0 = \hbar\omega$ is the ground state,

$|01\rangle = |\psi_1\rangle$; $|10\rangle = |\psi_2\rangle$; $E_1 = E_2 = 2\hbar\omega$ first excited state

b) Now imagine that this particle is **subject to an interaction**

$$H' = \alpha(xp_y - yp_x); \quad x = \sqrt{\frac{\hbar}{2m\omega}}(a_x + a_x^+); \quad p_x = i\sqrt{\frac{m\omega\hbar}{2}}(a_x^+ - a_x)$$

where α is a constant. Treating this interaction as a perturbation, find the **second order correction to the ground and the first order correction to the degenerate first excited state energies.**

$H' = i\sqrt{\frac{\hbar}{2\cancel{m\omega}}}\sqrt{\frac{\cancel{m\omega}\hbar}{2}}\left[(a_x + a_x^+)(a_y^+ - a_y) - (a_y + a_y^+)(a_x^+ - a_x)\right]$

$H = \frac{\alpha i\hbar}{2}\left[a_x a_y^+ - a_x a_y + a_x^+ a_y^+ - a_x^+ a_y - a_y a_x^+ + a_y a_x - a_y^+ a_x^+ + a_y^+ a_x\right]$

$E_0^1 = \boxed{\langle 00|H'|00\rangle = 0}$ since all operators do not preserve n_x, n_y:

$E_0^2 = \sum\limits_{n_x,n_y \neq 0,0} \frac{|\langle 00|H'|n_x n_y\rangle|^2}{\hbar\omega[1-(n_x+n_y+1)]} = -\sum\limits_{n_x,n_y \neq 0,0} \boxed{\frac{|\langle 00|H'|n_x n_y\rangle|^2}{\hbar\omega(n_x+n_y)} = 0}$

$\langle n_x n_y|H'|00\rangle = \frac{\alpha i\hbar}{2}\left[\langle n_x n_y|a_x^+ a_y^+|00\rangle - \langle n_x n_y|a_y^+ a_x^+|00\rangle\right] = 0$

$\langle\psi_1|H'|\psi_1\rangle = \langle 01|H'|01\rangle = 0 = \langle\psi_2|H'|\psi_2\rangle = \langle 10|H'|10\rangle$

$\langle\psi_1|H'|\psi_2\rangle = \langle 01|H'|10\rangle = \frac{i\hbar\alpha}{2}\langle 01|\,a_x a_y^+ + a_y^+ a_x\,|10\rangle$

$= \alpha i\frac{\hbar}{\cancel{2}}\sqrt{1}\sqrt{1}\langle 01|01\rangle \times \cancel{2} = i\hbar\alpha$

$\langle\psi_2|H'|\psi_1\rangle = \langle\psi_1|H'|\psi_2\rangle^* = -i\hbar\alpha$

$\begin{vmatrix} H_{11}' - E^1 & H_{12}' \\ H_{21}' & H_{22}' - E^1 \end{vmatrix} = \begin{vmatrix} -E^1 & i\hbar\alpha \\ -i\hbar\alpha & -E^1 \end{vmatrix} = 0 \Rightarrow (E^1)^2 = (\hbar\alpha)^2$

$\boxed{E_\pm^1 = \pm\hbar\alpha}$