PHYS.410

QUIZ # 4 Solution **Fall 221**

Q.

a) Explain using **few words/equations/graphs, as needed**, the main idea behind **WKB approximate solutions** to Schrodinger equation and **their limitations.**

WKB solve Schrodinger eq. for $\psi(x)$ in region where $V(x)$ is slowly varying, hence not discontinuous.

$$\psi^{WKB}(x) \begin{cases} = \dfrac{A}{\sqrt{p(x)}}\, e^{\frac{i}{\hbar}\int^{x} p(x')dx'} + \dfrac{B}{\sqrt{p(x)}}\, e^{-\frac{i}{\hbar}\int^{x} p(x')dx'} & \text{for } E > V(x) \\ = \dfrac{C}{\sqrt{k(x)}}\, e^{+\frac{1}{\hbar}\int^{x} k(x')dx'} + \dfrac{D}{\sqrt{k(x)}}\, e^{-\frac{1}{\hbar}\int^{x} k(x')dx'} & \text{for } E < V(x) \end{cases}$$

However, it is not valid close to turning points: $E = V(x)$ since $\psi(x)$ blows up.

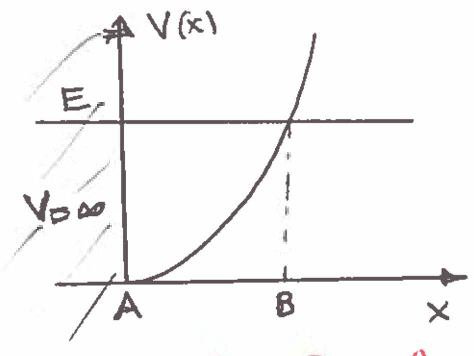
b) Explain using **few words/equations/graphs, as needed**, why do we need **connection formulae** and what is the adopted **strategy** in obtaining them? In particular write the **equation satisfied by the patching function** for one type of turning point that you **select and show on a graph.**

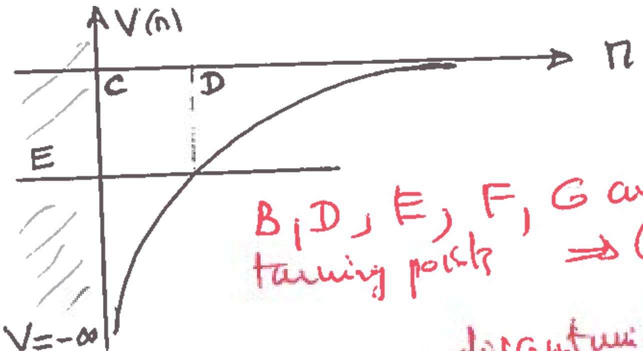
Since $\psi^{WKB}_{(x)}$ is not valid at turning points $V(x) = E$ then we need a patching function that is valid close to turning points where

$V(x) \simeq E + xV'(0) \Rightarrow \dfrac{d^2\psi}{dz^2} = z\psi(z) \Rightarrow \psi_p(z) = a\,Ai(z) + b\,Bi(z)$

matching $\psi^{WKB}_{(x)}$ and $\psi_p(x)$ near (but a bit away) from the turning point will lead to the connection formulae relating $\psi^{WKB}_{(x)}$ solution to left and right from this turning point.

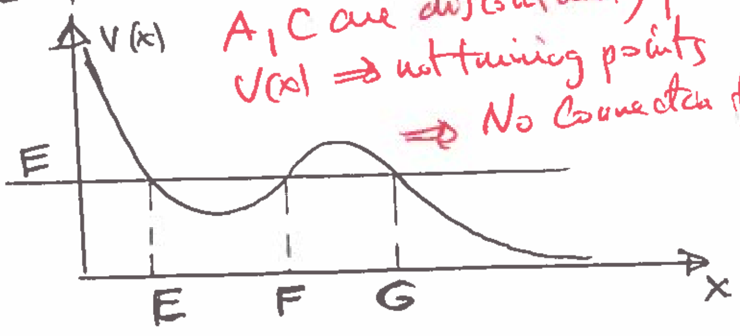
c) For the **potential curves** shown below, to which points **A, B, D, D, E, F and G** does the **connection formula apply**? If it **does not apply explain** why.





B, D, E, F, G are nice turning points ⇒ Connection formulae apply

A, C are discontinuity point of V(x) ⇒ not turning points ⇒ No Connection formulae



Connection formulae

⇓

need turning point to get Airy function

⇓

this will lead to Connection formulae