HW.1

Task 1.2

We can use Slater-Determinant to obtain the the wavefunction for many Fermions:

$$\psi_{Fermions}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(\vec{r}_1) & \psi_2(\vec{r}_1) & \ldots & \psi_N(\vec{r}_1) \\ \psi_1(\vec{r}_2) & \psi_2(\vec{r}_2) & \ldots & \psi_N(\vec{r}_2) \\ \vdots & \vdots & \vdots & \vdots \\ \psi_1(\vec{r}_N) & \psi_2(\vec{r}_N) & \ldots & \psi_N(\vec{r}_N) \end{vmatrix}$$

For Bosons we can use the same slater-determinant but with no negative signs. In other words, a *permenant*:

$$\psi_{Bosons}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_1(\vec{r}_1) & \psi_2(\vec{r}_1) & \ldots & \psi_N(\vec{r}_1) \\ \psi_1(\vec{r}_2) & \psi_2(\vec{r}_2) & \ldots & \psi_N(\vec{r}_2) \\ \vdots & \vdots & \vdots & \vdots \\ \psi_1(\vec{r}_N) & \psi_2(\vec{r}_N) & \ldots & \psi_N(\vec{r}_N) \end{vmatrix}_{Bosons}$$

Task 1.3

For two abelian anyons, their symmetric wavefunction will be the combination of a bosonic and fermionic ones:

$$a^\dagger_{\vec{r}_1} a^\dagger_{\vec{r}_2} |0\rangle = q(\vec{r}_1, \vec{r}_2) a^\dagger_{\vec{r}_2} a^\dagger_{\vec{r}_1} |0\rangle$$

$$\psi_{Anyons}(\vec{r}_1, \vec{r}_2) = \psi_{q\ Bosons}(\vec{r}_1, \vec{r}_2) + \psi_{q\ Fermions}(\vec{r}_1, \vec{r}_2)$$

$$\psi_{q\ \pm}(\vec{r}_1, \vec{r}_2) = \frac{1}{2}[\psi(\vec{r}_1, \vec{r}_2) \pm q(\vec{r}_2, \vec{r}_1)\psi(\vec{r}_2, \vec{r}_1)]$$

Where $q(\vec{r}_1, \vec{r}_2)$ will depends on the difference between $\vec{r}_1 - \vec{r}_2$. Moreover, I found this wavefunction for many particle abelian anyons, which I do not fully understand:

$$\psi_{Anyons}(z_1, \ldots, z_N) = \prod_{1 \le i < j \le N} (\vec{z}_i - z_j)^{n/m} P(z_1, \ldots, z_N) e^{-\sum_{j=1}^{N} |z_j|^2}$$

Which will give a factor of $e^{i\theta}, \theta = \pi\frac{n}{m}$ whenever we exchange 2 anyons. n, m are relatively co-prime constant integers, and $P(z_1, \ldots, z_N)$ is the symmetric polynomial.

Task 2.3

1. $c_1 c_2^\dagger c_2 c_1^\dagger \implies \langle 0_1, 0_2 | c_1 c_2^\dagger c_2 c_1^\dagger | 0_1, 0_2 \rangle = \langle 0_1, 0_2 | c_1 c_2^\dagger c_2 | 1_1, 0_2 \rangle = 0$
2. $c_1 c_2 c_2^\dagger c_1^\dagger \implies \langle 0_1, 0_2 | c_1 c_2 c_2^\dagger c_1^\dagger | 0_1, 0_2 \rangle = \langle 0_1, 0_2 | c_1 c_2 | 1_1, 1_2 \rangle = \langle 0_1, 0_2 | 0_1, 0_2 \rangle = 1$
3. $c_1 c_2 \implies \langle 0_1, 0_2 | c_1 c_2 | 0_1, 0_2 \rangle = 0$
4. $a_2 a_2^\dagger a_1^2 a_1^{\dagger^2} \implies \langle 0_1, 0_2 | a_2 a_2^\dagger a_1^2 a_1^{\dagger^2} | 0_1, 0_2 \rangle = \langle 0_1, 0_2 | a_2 a_2^\dagger a_1^2 | 2_1, 0_2 \rangle = \langle 0_1, 0_2 | a_2 a_2^\dagger | 0_1, 0_2 \rangle = 1$
5. $a_2 a_2^{\dagger^2} \implies \langle 0_1, 0_2 | a_2 a_2^{\dagger^2} | 0_1, 0_2 \rangle = \langle 0_1, 0_2 | a_2 | 0_1, 2_2 \rangle = \langle 0_1, 0_2 | 0_1, 1_2 \rangle = 0?$
6. $a_1 a_2 \implies \langle 0_1, 0_2 | a_1 a_2 | 0_1, 0_2 \rangle = 0$