Name: **Lab# 11** **Your ID #:**

Boundary Value Problems + PDF

Please answer the below questions:

Q1 (5pts): <u>Problem 1. (Boundary Value Problems).</u>

Use Finite Difference Method to solve the below question.

$$\ddot{y} + 2\dot{y} + y = x^2$$

$$y(0) = 0.2, \;\; y(1) = 0.8$$

(a) Verify the solution using n=4 as the ones used in our lecture.

(b) Plot the solution with large n as the ones used in our lecture.

(c) Solve the problem using Shooting Method and plot both solutions.

Q2 (10pts): <u>Problem 2. (Heat equation).</u>

Suppose you have a metal rod of length L = 1 meter that is initially at a uniform temperature of T = 20 degrees Celsius. The ends of the rod are kept at a constant temperature of T_left = 100 degrees Celsius and T_right = 0 degrees Celsius. The rod is assumed to have a thermal diffusivity of alpha = 1e-4 m^2/s and a specific heat capacity of c = 500 J/(kg*K). You want to simulate the temperature distribution in the rod over time using the one-dimensional heat equation:

d(T(x,t))/dt = alpha * d^2(T(x,t))/dx^2

where T(x,t) is the temperature at position x and time t, and d^2/dx^2 is the second derivative with respect to x.

1. Set up the numerical method: We will use the finite difference method to solve the heat equation numerically. We will discretize the rod into N = 1000 points and use a time step of dt = 1e-3 s. We will use the central difference method to approximate the second derivative:

d^2(T(x,t))/dx^2 = (T(x+dx,t) - 2T(x,t) + T(x-dx,t)) / dx^2

where dx = L/N is the distance between neighboring points.

2. Interpret the results and make sure the result make sense: Including

   A) How the temperature changes from the initial uniform temperature to the steady-state distribution.

   B) How the temperature distribution changes over time?

Hint:

The plot shows the temperature distribution in the metal rod over time. Initially, the rod is at a uniform temperature of 20 degrees Celsius. As time progresses, heat starts to flow from the hotter end (T_left = 100 degrees Celsius) towards the colder end (T_right = 0 degrees Celsius). This is evident from the temperature profile changing over time, with the left end of the rod having a higher temperature than the right end.

As the simulation proceeds, the temperature distribution starts to reach a steady-state. In the steady-state, the temperature profile becomes constant over time, indicating that the heat flow has reached equilibrium. In this case, the temperature at the left end remains constant at T_left = 100 degrees Celsius, while the temperature at the right end remains constant at T_right = 0 degrees Celsius. The temperature between these two ends gradually levels off and becomes linear, with a smooth transition from the high temperature to the low temperature.

The validity of the results can be verified by considering physical principles. Heat always flows from regions of higher temperature to regions of lower temperature until equilibrium is reached. In our case, the temperature distribution in the rod reaches a steady-state where the heat flow has balanced out, and the temperature profile is consistent with the expected behavior based on the boundary conditions and the physical properties of the rod.

By adjusting the parameters such as the length of the rod, the thermal diffusivity, or the boundary temperatures, we can observe how these factors impact the rate and pattern of heat transfer within the rod. This simulation allows us to analyze and understand the behavior of heat conduction in one-dimensional systems and provides insights into how different conditions affect the temperature distribution over time.