Name: **ID:**

PHYS 373 – Introduction to Computational Physics
Spring 2023 (Term 222)

Quiz 3

Date: Tuesday, Feb 28th, 2023

Q1 (6 Points): The following data is given:

x	*y*
2	3
4	6
6	10

a. **(4 Points)** Determine the parameters a and b that result in the least-squares fit for a power equation (i.e., $y=ax^b$) of the above data set.

b. **(2 Points)** Use the resulting power equation to predict x at $y=8$.

Solution:

(a) (6 Points) $y=ax^b$

We regress $\log_{10}(y)$ versus $\log_{10}(x)$ to give:

$$\log_{10} y = \log_{10} a + \mathrm{b}\log_{10} x$$

$$Y = \alpha + \mathrm{b}X$$

x	*y*	*X=log₁₀x*	*Y=log₁₀y*	X^2	*XY*
2	3	0.3010	0.4771	0.0906	0.1436
4	6	0.6021	0.7782	0.3625	0.4686
6	10	0.7782	1	0.6056	0.7782
		1.6813	**2.2553**	**1.0587**	**1.3904**

$$Construct\ \ \Phi : \Phi(a,b) = \sum_{i=0}^{n} (a + bx_i - f_i)^2$$

$$differentiate\ \ with\ \ respect\ to\ a and\ b$$

$$\frac{\partial\ \Phi(a,b)}{\partial\ a} = 0 = \sum_{k=1}^{N} 2(a + bx_k - y_k)$$

$$\frac{\partial\ \Phi(a,b)}{\partial\ b} = 0 = \sum_{k=1}^{N} 2(a + bx_k - y_k)x_k$$

Normal Equations

$$N\,a + \left(\sum_{k=1}^{N} x_k\right)b = \left(\sum_{k=1}^{N} y_k\right)$$

$$\left(\sum_{k=1}^{N} x_k\right)a + \left(\sum_{k=1}^{N} x_k^2\right)b = \left(\sum_{k=1}^{N} x_k\, y_k\right)$$

The results can be summarized as follows:

3α + 1.6813b = 2.2553

1.6813 α +1.0587b = 1.3904

$Y = 0.1434 + 1.0856X$ **, α = 0.1434 (Intercept), b = 1.0856 (Slope)**

$\log_{10} y = 0.1434 + 1.0856 \log_{10} x$

Therefore, $a = 10^{0.1434} = 1.3912$ and $b = 1.0856$, and the power model is

$y = 1.3912x^{1.0856}$

(b) (2 Points) Use the resulting power equation to predict *x* at *y=8*.

$8 = 1.3912x^{1.0856} \rightarrow x^{1.0856} = 5.7504 \rightarrow 1.0856 \log x = \log 5.7504 = 0.7597 \rightarrow$

$\log x = \dfrac{0.7597}{1.0856} = 0.6998 \rightarrow x = 10^{0.6998} = 5.0096$

Q2(4 Points): Use Gaussian elimination to solve the following system of equations. Check the solution. Show all details

$$\begin{bmatrix} -4 & 1 & 8 & 3 \\ 2 & 1 & 4 & 1 \\ -3 & 2 & 6 & 4 \\ 2 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -2 \\ 2 \end{bmatrix}$$

Solution:

1.0000
-8.5000
0.0000
4.5000
Residue R=|AX-B| = 0