Name: **ID:**

PHYS 373 – Introduction to Computational Physics
Spring 2023 (Term 222)

Quiz 1

Date: Tuesday, Feb 7th, 2023

1. **(2.5 Points) Obtain the first three non-zero terms of the Taylor series expansions of *sin(2x)* about the center of expansion *c = 0.5π***

$$f(x)=\sin(2x) \qquad f(0.5\pi)=0$$
$$f'(x)=2\cos(2x) \qquad f'(0.5\pi)=-2$$
$$f''(x)=-4\sin(2x) \qquad f''(0.5\pi)=0$$
$$f^{(3)}(x)=-8\cos(2x) \qquad f^{(3)}(0.5\pi)=8$$
$$f^{(4)}(x)=16\sin(2x) \qquad f^{(4)}(0.5\pi)=0$$
$$f^{(5)}(x)=32\cos(2x) \qquad f^{(5)}(0.5\pi)=-32$$

$$TS=-2(x-\frac{\pi}{2})+8\frac{(x-\frac{\pi}{2})^3}{3!}-\frac{32(x-\frac{\pi}{2})^5}{5!}$$

2. **(2.5 Points) Estimate the value of *sin(3)* using the expansion of part 1.**

$$TS: f(1.5)=-2(1.5-\frac{\pi}{2})+8\frac{(1.5-\frac{\pi}{2})^3}{3!}-\frac{32(1.5-\frac{\pi}{2})^5}{5!}$$
$$=0.14159-0.00047+0.00000=0.14112$$

3. **(2.5 Points) How many terms of the Maclaurin series are needed to approximate $f(x) = e^{-x}$ over the interval [0,1] with error less than 0.01**

To solve this problem we use c = 0

$$R_{n+1} = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-c)^{n+1} = \frac{(-1)^{n+1}e^{-\xi}}{(n+1)!}(x)^{n+1}$$

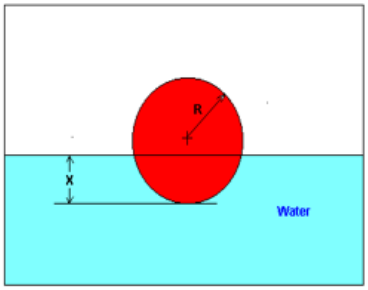
$$|R_{n+1}| \le \frac{(1)^{n+1}e^{0}}{(n+1)!} \le \frac{1}{(n+1)!} \le 0.01$$

$$by\ trial: \quad n=2, \frac{1}{(n+1)!} = 0.167, \quad n=3, \frac{1}{(n+1)!} = 0.0417$$

$$n=4, \frac{1}{(n+1)!} = 0.0083 < 0.01 \Rightarrow 5\ terms\ are\ needed$$

4. **(2.5 Points)** The equation that gives the depth 'x' to which the ball is submerged under water is given by

$$f(x) = x^3 - 0.165x^2 + 3.993\text{x}10^{-4}$$



Use the Bisection method of finding roots of equations to find the depth 'x' to which the ball is submerged under water. Conduct three iterations to estimate the root of the above equation.

Absolute Relative Approximate Error %

$$|\epsilon_a| = \left|\frac{x_m^{new} - x_m^{old}}{x_m^{new}}\right| \times 100$$

Iteration	a	b	c	F(c)	Error %
1	0.00000	0.11	0.055	$6.655\text{x}10^{-5}$	-------
2	0.055	0.11	0.0825	$-1.622\text{x}10^{-4}$	33.33
3	0.055	0.0825	0.06875	$-5.563\text{x}10^{-5}$	20.00