Taylor

(1) $$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k \quad , \quad R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$$ (2)

for simplification

$f(x)$ $\quad$ $f(x_0)$

$f^{(1)}(x)$ $\quad$ $f^{(1)}(x_0)$

$f^{(2)}(x)$ $\quad$ $f^{(2)}(x_0)$

$\vdots$ $\quad$ $\vdots$

choose ξ so that it maximizes

$$\text{error}_{n+1} \leq R_n$$

! $c = 0$ Maclaurin series

! $n \equiv$ # terms

- exact error $= f(x) - |P_n(x)|$, $R_n(x)$

Actual $\quad$ taylor series $\quad$ max error (remainder)

Solving Nonlinear eq

Bisection

write these to easily follow

1	1 v	2 ^	1.5 ^	2.375
2	1 v	1.5 ^	1.25 v	-1.796
3	1.25 v	1.5 ^	1.375 ^	0.162

$\vdots$

(3) $$n \geq \text{ceil}\left[\frac{\log(\overbrace{b-a}^{\text{interval guessed}}) - \log(\underbrace{\varepsilon}_{\text{error}})}{\log(2)}\right]$$

! can be used to estimate expressions like $\sqrt{2}$
just find the root of $f(x) = x^2 - \sqrt{2}$

(make it equal to a variable $X = \sqrt{2} \leftarrow X^2 - 2 = 0$)

Newton Raphson

(4) $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad i \geq 0$$

Secant

(5) $$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Systems of Linear Equations

$$\left[\begin{array}{c|c} \vdots & \\ 0 & 0 \end{array}\right] \quad \left[\begin{array}{c|c} & \\ 0 & 1 \end{array}\right] \quad \left[\begin{array}{c|c} & \\ 1 & 1 \end{array}\right]$$

∞ (Det $= 0$) $\quad$ ϕ (Det $= 0$) $\quad$ unique (Det $\neq 0$)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad x = \frac{-B}{A}$$

Gaussian elimination $\quad$ Gauss-Jordan

Least square fitting

(6) $$\phi = \sum \left(F(x_i) - y_i \right)^2$$

F — fit function

$\left.\begin{matrix} x_i \\ y_i \end{matrix}\right\}$ given points

$$\frac{\partial \phi}{\partial a} = 0$$
$$\frac{\partial \phi}{\partial b} = 0$$
$$\vdots$$

parameters of the fit function

then solve this sys. of eq

! linearization (for simplicity)

write original y in a linear form and rename...

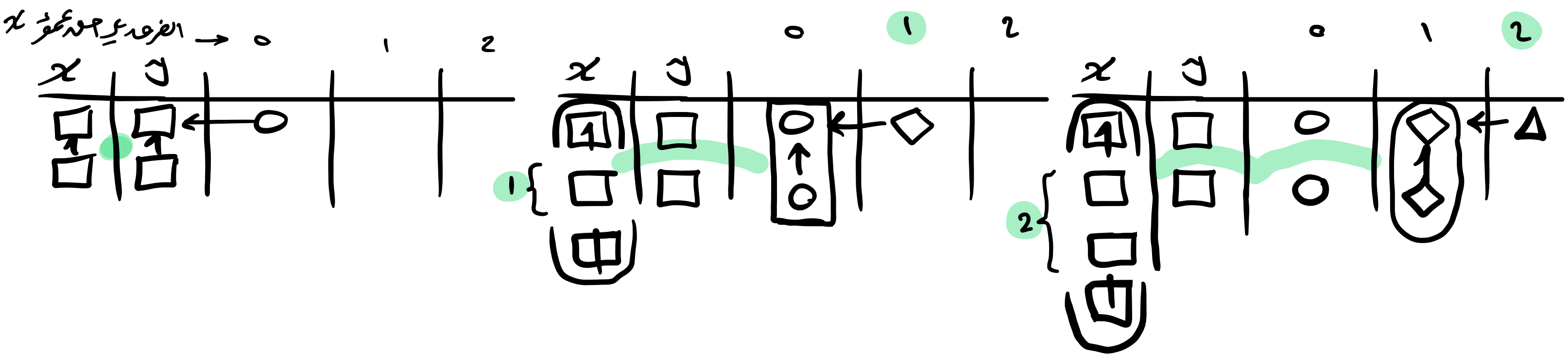
$$\phi = \sum \left(F(x_i) - Y_i \right)^2$$

$$y_i = a x_i^{\,b}$$
$$\ln y_i = \ln a + b \ln x$$
$$F(X_i) = Y_i = c + b X_i$$

easy → compared to $a x_i^{\,b}$

Interpolation

- **Newton Divided Difference Table**



(7) $$f_n(x) = y_0 + \text{①}(x - x_0) + \text{◇①}(x - x_0)(x - x_1) + \Delta \cdots$$

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Lagrange Interpolation

(8) $P_2(x) = \sum_{n=0}^{2} f(x_i)\,\ell_i(x)$, $\ell_i = \sum \frac{x - \overbrace{!x_i}^{\text{Not } x_i}}{x_i - !x_i}$

ℓ index $\rightarrow$ 0 1 2

x	1	3	4
y	7	5	6

الـ ℓ تنجاب بس بقيم الاكس !

$$\ell_0(x) = \frac{x-3}{1-3}\,\frac{x-4}{1-4}$$

x	1	3	4
y	7	5	6

$$\ell_1(x) = \frac{x-1}{3-1}\,\frac{x-4}{3-4}$$

x	1	3	4
y	7	5	6

$$\ell_2(x) = \frac{x-1}{4-1}\,\frac{x-3}{4-3}$$

x	1	3	4
y	7	5	6

$$P_2(x) = 7\ell_0 + 5\ell_1 + 6\ell_2$$

Integral

$h = \frac{b-a}{n}$

(10) **Trap** $\sum \frac{1}{2}\left(\text{part length}\right)\left(f(x_n) + f(x_{n+1})\right)$ $\qquad \frac{(b-a)^3}{12n^2} \max_{[a,b]} |f''(x)|$

$$S : h\left[\frac{1}{2}\left(f(x_0) + f(x_f)\right) + \underbrace{\sum_{1}^{f-1} f(x)}_{\text{inner points ONLY}}\right]$$

inner points ONLY (points b/w 1st & last)

(11) **1/3** $\frac{h}{3}\sum s\, f(x_i)$

$s = \underline{1}\ 4\ 2\ 4\ 2\ 4\ 2\ 4\ \underline{1}$

(12)

3/8 $\frac{3h}{8}\sum \varrho f(x_i)$

$\varrho \equiv \underline{\underline{1}}\ 3\ 3\ 2\ 3\ 3\ 2 \cdots \underline{\underline{1}}$

ODE

Euler $y_{i+1} = y_i + h f(x_i, y_i)$

Heun

predicto $y^0_{i+1} = y_i + h f(x_i, y_i)$

Corecto $y^{x+1}_{i+1} = y_i + \frac{h}{2}\left[f(x_i, y_i) + f(\underbrace{x_{i+1}}_{x_i+h}, y^x_{i+1})\right]$

Mid

$y_{i+1/2} = y_i + \frac{h}{2} f(x_i, y_i)$

$y_{i+1} = y_i + h\, f(\underbrace{x_{i+1/2}}_{= x_i + \frac{h}{2}}, y_{i+1/2})$

RK2

$K_1 = h f(t, x) \leftarrow$ u start w/ some condition

$K_2 = h f(t+h, x+K_1)$

$x(t+h) = x(t) + \frac{1}{2}(K_1 + K_2)$

RK4

$K_1 = f(x_i, y_i)$

$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1 h)$

$$K_2 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}K_1 h)$$

$$K_3 = f(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}K_2 h)$$

$$K_4 = f(x_i + h, y_i + K_3 h)$$

$$y_{i+1} = y_i + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

Shooting

. . .

System ODE

$$F = \begin{bmatrix} \dot{y}_1(x) \\ \dot{y}_2(x) \end{bmatrix}$$

$Y \equiv$ the y values (solutions) of the indicated step

example

$$F(Y, x) = \begin{bmatrix} y_2(x) \\ 1 - y_1(x) \end{bmatrix} \quad Y(x) = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}$$

solve for two steps of Euler $h=0.1$

Euler $Y_{i+1} = Y_i + h F(Y_i)$

$$Y_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Y_1 = Y_0 + h\,F(Y_0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} + 0.1 \begin{bmatrix} 1 \\ 1+1 \end{bmatrix} = \begin{bmatrix} -0.9 \\ 1.2 \end{bmatrix} \begin{matrix} y_1(0.1) \\ y_2(0.1) \end{matrix}$$

$$Y_2 = Y_1 + h\,F(Y_1) = \begin{bmatrix} -0.9 \\ 1.2 \end{bmatrix} + 0.1 \begin{bmatrix} 1.2 \\ 1+0.9 \end{bmatrix} = \begin{bmatrix} -0.78 \\ 1.39 \end{bmatrix}$$

$F(Y_i) \equiv$ sub the values of $\underset{\sim}{y}_1, \underset{\sim}{y}_2, \underset{\sim}{y}_3, \vdots$ from the Y_i