COE 530
Quantum Computers and Architecture

Lecture 17

Quantum Benchmarking

References:
Hands-on Introduction to Randomized Benchmarking

Benchmarking quantum gates and circuits

Quantum Gate Benchmarking

- Quantum gate benchmarking assesses the performance and fidelity of quantum gates, crucial for reliable quantum computation
- Why?
  - Ensures gate operations meet fault-tolerance thresholds
  - Identifies and quantifies errors affecting quantum computations
- Common Techniques:
  - Randomized Benchmarking (RB)
  - Deterministic Benchmarking (DB)

Randomized Benchmarking

- Applies sequences of randomly chosen Clifford gates to a quantum system and measures the decay in fidelity to estimate average error rates due to imperfect gates
- This can be accomplished by simulating a quantum operation that is equivalent to identity

- Starting from state ρ, we apply a sequence of Clifford operations

$$\rho \mapsto (C_m \circ C_{m_1} \circ \cdots \circ C_1)(\rho)$$

- Then apply an operation C_{m+1} to get back to the initial state, s.t,

$$C_{m+1} = \left(C_m \circ C_{m_1} \circ \cdots \circ C_1\right)^\dagger$$
$$= C_1^\dagger \circ \cdots \circ C_m^\dagger$$

Randomized Benchmarking

- In practice, each quantum operation will carry some associated error

$$S_{i_m} = \bigodot_{j=1}^{m+1} \Lambda_{i_j,j} \circ C_{j_i}$$

Where $\Lambda_{i_j,j}$ is the error gate associated to Clifford operation

- As a result, the final state $\tilde{\rho}$ is not equivalent to the starting state ρ due to the error associated with each quantum operation

Randomized Benchmarking

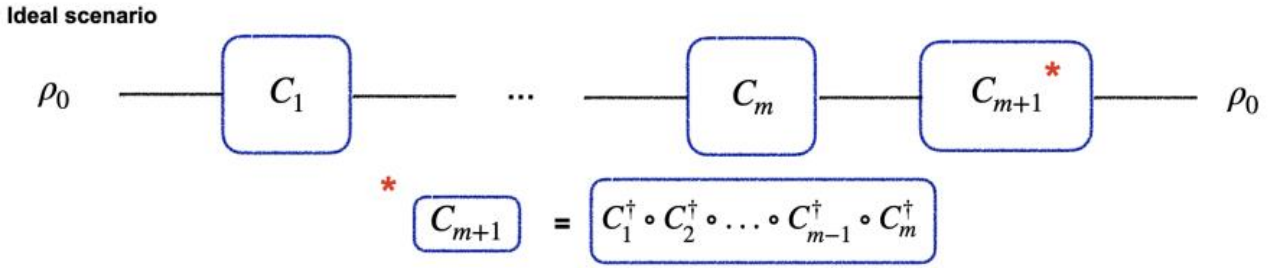


Figure 1: When the gates correspond to their ideal implementations, the RB circuit becomes equivalent to an identity operation.



Figure 2: The noisy implementation of the gates is modeled by replacing their ideal action by the composition with an error map $\Lambda_{i_j,j}$. Here i_j labels the specific gate to which the noise channel is applied to, and j denotes the time instant in which it is applied. Note that for the sequential circuit depicted in the figure, the time instance coincides with the position of the operation in the circuit. The input state is denoted by ρ_0. The state $\tilde{\rho}$ is the output state, which will differ from ρ_0 due to the noisy gates.

Standard RB Protocol

- Procedure:
  1. Prepare the qubit in a known initial state
  2. Apply a sequence of randomly selected Clifford gates
  3. Apply an inversion gate to return the system to the initial state
  4. Measure the final state fidelity
  5. Repeat for varying sequence lengths and average results
- **Outcome:** Exponential decay of fidelity with sequence length, from which average gate error rates are extracted

RB Results – example

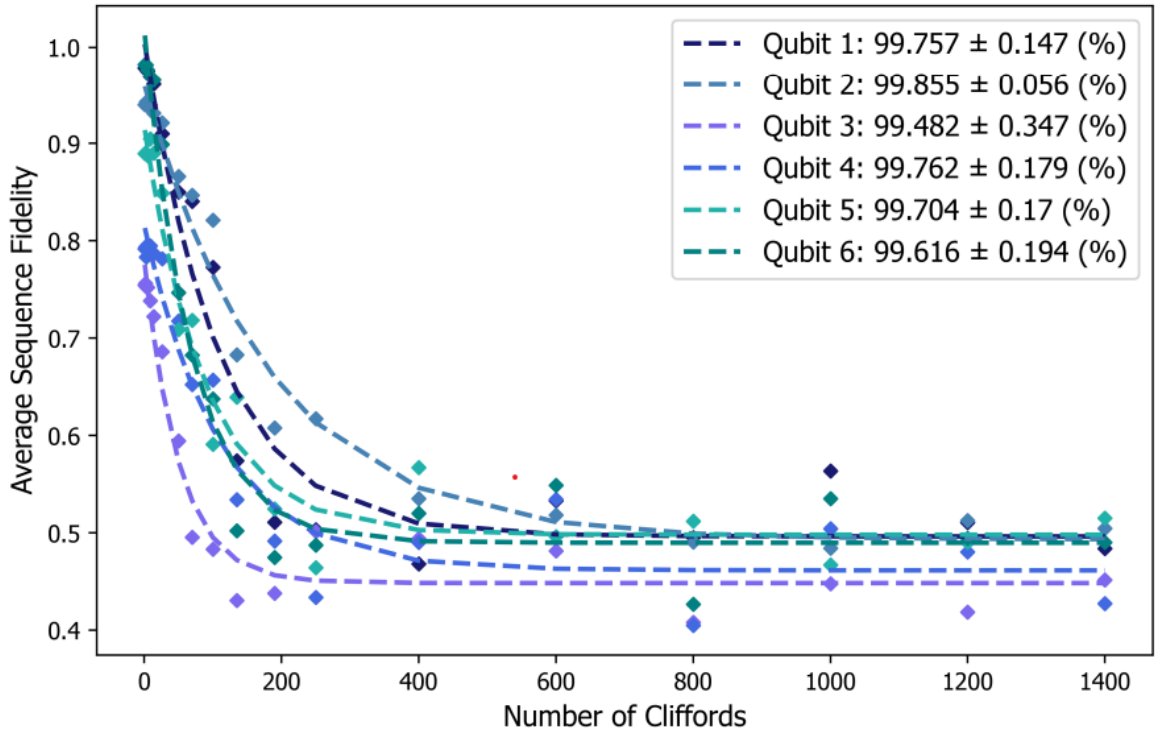


Figure 4: Average sequence fidelities as a function of the number of Clifford gates for all six individual qubits. Each decay profile corresponds to the result of the standard RB applied to benchmark the single-qubit elementary gates applied to one of the qubits in the system, as indicated in the legend. The legend also includes the corresponding single-qubit average fidelity for the operations implemented on the corresponding qubit together with the 2σ margin of error of each fidelity estimate.

Advantages of Randomized Benchmarking

- **Advantages**
  - Scalable to multi-qubit systems
  - Robust against state preparation and measurement (SPAM) errors
  - Provides a global error rate for gate sets
- **Challenges**:
  - Less sensitive to coherent (systematic) errors
  - May underestimate certain error types, affecting fault-tolerance assessments

Varients of RB

- **Interleaved RB**: Measures the fidelity of a specific gate by interleaving it within random sequences
- **Simultaneous RB**: Assesses crosstalk by applying RB to multiple qubits simultaneously
- **Correlated RB**: Detects correlated errors across qubits
- **Gate-set Shadow Protocol**: Integrates RB with shadow tomography for comprehensive characterization

Determinstic Benchmarking (DB)

- DB utilizes a carefully chosen small set of gate-pair sequences that specifically target the real noise affecting single-qubit gates

Procedure:

1. Prepare the qubit in a known initial state.
2. Apply a fixed sequence of gates designed to amplify specific errors.
3. Measure the final state fidelity.
4. Analyze results to extract coherent and incoherent error parameters.

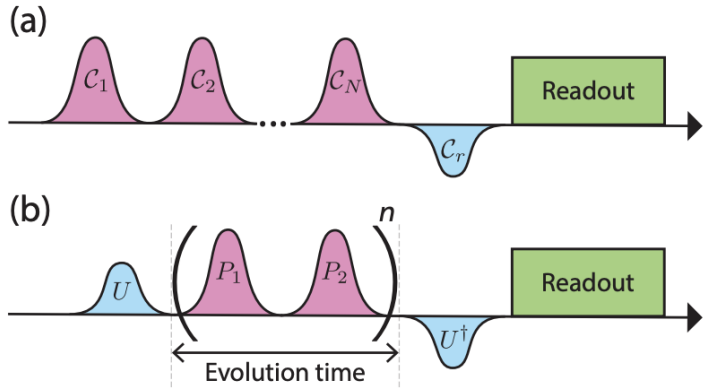


Figure 5: Schematic representations of the gate sequences for (a) randomized benchmarking (RB) and (b) deterministic benchmarking (DB), respectively. RB uses N random Clifford gates C_i, followed by a single recovery gate $C_r = C_N^\dagger \cdots C_1^\dagger$. DB prepares an initial state via $U|0\rangle$, applies n repetitions of a fixed, deterministic two-pulse sequence $P_1 P_2$, and unprepares the initial state via $U^\dagger$.

Randomized vs. determinstic

Feature	Randomized Benchmarking (RB)	Deterministic Benchmarking (DB)
Error Sensitivity	Primarily stochastic errors	Both coherent and incoherent errors
Sequence Type	Random Clifford sequences	Fixed, deterministic sequences
SPAM Error Robustness	High	Moderate
Measurement Requirements	High	Low
Error Characterization	Global error rates	Detailed error decomposition

System-Level Benchmarking

- Evaluate the performance of an entire quantum system, instead of focusing on individual components like single gates or qubits
- System-level benchmarking tests how well the quantum device performs **realistic or algorithmic tasks**, providing a more application-oriented measure of its practical utility

Aspect	**Gate-Level**	**System-Level**
Focus	Individual gate fidelity	Overall device performance
Methods	Randomized Benchmarking, T1/T2	Quantum Volume, full algorithm runs
Sensitivity to SPAM errors	Usually robust	Not robust (SPAM included)
Use Case	Hardware tuning	User-facing performance evaluation

Quantum Volume

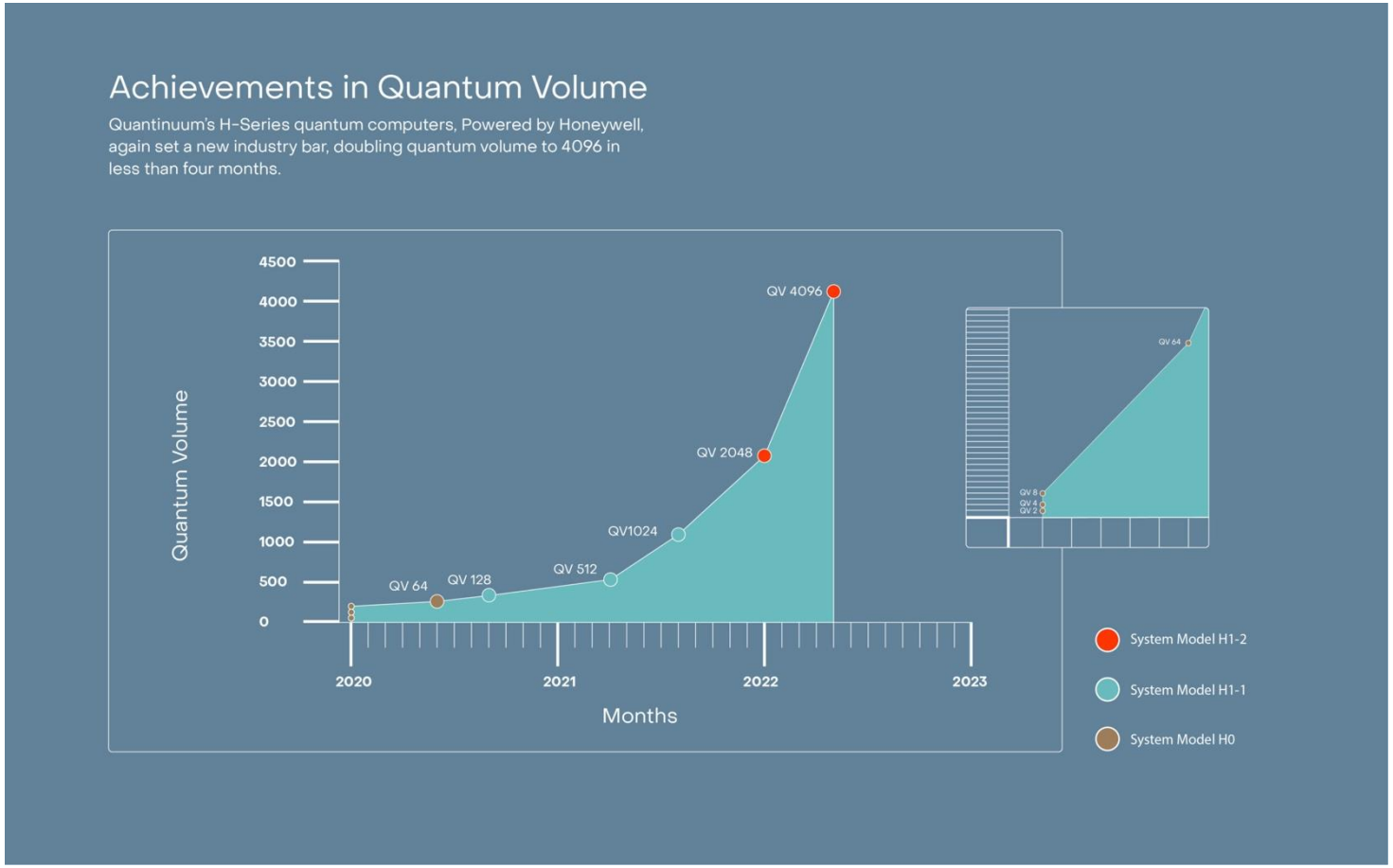
- Quantum Volume (QV) is a holistic benchmark introduced by IBM to quantify the computational power of a quantum processor
- QV takes into account **multiple factors** that influence real-world performance
- Unlike metrics that look only at gate fidelity or number of qubits, quantum volume reflects how effectively a device can run complex quantum circuits reliably
- QV V_Q is defined as

$$V_Q = 2^d$$

- Where d is the largest square quantum circuit (depth = width) the device can successfully execute with high fidelity. This means:
  - The circuit has **d qubits**, and
  - The circuit has **d layers (depth)** of random two-qubit gates, and
  - It succeeds (i.e., outputs a statistically meaningful result) above a certain **threshold fidelity** (typically > 2/3)

Quantum Volume – Importance

- QV reflects **the largest problem size** (in terms of depth and qubits) that the quantum computer can handle **reliably**. It's especially useful for comparing devices with the same number of qubits but different error rates or topologies



Quantum Benchmarking Initiative*

- DARPA's Quantum Benchmarking Initiative (QBI) aims to determine if it's possible to build an industrially useful quantum computer much faster than conventional predictions

- **In the simplest terms, QB seeks to answer the question: If a fully functioning quantum computer magically appeared, what would it make possible that a standard computer cannot accomplish?**

- https://www.darpa.mil/research/programs/quantum-benchmarking-initiative