جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

College of Computing and Mathematics

Information and Computer Science Department

ICS 560: Foundations of Quantum Computing

Semester 2025/2026 (251)

Midterm Exam

Date: Friday November 21 2025 **Time allowed: 120 minutes**

Name: Key Answer. **ID #** ____________ **Sec. No.** 1

Instructions:

- **Remember to write your name and KFUPM_ID.**
- **Clearly list any assumptions you make while answering the written questions.**
- **This exam contributes 20% towards the total marks for the course.**
- **Any clear evidence of cheating will result in zero grade for ALL the students involved.**
- **Attempt all questions.**

Marks distribution	
Part 1	20 points
Part 2	20 points
Total	/40
Total	**/20**

Cheat sheet

Name: .... **ID#:** ....

A B Fill the bubble fully. Use a dark pen or pencil. Erase cleanly for changes. D

1.	○ A	○ B	● C	○ D	11.	○ A	● B	○ C	○ D
2.	○ A	● B	○ C	○ D	12.	○ A	○ B	○ C	● D
3.	○ A	● B	○ C	○ D	13.	● A	○ B	○ C	○ D
4.	○ A	○ B	○ C	● D	14.	○ A	○ B	○ C	● D
5.	○ A	○ B	● C	○ D	15.	○ A	○ B	● C	○ D
6.	○ A	○ B	● C	○ D	16.	● A	○ B	○ C	○ D
7.	○ A	● B	○ C	○ D	17.	○ A	● B	○ C	○ D
8.	○ A	○ B	○ C	● D	18.	○ A	● B	○ C	○ D
9.	○ A	○ B	● C	○ D	19.	○ A	● B	○ C	○ D
10.	● A	○ B	○ C	○ D	20.	○ A	○ B	○ C	● D

Part 1 (MCQ): (20 points; 1 point each) Answer the following questions.

1. Which of the following is **not** a property of a vector space over complex numbers?

    A. Commutativity of multiplicative

    B. Associativity of scalar multiplication

    C. Non-commutativity of vector addition

    D. Existence of additive inverse

2. The transpose of a matrix $\boldsymbol{A}$ is denoted as:

    A. $\boldsymbol{A}^*$

    B. $\boldsymbol{A}^T$

    C. $\boldsymbol{A}^\dagger$

    D. $\overline{\boldsymbol{A}}$

3. The conjugate of a complex matrix $\boldsymbol{A}$ is obtained by:

    A. Transposing $\boldsymbol{A}$

    B. Taking complex conjugate of each entry

    C. Multiplying by i

    D. Subtracting imaginary part

4. The adjoint (dagger) of a complex matrix $\boldsymbol{A} = \boldsymbol{A}[\boldsymbol{j}, \boldsymbol{k}]$ is:

    A. $\boldsymbol{A}[\boldsymbol{k}, \boldsymbol{j}]$

    B. $\overline{\boldsymbol{A}[\boldsymbol{j}, \boldsymbol{k}]}$

    C. $\overline{\boldsymbol{A}[\boldsymbol{k}, \boldsymbol{k}]}$

    D. $\overline{\boldsymbol{A}[\boldsymbol{k}, \boldsymbol{j}]}$

5. A **linear map** between complex vector spaces V_1 and V_2 satisfies:

    A. $f(V_1 + V_2) = f(V_1)f(V_2)$

    B. $f(cV_1) = f(V_1)$ for some $c > 0$

    C. $f(V_1 + V_2) = f(V_1) + f(V_2)$ and $f(cV_1) = cf(V_1)$ for some $c > 0$

    D. None of the above

6. In a deterministic classical system, the state evolution is represented by:

   A. A stochastic matrix

   B. A unitary matrix

   C. A Boolean adjacency matrix

   D. A probability vector.

7. In a probabilistic system, transition likelihoods are represented by:

   A. Hermitian matrices

   B. Doubly stochastic matrices

   C. Identity matrices

   D. None of the above

8. A doubly stochastic matrix has all entries:

   A. Equal to 1

   B. Complex entries

   C. None of the above

   D. Real numbers between 0 and 1 with rows and columns summing to 1.

9. The modulus squared of a complex number gives:

   A. Phase

   B. Amplitude

   C. Probability

   D. Real part.

10. **Interference** in quantum systems refers to:

    A. Complex numbers cancelling each other

    B. Matrix addition

    C. Probability normalization

    D. Vector transformation

11. The modulus squared of entries of a unitary matrix forms:

A. Identity matrix

B. Doubly stochastic matrix

C. Hermitian matrix

D. Complex exponential matrix

12. A quantum state is represented as:

A. Real-valued vector

B. Hermitian operator

C. Scalar value

D. Complex column vector

13. The probability of finding a particle at position x_i is:

A. $|c_i|^2 / \sum |c_i|^2$

B. $|c_i|^2$

C. $|c_i| / \sum |c_i|$

D. $c_i \overline{c_i}$

14. Which of the following matrices is Hermitian?

A. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & i \\ i & 2 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$

15. The eigenvalues of a Hermitian matrix are always:

A. Complex
B. Purely imaginary
C. Real
D. Zero

16. For a valid qubit, the condition is:

A. $\alpha\bar{\alpha} + \beta\bar{\beta} = 1$

B. $|\alpha| \quad + |\beta| \quad = 1$

C. $\alpha^2 + \beta^2 = 1$

D. $\alpha\bar{\alpha} - \beta\bar{\beta} = 1$

17. The Hadamard gate (H) acts as:

A. Bit swapper

B. Superposition generator

C. Phase shifter

D. NOT gate

18. The Phase gate (***S***) corresponds to a rotation of:

A. 90° about the x-axis

B. 90° about the z-axis

C. 90° about the y-axis

D. 45° about the z-axis

19. The **T gate** is equivalent to:

A. S^2

B. $Z^{1/4}$

C. Z^4

D. Z^2

20. In the Bloch sphere parameterization, the angle θ determines:

A. The longitude (phase) of the state

B. The rotation about the z-axis

C. The overall global phase

D. The probability amplitude ratio between $|0\rangle$ and $|1\rangle$

Part II: (20 points) **show all necessary steps to answer the following questions**

1. Consider the qubit $|\omega\rangle = -\frac{i}{\sqrt{2}}|00\rangle + \frac{1}{2}|01\rangle - \frac{\sqrt{3}-i}{4}|11\rangle$. Measure the right qubit in the state $|1\rangle$ and what quantum state does the system collapse to after the measurement.

$$P_{+}(1) = \left|\frac{1}{2}\right|^2 + \left|\frac{\sqrt{3}-i}{4}\right|^2 = \frac{1}{4} + \frac{4}{16} = \frac{1}{2}$$

Collapse to

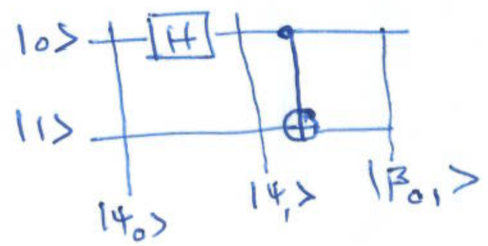
$$\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}|01\rangle - \frac{\frac{\sqrt{3}-i}{4}}{\frac{1}{\sqrt{2}}}|11\rangle$$

$$= \frac{\sqrt{2}}{2}|01\rangle - \frac{\sqrt{2}(\sqrt{3}-i)}{4}|11\rangle$$

$$= \frac{1}{\sqrt{2}}|01\rangle - \frac{\sqrt{3}-i}{2\sqrt{2}}$$

2. Describe a circuit that will generate the following Bell states.

$$|B_{01}\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$



① $|\psi_0\rangle = |01\rangle$

② $|\psi_1\rangle = H \otimes I\, |\psi_0\rangle$

$= |+\rangle|1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle)$

③ $CNOT\, |\psi_1\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

3. Determine the polar angle $\boldsymbol{\theta}$ and the phase rotation $\boldsymbol{\varphi}$ of the following qubit.

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{\sqrt{3}i}{2}|1\rangle$$

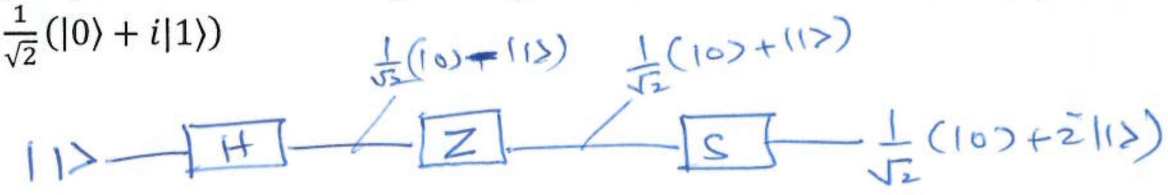
$$= \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2} e^{i\pi/2}|1\rangle$$

$$\cos\frac{\theta}{2} = \frac{1}{2} \quad \sin\theta/2 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{\theta}{2} = \frac{\pi}{3} \Rightarrow \theta = \frac{2\pi}{3}$$

$$\varphi = \frac{\pi}{2}$$

4. Design a quantum circuit using basic gates that transforms the state $|1\rangle$ to the state $\frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$



$$S \otimes Z \otimes H\,|1\rangle$$

$$= S \otimes Z\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$$

$$= S\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$$

5. Find the matrix representation of the Controlled$-$ U gate, if $U = H$

$C\text{-}H|00\rangle = |00\rangle$

$C\text{-}H|01\rangle = |01\rangle$

$C\text{-}H|10\rangle = |1\rangle H|0\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |11\rangle) \rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

$C\text{-}H|11\rangle = |1\rangle H|1\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |11\rangle)$

$$C\text{-}H = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$